PROCEEDINGS OF SCIENCE

Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ with ISR events at *BABAR* and calculation of its contribution to the $(g-2)_\mu$

Fabio Anulli[a,1,*]

[a] *https://orcid.org/0000-0003-2734-130X*

E-mail: fabio.anulli@roma1.infn.it

We present an analysis of the process $e^+e^- \to \pi^+\pi^-\pi^0$ performed at *BABAR* using the initial-state radiation (ISR) technique. The analysis is based on the full *BABAR* data set of 469 fb^{-1}, recorded at and near the $\Upsilon(4S)$ resonance. From the fit to the measured 3π mass spectrum we determine the products $\Gamma(V \to e^+e^-)\mathcal{B}(V \to 3\pi)$, where V is the $\omega(780)$ or the $\phi(1020)$ resonance, and $\mathcal{B}(\rho \to 3\pi)$. The latter isospin-breaking decay is observed with 6σ significance. The $e^+e^- \to \pi^+\pi^-\pi^0$ cross section is measured from 0.62 to 3.5 GeV with unprecedented precision. The measured cross section is used to calculate the leading-order hadronic contribution to the muon magnetic anomaly from this exclusive final state with improved accuracy. We also present preliminary results on a study, performed on the same data set, of the $K^+K^-\pi^0\pi^0\pi^0$, $K^0_S K^\pm\pi^\mp\pi^0\pi^0$ and $K^0_S K^\pm\pi^\mp\pi^+\pi^-$ final states produced in e^+e^- collisions via ISR.

41st International Conference on High Energy physics - ICHEP2022
6-13 July, 2022
Bologna, Italy

[1]On behalf of the *BABAR* Collaboration.
*Speaker

https://pos.sissa.it/

PoS(ICHEP2022)786

1. Introduction

The comparison between the experimental and the theory-predicted values of the anomalous magnetic moment of the muon $a_\mu = (g_\mu - 2)/2$ is a powerful test of the Standard Model validity. To precisely calculate the value of a_μ, quantum fluctuation corrections at all orders, from all known interactions, must be considered. The whole uncertainty is essentially produced by the small QCD corrections, mainly vacuum polarization (VP) with hadrons, as low-energy hadronic processes cannot be calculated in perturbation theory. The leading-order VP contribution to a_μ is however reliably determined via dispersion relations connecting it to the cross section of $e^+e^- \to$ hadrons:

$$a_\mu^{LO,VP} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int \frac{R(s) \times \hat{K}(s)}{s^2} ds, \tag{1}$$

where $R(s)$ is the bare hadronic cross section at the center-of-mass (CM) energy squared s, normalized to the pointlike muon pair cross section, and $\hat{K}(s)$ is a known function that is of order unity. Recent theoretical determinations of a_μ give similar results and have been summarized in Ref. [1], where the common result $a_\mu^{LO,VP} = (693.1 \pm 4.0) \times 10^{-10}$ have been proposed.

The present experimental world average value of a_μ [2] exceeds the theoretical one by more than four standard deviations, possibly hinting at new physics. The experimental determination is expected to improve significantly within a few years thanks to the new measurements at the running Muon g-2 experiment at FermiLab and in the future at the J-PARC facility. Therefore, more precise measurements of the hadronic cross sections would be desirable to have a similar improvement also on the theoretical side.

We report in the following the most recent results obtained by the BABAR experiment [3] on the measurement of exclusive e^+e^- cross sections at low energies. These measurements make use of the so-called initial-state radiation (ISR) method [4], with e^+e^- collisions at a nominal CM energy near the peak of the $\Upsilon(4S)$ resonance, that is $\sqrt{s} \sim 10.6$ GeV. In this experimental approach, the $e^+e^- \to f$ cross section is deduced from a measurement of the radiative process $e^+e^- \to f + \gamma_{ISR}$, where the photon is emitted by the e^+or the $e^-$with a CM energy $E^*_\gamma = x\sqrt{s}/2$, and f can be any final state produced at the reduced CM energy squared $E^2_{CM} = s(1-x)$:

$$\frac{d\sigma_{e^+e^-\to f\,\gamma_{ISR}}}{dm}(s,m) = \frac{2m}{s} W(s,m)\sigma_{e^+e^-\to f}(m), \tag{2}$$

where $m = E_{CM}$ is experimentally obtained from the reconstructed mass of the hadronic system. The so-called radiator function $W(s,m)$, is the probability density to radiate a photon with a fraction x of the beam energy and is known with a precision better than 1% [5, 6].

2. Measurement of the $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

The process $e^+e^- \to \pi^+\pi^-\pi^0$ provides the second largest contribution to the hadronic cross section in the energy region below 1.1 GeV, after $e^+e^- \to \pi^+\pi^-$. Currently the accuracy of its contribution to a_μ is about 3%.

The event selection makes use of the good solid angle coverage of the detector and the well known beam energies. The event is fully reconstructed requiring exactly two good quality opposite-sign tracks and at least three photons detected. The highest-energy photon is assumed to be the

ISR photon and is required to have an energy of at least 3 GeV, while at least one pair of the other photons must be compatible with being produced in a π^0 decay. The candidate events are subjected to a kinematic fit under the hypothesis $e^+e^- \to \gamma_{ISR}\pi^+\pi^-\pi^0$, with requirement of energy and momentum conservation and π^0 mass constraint on the π^0 candidate. The χ^2 of the fit is used for selecting the signal events. The main backgrounds come from other ISR processes, in particular $e^+e^- \to \gamma_{ISR}\pi^+\pi^-$, $\gamma_{ISR}\pi^+\pi^-\pi^0\pi^0$, and the non-ISR processes $e^+e^- \to q\bar{q}$, where q is a light quark u, d, s, and $e^+e^- \to \tau^+\tau^-$, whose estimate have been obtained with dedicated studies.

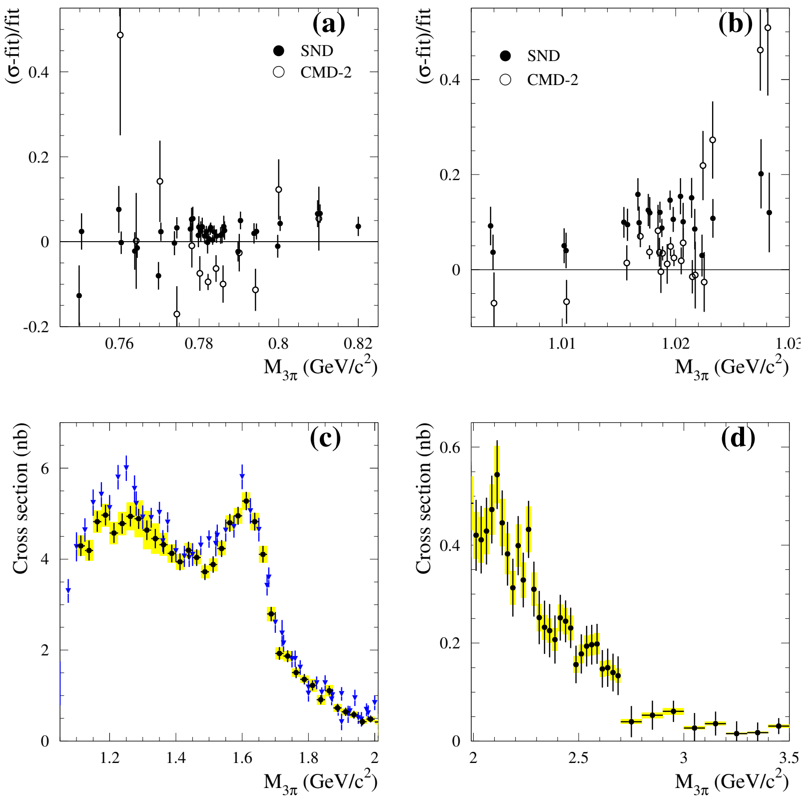

Figure 1: (top row) Relative difference between SND and CMD-2 data and *BABAR* $e^+e^- \to 3\pi$ cross section in the ω and ϕ regions. Only statistical uncertainties are shown. The systematic uncertainties amount to 3.4%, 1.3% and 1.3% for SND, CMD-2 and *BABAR*, respectively, in the ω-meson region, and to 5%, 2.5% and 1.3% in the ϕ-meson region. (bottom row) The *BABAR* $e^+e^- \to \pi^+\pi^-\pi^0$ cross section (circles) in comparison with the SND result [11] (triangles) for $m_{3\pi} > 1.1$ GeV$/c^2$. The contribution of the J/ψ meson is subtracted. The error bars represent the statistical uncertainties for both data set, while the shaded box shows the *BABAR* systematic uncertainty. The systematic uncertainty on SND data amount to 4.4%.

The 3π mass spectrum can be described as the sum of contributions of five resonances: $\rho = \rho(770)$, $\omega = \omega(780)$, $\phi = \phi(1020)$, $\omega' = \omega(1420)$ and $\omega'' = \omega(1650)$. In order to precisely measure the cross section, detector resolution effects need to be unfolded from the measured 3π invariant-mass spectrum in the energy region below 1.1 GeV, where the ω and ϕ resonances dominate by several order of magnitude. The folding matrix is obtained from Monte Carlo simulation, corrected for Data-MC differences. The latter are determined taking advantage of the well known values of the ω and ϕ resonance parameters. Additional Gaussian and Lorentzian smearing of the resolution is considered. Four different fitting models are tested, which include or not the Lorentzian smearing, and allow or not for the rare $\rho \to 3\pi$ decay. The results show that the $\rho \to 3\pi$ decay is necessary to describe the data. We measure a branching fraction $\mathcal{B}(\rho \to \pi^+\pi^-\pi^0) = (0.88 \pm 0.23 \pm 0.30) \times 10^{-4}$ with a significance greater than 6σ, consistent with a previous result from SND [7].

From the unfolded mass spectrum, the cross section $\sigma_{\pi^+\pi^-\pi^0}(m)$ is calculated for each mass interval up to 1.1 GeV/c^2 as:

$$\sigma_{e^+e^- \to \pi^+\pi^-\pi^0}(m) = \frac{(\mathrm{d}N/\mathrm{d}m)^{unfolded}}{\varepsilon R\, \mathrm{d}\mathcal{L}/\mathrm{d}m}\,; \tag{3}$$

where ε is the overall efficiency of the event selection process, R is a radiative correction factor, and $\mathrm{d}\mathcal{L}/\mathrm{d}m$ is the so-called ISR differential luminosity, obtained multiplying the total integrated luminosity L by the radiator function $W(s, m)$. Figure 1(top row) show the relative difference between SND [7, 8] and CMD-2 [9, 10] data and the BABAR cross section, around the ω and ϕ resonance peaks. Some significant difference is observed that is not fully covered by the systematic uncertainties (not reported in the error bars). Above 1.1 GeV/c^2 no narrow structures are present and resolution effects do not affect significantly the mass spectrum. Therefore, there is no need for unfolding and the measured mass spectrum can be used to extract the cross section, which is shown in Fig. 1, compared with the only previous measurement from SND [11]. Significant differences between the two measurements are seen near 1.25 and 1.5 GeV/c^2.

The calculation of the contribution to a_μ through Eq. (1) requires the bare cross section $\sigma_0(e^+e^- \to 3\pi)$, which is obtained from the measured cross section removing the effects of vacuum polarization: $\sigma_0(e^+e^- \to 3\pi)(s) = \sigma_{3\pi}(s)|1 - \Pi(s)|^2$. The values of $a_\mu^{3\pi}$calculated in each mass interval are reported in Ref. [12]. In particular, for $m_{3\pi} < 2.0$ GeV/c^2 it results $a_\mu^{3\pi} = (45.86 \pm 0.14 \pm 0.58) \times 10^{-10}$, which is in agreement with calculations based on previous $e^+e^- \to 3\pi$ cross-section measurements (see [1] and references therein), but has better accuracy.

3. Study of the processes $e^+e^- \to 2K3\pi$

An analyses of the $K^+K^-\pi^0\pi^0\pi^0$, $K_S^0K^\pm\pi^\mp\pi^0\pi^0$ and $K_S^0K^\pm\pi^\mp\pi^+\pi^-$ final states produced in e^+e^- collisions via ISR, has been recently performed by BABAR [13]. The only other information on final states with two kaons and three pions comes from a BABAR study of $e^+e^- \to K^+K^-\pi^+\pi^-\pi^0$ [14]. These reactions have sizable cross sections around 2 GeV, whose measurement could shed some light on the deviation seen between the sum-of-exclusive cross section and the calculation in perturbative QCD in that energy region. In addition, direct measurement of the final states reduces the need of isospin relations to account for the missing channels in the a_μ calculation. Finally, the study of the intermediate states allows to test QCD models and to search for new states.

With an analysis procedure similar to that of the three-pion final state, the total cross section for each final state is measured and shown in Fig. 2 (top row). The systematic uncertainties amount to about 10%. The contributions of the main intermediate states for each process are then extracted, as described in detail in Ref. [13].

The cross sections show hints of possible peaks around 2.17 and 2.4 GeV/c^2, which have been studied with refined binning. The bump at 2.4 GeV/c^2, particularly visible in $e^+e^- \to$

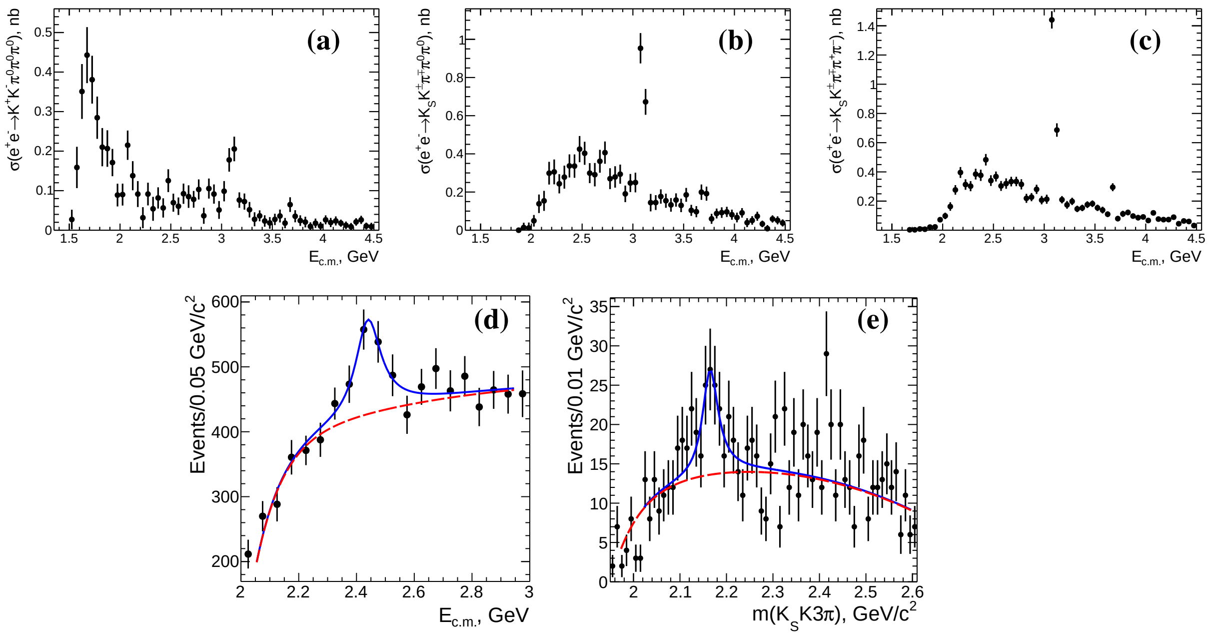

Figure 2: The measured (a) $e^+e^- \to K^+K^-\pi^0\pi^0\pi^0$, (b) $e^+e^- \to K^0_S K^\pm\pi^\mp\pi^0\pi^0$, and (c) $e^+e^- \to K^0_S K^\pm\pi^\mp\pi^+\pi^-$ cross sections. The uncertainties are statistical only. (d) The sum of events from $e^+e^- \to K^0_S K^\pm\pi^\mp\pi^+\pi^-$, $K^+K^-\pi^+\pi^-\pi^0$, and $\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0\pi^0$ processes around the bump at 2.4 GeV. (e) The $m(K^0_S K3\pi)$ invariant mass distribution around 2.17GeV$/c^2$. The result of the fits described in the text are superimposed to the event distributions.

$K^0_S K^\pm\pi^\mp\pi^+\pi^-$, was already seen in the $e^+e^- \to K^+K^-\pi^+\pi^-\pi^0$ and $e^+e^- \to 2(\pi^+\pi^-)3\pi^0$ processes. The events from all these reactions are summed in Fig. 2(d) and fitted with a Breit-Wigner plus a polynomial function; the signal has a significance of 3.5 σ, with parameters, $m = 2.44 \pm 0.02$ GeV$/c^2$ and $\Gamma = 107 \pm 49$ MeV, consistent with the structure called $X(2400)$ [15]. This structure, can also been explained as a threshold behaviour of the process $e^+e^- \to \phi f_0(1370)$.

The events in the mass region around 2.17 GeV$/c^2$, with the additional requirement $m(\pi^+\pi^-) < 0.7$ GeV$/c^2$ to remove a possible contribution from the $\rho(770)$, are shown in Fig. 2(e) together with the result of the fit with a BW and a polynomial function. The signal has a significance of 3.9 σ. The resonance parameters result to be $m = 2.164 \pm 0.006$ GeV$/c^2$ and $\Gamma = 41 \pm 20$ MeV, consistent with a new decay channel for the $\phi(2170)$ resonance.

4. Conclusions

We presented a recent *BABAR* measurement of the $e^+e^- \to \pi^+\pi^-\pi^0$ cross section and the corresponding contribution to a_μ, which is in agreement with calculations based on previous measurements, but is more precise by about a factor of about two.

We also presented preliminary results on the measurement of the cross section for three $e^+e^- \to 2K3\pi$ processes, together with the study of the many intermediate states. In particular we have shown hints for a new possible decay of the recently discovered $\phi(2170)$ resonant state.

References

[1] T. Aoyama *et al.*, *The anomalous magnetic moment of the muon in the Standard Model*, Phys. Rept. **887** (2020) 1.

[2] B. Abi *et al.* [Muon g-2 Collab.], *Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm*, Phys. Rev. Lett. **126**, (2021) 141801.

[3] B. Aubert *et al.* [BABAR Collab.], *The BABAR Detector*, Nucl. Instr. Meth. A **479**, (2002) 1. *The BABAR Detector: Upgrades, Operation, Performance*, Nucl. Instr. Meth.A **729**, (2013) 615.

[4] G. Bonneau and F. Martin, *Hard-photon emission in e^+e^- reactions*, Nucl. Phys. B **27**, (1971) 381.

[5] H. Czyż, A. Grzelińska, J.H. Kühn and G. Rodrigo, *Electron-positron annihilation into three pions and the radiative return*, Eur. Phys. J. C **47**, 617 (2006).

[6] H. Czyż and J.H. Kühn, *Four pion final states with tagged photons at electron positron colliders*, Eur. Phys. J. C **18**, 497 (2001).

[7] M.N. Achasov *et al.* [SND Collab.], *Study of the process $e^+e^- \to \pi^+\pi^-\pi^0$ in the energy region $\sqrt{s}$ below* 0.98 GeV, Phys. Rev. D **68**, 052006 (2003).

[8] M.N. Achasov *et al.* [SND Collab.], *Measurements of the parameters of the $\phi(1020)$ resonance through studies of the processes $e^+e^- \to K^+K^-$, K_SK_L, and $\pi^+\pi^-\pi^0$*, Phys. Rev. D **63**, 072002 (2001).

[9] R.R. Akhmetshin *et al.* [CMD-2 Collab.], *Reanalysis of hadronic cross-section measurements at CMD-2*, Phys. Lett. B **578**, 285 (2004).

[10] R.R. Akhmetshin *et al.* [CMD-2 Collab.], *Study of $\phi \to \pi^+\pi^-\pi^0$ with CMD-2 detector*, Phys. Lett. B **642**, 203 (2006).

[11] M.N. Achasov *et al.* [SND Collab.], *Study of dynamics of the process $e^+e^- \to \pi^+\pi^-\pi^0$ in the energy range* $1.15 - 2.00$ GeV, Eur. Phys. J. C **80**, 993 (2020).

[12] J.P. Lees *et al.* [BABAR Collab.], *Study of the process $e^+e^- \to \pi^+\pi^-\pi^0$ using initial state radiation with BABAR* Phys. Rev. D **104**, 112003 (2021).

[13] J.P. Lees *et al.* (BABAR Collaboration), *Study of the reactions $e^+e^- \to K^+K^-\pi^0\pi^0\pi^0$, $e^+e^- \to K^0_SK^\pm\pi^\mp\pi^0\pi^0$, and $e^+e^- \to K^0_SK^\pm\pi^\mp\pi^+\pi^-$ at center-of-mass energies from threshold to 4.5 GeV using initial-state radiation*, arXiv:2207.10340[hep-ex]. Article accepted for publication in Phys. Rev. D.

[14] B. Aubert *et al.* [BABAR Collab.], *The $e^+e^- \to 2(\pi^+\pi^-)\pi^0$, $2(\pi^+\pi^-)\eta$, $K^+K^-\pi^+\pi^-\pi^0$ and $K^+K^-\pi^+\pi^-\eta$ cross sections measured with initial-state radiation*, Phys. Rev. D **76**, 092005 (2007). [Erratum Phys. Rev. D **7**7, 119902 (2008)]

[15] C.P. Shen and C.Z. Yuan, *Combined fit to BABAR and Belle data on $e^+e^- \to \phi\pi^+\pi^-$ and $\phi f_0(980)$*, Chin. Phys. C **34**, 1045 (2010).