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Final-state interactions in the CP asymmetry of D meson two-body decays

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The Kobayashi-Maskawa (KM) mechanism predicts that a single parameter must be responsible for CP-violating phenomena in different quark flavour sectors of the Standard Model (SM). Despite this minimal picture, challenged by non-SM physics, the KM mechanism has been so far verified in the bottom and strange sectors, but lacks tests in the complementary charm sector. For the sake of this, urgent theoretical progress is needed in order to provide an estimate in the SM of the recent measurement by LHCb of direct CP violation in charm-meson two-body decays, which will be significantly improved by new data expected from LHCb and Belle II. Rescattering effects are particularly relevant for a meaningful theoretical account of the amplitudes involved in such observable, as signaled by the presence of large strong phases. We discuss the computation of the latter effects based on dispersion relations.

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1. Introduction

Charm represents physics of the up-type and is then complementary to the down-type sector, which is comparatively better known. In particular, charm offers the opportunity to understand QCD at intermediate regimes, namely, in between the light flavours and the bottom, in both of which cases there are consolidated theoretical tools. Moreover, with charm one can also access flavour-changing neutral currents (FCNCs) of the up-type, where a more effective Glashow-Iliopoulos-Maiani (GIM) mechanism applies, which constitutes an opportunity for clear identification of BSM contributions. In the case of CP violation (CPV), the KM mechanism in the SM introduces a single CPV phase that must be responsible for CPV across different flavour sectors; this mechanism has been tested in the bottom and strange sectors (e.g., [1]), but tests in the charm sector are still missing.

In this respect, CPV in the charm sector has been established very recently by LHCb [2], which measured the difference $\Delta A_{CP}^{exp} \simeq -2\times 10^{-3}$ of CP asymmetries in final states involving two charged kaons $A_{CP}^{D^0\to K^+K^-}$, minus the one involving two charged pions $A_{CP}^{D^0\to\pi^+\pi^-}$, where:[1]

$$A_{CP}^{i\to f} \equiv \frac{|\langle f|T|i\rangle|^2 - |\langle \overline{f}|T|\overline{i}\rangle|^2}{|\langle f|T|i\rangle|^2 + |\langle \overline{f}|T|\overline{i}\rangle|^2} = \Sigma_j \left[p_j \, \sin(\Delta\delta_j) \, \sin(\Delta\phi_j) \right] , \tag{1}$$

T being the transition matrix. There is an active experimental research program, as attested by very recent results [3], with available bounds on CPV for many other channels. Much progress is expected in the years to come, thanks to LHCb and Belle II, which will largely improve the sensitivity to sources of CPV; BESIII also has an active research program in charm physics.

On the other hand, theory has to match the observed experimental progress. In order to have a non-vanishing CP asymmetry, one needs both differences of weak ($\Delta\phi$) and strong ($\Delta\delta$) phases, as indicated schematically in the right-hand side of Eq. (1); therein, the sum consists of all possible interference terms j among pairs of amplitudes that have simultaneously different weak ($\Delta\phi_j \neq 0 \pmod{\pi}$) and strong phases ($\Delta\delta_j \neq 0 \pmod{\pi}$), and p_j scales like the ratio of a CP-odd over a CP-even amplitudes. As previously stated, in the SM the weak phases come from the Cabibbo-Kobayashi-Maskawa matrix V, and we introduce the following set of local effective interactions that give the dominant description of $|\Delta C| = 1$ transitions:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \left[\sum_{i=1}^{2} C_i(\mu) \left(\lambda_d Q_i^d + \lambda_s Q_i^s \right) - \lambda_b \sum_{j=3}^{6} C_j(\mu) Q_j \right] + h.c. \,, \quad \lambda_q = V_{cq}^* V_{uq} \,, \tag{2}$$

where $q = d, s, b$; $Q_i^{d,s}$ for $i = 1, 2$ (Q_j for $j = 3, \ldots, 6$) correspond to current-current (respectively, gluonic penguin) operators. The Wilson coefficients (WCs) C_i encode the short-distance dynamics from above the scale $\mu \sim 2$ GeV; we have that $|C_{1,2}| \sim O(1)$, while $|C_{3,\ldots,6}| \lesssim O(0.06)$ [4], such that, in presence of rescattering discussed below, CPV in charm decays mainly results from current-current operators. The strong phases originate from non-perturbative QCD effects; a dynamical mechanism for their generation is the rescattering of on-shell particles, in particular pion and kaon pairs. The main interest of this work is the determination of these non-perturbative effects, and their impact on the prediction of the CP asymmetry.

[1]Time-integration is left implicit; the contribution of indirect CPV is negligible [2].

2. Rescattering effects

In describing $D \to (\pi\pi, KK)$, to first order in weak interactions a discontinuity equation can be written for the transition amplitudes analytically extended to the complex plane (of the invariant mass squared s of the pseudo-scalar pair). The discontinuity is set by the rescattering of the light particles that are stable under strong interactions, with the right-hand cut starting at the threshold for production of pion pairs; for an introduction, see [5]. The strong dynamics is non-perturbative in nature and has some useful properties: it conserves flavour, CP, isospin and G-parity. The rescattering among light, stable final states gives origin to the strong phases necessary for a non-vanishing CP asymmetry. In the elastic limit, such a phase in the weak decay can be extracted directly from the phase shift in the rescattering of pions (see Watson's theorem below). More can be learnt about the rescattering by exploiting its analyticity in the relevant kinematical variables, relating the dispersive/real and absorptive/imaginary parts of the rescattering amplitudes. The comments about analyticity made at the beginning of this paragraph translate into the (unsubtracted) dispersion relation (DR):

$$\mathrm{Re}[\Omega(s)] = \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}[\Omega(s')]}{s'-s} ds' = \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} \frac{T_0^{0*}(s')\Sigma(s')\Omega(s')}{s'-s} ds' \,, \tag{3}$$

where $T_0^0(s)$ appears in the strong S sub-matrix:

$$S_{ij} = \delta_{ij} + iT_{ij} = \delta_{ij} + 2i\Sigma_i^{1/2}(s)(T_0^0)_{ij}\Sigma_j^{1/2}(s) \,, \tag{4}$$

and $\Sigma(s)$ contains kinematical factors (Θ is the Heaviside distribution):

$$\Sigma(s) = \mathrm{diag}(\sigma_\pi(s)\Theta(s-4M_\pi^2), \sigma_K(s)\Theta(s-4M_K^2)) \,, \quad \sigma_i(s) = (1-4M_i^2/s)^{1/2} \,. \tag{5}$$

In the elastic limit, one can solve this integral equation explicitly [6]. The partial-wave transition amplitude is then given by:

$$A_J^I(s) = \overline{A}_J^I(s) \exp\left\{i\,\delta_J^I(s)\right\} \underbrace{\exp\left\{\frac{s-s_0}{\pi} \,⨍_{4M_\pi^2}^{\infty} \frac{dz}{z-s_0}\frac{\delta_J^I(z)}{z-s}\right\}}_{\text{Omnès factor } |\Omega^{(I)}|} \,, \tag{6}$$

where $s = m_D^2$ and $J = 0$ in our present situation; the slashed integral represents its principal value; $s_0 \le 4M_\pi^2$ is the subtraction point (we consider a single subtraction). This result displays a complex phase following the result of Watson's theorem, i.e., the phase of the pion form factor in the weak transition is equal to the phase shift δ_J^I of $\pi\pi \to \pi\pi$ (mod 180^o) in the elastic region above $\pi\pi$ threshold. It is also proportional to an overall factor, called Omnès factor $|\Omega^{(I)}|$, that depends on the phase shift; this is a positive factor that can deviate substantially from one. The phase shift and the Omnès factor encode the effects of rescattering, and are necessary for a good qualitative and quantitative description of the amplitudes. There is also an overall polynomial ambiguity $\overline{A}_J^I(s)$ that is left unknown; for its determination, we need some physical insight: for instance, in the case of kaon decays it can be determined from chiral perturbation theory (ChPT) [7, 8].

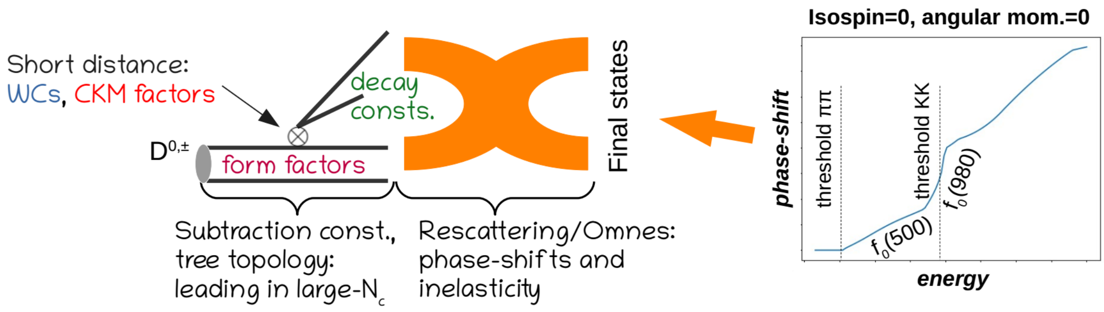

Figure 1: Ingredients in a two-channel analysis of $D \to (\pi\pi, KK)$ based on large-N_c counting, corrected by rescattering taken into account via the use of subtracted DRs. The graphic on the right illustrates the input for isospin-0, angular momentum-0, and distinguishes the resonances whose imprints manifest in the phase shift. The subtraction constant requires the use of decay constants, form factors and contains information about the weak dynamics, namely, sources of CPV coming from the CKM matrix, and the Wilson coefficients.

In the present case, the required analysis is non-elastic [9] because in particular we are above the threshold for production of kaon pairs. We have then a set of integral equations related by unitarity. These equations have to be solved numerically [10], as no explicit form of the solution is known in general. We are going to include in our analysis only pion and kaon pairs, for which we dispose of abundant data, and neglect further channels in this work. Having pions and kaons, we need as inputs two phase shifts and one inelasticity, which accounts for the transition between pion and kaon pairs; we use available parameterizations for them [11–14].

As in the elastic case, we also need some physical input for the polynomial ambiguities. We employ large-N_c counting for their determination:

$$\overline{A}_J^I(s) \propto f_{\pi^+} \times f_0^{D\pi}(m_\pi^2) \times (s - m_\pi^2)\,, \quad s = m_D^2\,, \tag{7}$$

with an analogous expression applying for kaons, which is the product of form factors (FFs) $f_0^{D\pi}$ and decay constants (DCs) f_{π^+}; both FFs and DCs contain sub-leading effects in large-N_c. Note that rescattering is also sub-leading in large-N_c: this whole procedure consists in improving the leading large-N_c estimation of Eq. (7) in a data-driven way with contributions that are of higher order in $1/N_c$. The full picture is the one of Eqs. (6) and (7); see also Fig. 1. The final ingredient is the short-distance part, in which the WCs collect perturbative QCD effects [4].

Based on isospin symmetry, we exploit the charged decay modes $D^+ \to \pi^+\pi^0$ and $K^+K_S^0$ to extract $|\Omega^{(I)}|$ for the isospin-1 (of kaon pairs) and -2 (of pion pairs) modes, that we assume to be elastic. The isospin-1 and -2 phases remain undetermined following this procedure; since they are also not known from data on rescattering at the scale of the charm meson, these phases are adjusted to achieve branching ratios of the D^0 decays discussed below close to their experimental values.

For both the branching ratios and CP asymmetries, the necessary physical inputs are the WCs, DCs, FFs, and rescattering factors, as illustrated in Fig. 1. With these, one has an expression for the (CP-even) isospin amplitudes, and then one determines the branching ratios $\mathcal{B}r^{theo}$. They come in agreement with the experimental results, as these theoretical branching ratios are about $(1.1 - 1.2) \times \mathcal{B}r^{exp}$. In other words, the amplitudes come right when compared to results of a global fit extraction of the same amplitudes, e.g., [15]. Let us also note that rescattering is another source of breaking of $SU(3)$-symmetry relating pions and kaons at the charm meson scale, that turns out to be of the same size as the breaking from the DCs and FFs, i.e., about 20%.

Similarly, for the CP asymmetries one determines the relevant isospin amplitudes, including the CP-odd ones, and then the CP asymmetries, resulting in the following schematic form for the difference of CP asymmetries:

$$\Delta A_{CP}^{theo} = O(0.1) \times \frac{\text{Jarlskog}}{|\lambda_d|^2} \sim O(\text{few}) \times 10^{-4} \,. \tag{8}$$

Note that there is an overall suppression given by the combination of the Jarlskog and the modulus of λ_d (both rephasing-invariant quantities): Jarlskog$/|\lambda_d|^2 = 6.2 \times 10^{-3}$. It turns out that the overall prefactor, containing differences of strong phases and Omnès factors (other than DCs, FFs, and current-current WCs), is substantially below $O(1)$, being instead $O(0.1)$, and one gets then a prediction which is too small compared to the actual measurement.

Finally, let us note that these individual CP asymmetries result from a sum of interference terms among different amplitudes. We don't observe a substantial cancellation among these different contributions. Moreover, the contributions from pions and kaons add up together, and come with the sign seen in the experimental measurement. It seems thus difficult to reproduce the measured CP asymmetry in this approach given the ingredients pointed out above.

3. Conclusions and future steps

We have discussed a data driven approach, which is based on the use of dispersion relations to take into account rescattering in the isospin-0 mode, with subtraction constant given by large-N_c; we have also employed the charged decay modes for extracting isospin-1 and -2 quantities. Only pion and kaon pairs are included in the analysis, and further inelasticities are omitted. We have shown preliminary results according to which the branching ratios come in agreement with measurements, while the CP asymmetries come too small compared to the experimental value [2, 3]. The error budget based on different parameterizations of the isospin-0 phase shifts and inelasticity will be presented in the coming publication. In the future, we plan to include further inelasticities, moving beyond a two-channel analysis.

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