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Double charm tetraquark in DD^* scattering from lattice QCD

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The LHCb collaboration recently discovered a doubly charmed tetraquark T_{cc} with flavor $cc\bar{u}\bar{d}$ just 0.36(4) MeV below D^0D^{*+} threshold. This is the longest lived hadron with explicitly exotic quark content known to this date. We present the first lattice QCD study of DD^* scattering in this channel, involving rigorous determination of pole singularities in the related scattering amplitudes that point to the existence of T_{cc}. Working with a heavier than physical light quark mass, we find evidence for a shallow virtual bound state pole in the DD^* scattering amplitude with $l = 0$, which is likely related to T_{cc}.

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1. Introduction

Conventional quark-matter particles are composed of three quarks (baryons) or quark-antiquark pairs (mesons). In the past two decades, various particle collider experiments have discovered exotic particles that consist of more than three quarks and antiquarks. Among these, the longest-lived and the most unique is T_{cc}, discovered by the LHCb experiment at CERN in July 2021 [1, 2]. A successful theoretical description of the properties of this particle and its likes can have important implications in our understanding of the existence of hadrons. The main aim of such studies is to reveal whether the mechanisms responsible for its existence are analogous to those that bind protons and neutrons to nuclei.

The doubly charmed tetraquark, T_{cc}, was observed as a very narrow feature in the $D_0 D_0 \pi^+$ final states. This suggests its explicit four quark content ($cc\bar{u}\bar{d}$) and favors $I(J^P) = 0(1^+)$, where I is the isospin, J is the total angular momentum and P is the parity. Its narrow width ($\sim$ 48 keV) indicates its long life time. Its binding energy with respect to the energy of the lowest two-hadron($D_0 D^{+*}$) threshold is found to be very small (~ -360 keV/c^2). Several theoretical studies indicate attraction between the constituents within such doubly heavy systems, suggesting the existence of deeply bound states in doubly bottom tetraquark systems [3–8]. However, $cc\bar{u}\bar{d}$ system is found to be on the verge of being (un)bound [3, 4], demanding a careful investigation using first principles.

In order to theoretically describe T_{cc} from first-principles, one has to establish a related pole singularity in the corresponding scattering amplitude $t(E_{cm})$. This is very crucial for a near-threshold state like T_{cc}. Lattice formulation of QCD is the only *ab-initio* non-perturbative approach that can be utilized to study the physics in hadronic regime with quantifiable uncertainties. Although there are a handful of lattice QCD calculations of T_{cc} [4, 9, 10] determining the finite-volume spectrum, there has been no determination of a pole singularity in the DD^* scattering amplitude related to T_{cc}. In this talk, we have presented a recent exploratory lattice QCD calculation of ours, which indicated a clear evidence for the existence of this exotic particle directly from the fundamental quantum field theory [11].

In this work, we investigate the possible existence of a state with flavor $cc\bar{u}\bar{d}$, $I = 0$, and $J^P = 1^+$ near the DD^* threshold. To this end, we work on a pair of lattice QCD ensembles with $N_f = 2 + 1$ flavors of $O(a)$-improved Wilson quarks and spatial volumes [($\sim$ 2 fm)^3 and ($\sim$ 2.7 fm)^3], generated by the CLS consortium [12, 13]. The lattice spacing in these ensembles is $a = 0.08636(98)(40)$ fm, whereas the pion mass is measured to be $m_\pi = 280(3)$ MeV. With this heavier than physical pion mass, D^* is stable to strong decay and hence the $DD\pi$ and D^*D^* thresholds are sufficiently higher in energy so that any effects pertaining to these channels can be ignored. $t(E_{cm})$ near threshold is extracted from the finite-volume spectra following the formalism first proposed by M. Lüscher [14]. We demonstrate the existence of a hadronic pole related to T_{cc} and make a first attempt to investigate its heavy quark mass dependence by extracting $t(E_{cm})$ for two values of the charm quark mass, one slightly heavier than physical ($m_c^{(h)}$) and one slightly lighter ($m_c^{(l)}$).

2. Technical details

Hadron spectroscopy programs in lattice QCD such as ours proceed through the computation of two point correlation matrices on ensembles with elements of the form

$$C_{ij}(t) = \sum \langle O_i(t_{src}+t) O_j^{\dagger}(t_{src}) \rangle, \tag{1}$$

where the sum is over a set of source timeslices t_{src} and O_i are interpolating operators that imitate properties of multi-hadron final states in the finite-volume. Two-hadron interpolators O_i are constructed in four moving frames with total momenta p=0, 1, $\sqrt{2}$, and 2 in finite-volume irreducible representations (Λ) in the respective frames. We build these operators such that each hadron is also separately projected to a definite momentum (p_1, p_2; $p = p_1 + p_2$) as [15]

$$\begin{aligned} O^{DD^*} &= \sum_{k,j} A_{kj}\, D(\vec{p}_{1k}) D_j^*(\vec{p}_{2k}) \;,\; \vec{p}_{1k} + \vec{p}_{2k} = \vec{P} \\ &= \sum_{k,j} A_{kj} [(\bar{u}\Gamma_1 c)_{\vec{p}_{1k}} (\bar{d}\Gamma_{2j} c)_{\vec{p}_{2k}} - (\bar{d}\Gamma_1 c)_{\vec{p}_{1k}} (\bar{u}\Gamma_{2j} c)_{\vec{p}_{2k}}]. \end{aligned} \tag{2}$$

Such a projection at both ends of $C_{ij}(t)$ is made possible by the versatile technique known as 'distillation' [16, 17] that is widely used by many lattice groups performing hadron spectroscopy.

The spectral decomposition of the correlator are given by $C_{ij}(t) = \sum_{n=0}^{\infty} \langle \Omega | O_i | n \rangle \langle \Omega | O_j | n \rangle^{\dagger}$ $e^{-E_n t}$, where Ω is the vacuum and E_n is the energy of the n^{th} eigenstate. The correlation matrices are analyzed using variational procedures to extract the time evolution of states in the finite-volume. These follow solving for a generalized eigenvalue problem, $C(t)v^{(n)}(t) = \lambda^{(n)}(t)C(t)v^{(n)}(t)$. The eigenvalues at asymptotically large times are expected to have saturated with the lowest level and behave as $\lambda^{(n)}(t) \propto e^{-E_n t}$, from which the energy spectrum in the finite-volume is extracted. The eigenvectors $v^{(n)}(t)$ are related to the operator-state overlaps $Z_i^n = \langle \Omega | O_i | n \rangle$.

3. Results

Finite-volume spectrum: In Figure 1, we present the finite-volume spectrum as determined for $m_c^{(h)}$. The nonzero negative energy splittings between the interacting and the noninteracting energy levels clearly suggest an attractive interaction between the D and the D^* mesons. These energy shifts provide the necessary information to extract the infinite-volume DD^* scattering amplitudes. We observe similar negative energy shifts in $m_c^{(l)}$, indicating possible attractive interaction.

Scattering analysis: The infinite volume physics related to two particle scattering or the scattering amplitude t is encoded in the finite-volume energy splittings between interacting and non-interacting spectrum. The scattering amplitude t in $S = e^{2i\delta} = 1 + i\frac{4p}{E_{cm}}t$ depends on energy (E_{cm}), the partial wave l and $J = |s-l|, .., |s+l|$, where $s = 1$ for DD^* system. The s-wave amplitude, which dominates near threshold and is relevant for T_{cc}, is parametrized using an effective range expansion (ERE). We determine the energy dependence by optimizing the ERE parameters such that for all the energy levels considered in the analysis, the Lüscher's relation is simultaneously satisfied. We find the scattering length $a_0^{(1)} = 1.04(29)$ fm and the effective range $r_0^{(1)} = 0.96(^{+0.18}_{-0.20})$ fm for $m_c^{(h)}$.

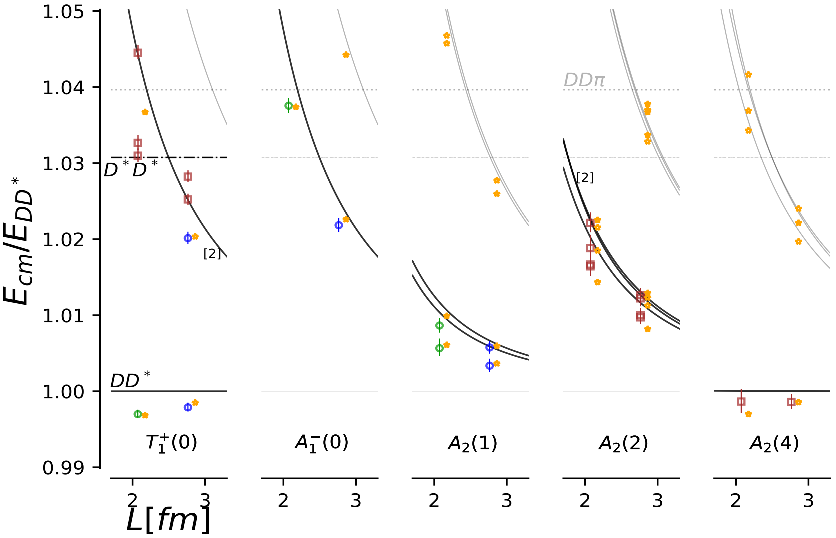

Figure 1: The finite-volume energy spectrum, in selected Λs relevant for a study of T_{cc}, presented in units of the energy of the elastic threshold DD^* as a function of the spatial extent L of the lattice QCD ensemble. The gray horizontal lines indicate various multi-hadron thresholds, whereas the black and gray curves represent the noninteracting finite-volume levels. Large unfilled circles and squares indicate the simulated interacting finite-volume spectra, in which the latter are not considered in the final scattering analysis. The nonzero negative energy splittings evident between the interacting and the noninteracting spectrum indicate attractive interaction between the D and D^* mesons. The orange stars (shifted horizontally for clarity) are the analytically reconstructed finite-volume levels based on the extracted scattering amplitudes determined from the large circles.

On the left of Fig. 2, we present the results from this analysis performed on a set of chosen energy levels related to DD* scattering leading to $J^P = 1^+$ for $m_c^{(h)}$. Here p is the momentum of the D/D^* meson in the center-of-momentum frame, which is related to energy E_{cm} and δ_0 is the related scattering phase shift. The cyan and orange curves are constraints $(\pm\sqrt{-p^2})$ for existence of 'hadronic' poles below the threshold. Data points map E_{cm} dependence of $pcot(\delta_0)\ [E_{cm}]$, which is related to $t(E_{cm}) \propto (pcot(\delta_0) - ip)^{-1}$. The energies where the parametrized $pcot(\delta_0)\ [E_{cm}]$ crosses the constraint curves are solutions of poles in t related to sub-threshold hadrons we observe in nature. We observe the parametrized energy dependence of t indicates pole solutions on the cyan curves, which suggests the existence of a virtual bound state pole with binding energy $\delta m_{T_{cc}} = E^p_{cm} - m_D - m_{D^*} = -9.9^{+3.6}_{-7.1}$ MeV. This result is robust to a variety of fits we have performed, as further detailed in the Supplemental of Ref. [11]. The finite-volume spectrum reconstructed from the fitted scattering amplitude is shown in Figure 1 as orange stars, demonstrating the quality of the fit. We make similar observation of a virtual bound state pole at $m_c^{(l)}$, but with a deeper binding energy.

The notion of virtual bound state can be understood easily from simply Quantum Mechanics as a feature that happens when an attractive potential is not deep enough to hold a bound state. In terms of t, it is a pole in the real axis of unphysical Riemann sheet below the threshold at a binding momentum $p = -i|p_B|$. An example of such a pole is in ${}^1S_0$ nucleon-nucleon channel [18, 19]. We demonstrate and provide elaborate discussion on virtual bound poles in Ref. [11].

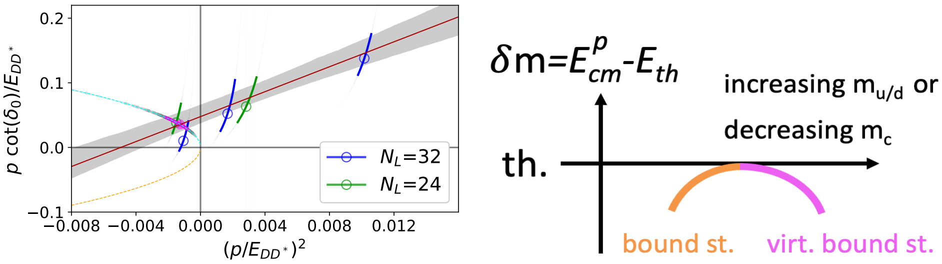

Figure 2: **Left**: $p\cot\delta_{l=0}^{(J=1)}$ for DD^* scattering at the heavier charm quark mass (red line) and $ip = +|p|$ (cyan line) versus p^2, all normalized to $E_{DD^*} \equiv m_D + m_{D^*}$. The virtual bound state occurs at the momenta indicated by the magenta octagon, where two curves intersect. **Right**: Heavy and light quark mass dependence of the binding energy (δm) of an s-wave (virtual) bound state in a purely attractive potential.

In order to relate this virtual bound state observed in the DD^* scattering amplitude with the real world, we model the s-wave interaction between the D and D^* mesons with simple attractive potentials and look for Quantum Mechanical solutions in it. We find that with increasing heavy quark mass and decreasing light quark mass, the attractive potential is strengthened such that a virtual bound state transforms to form a bound state pole. This behaviour is depicted in the right of Fig. 2 and is observed with any general attractive potentials as demonstrated in Ref. [11]. Given this picture, T_{cc} for the case of unphysically heavy u/d masses like in our study, would have become a virtual bound state and hence is related to the pole we find in our study. We also find such a near-threshold virtual bound state pole also at $m_c^{(l)}$, with slightly deeper binding. This is in agreement with the heavy quark mass dependence of pole position as presented in the right of Fig. 2.

4. Conclusions

In this talk, we present the first lattice QCD investigation of DD^* scattering that could extract the near-threshold scattering amplitudes in the flavor channel $cc\bar{u}\bar{d}$ with isospin $I = 0$. We follow the finite-volume formulation proposed by M. Lüscher to extract the scattering amplitude. With the unphysically heavy light quark mass setup, we find that the doubly charm tetraquark with $J^P = 1^+$ features as a virtual bound state immediately below the DD^* threshold. We also find that the strength of the binding for this virtual bound state decreases with increasing heavy quark mass. This observed behaviour of the hadronic pole is in agreement with expected Quantum Mechanical solutions for simple attractive potential between the D and D^* mesons. Based on these expectations we provide arguments on the how this lattice observed virtual bound state pole is related to T_{cc}.

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