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# High precision calculations for the MUonE experiment

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The anomalous magnetic moment of the muon $a_\mu = (g-2)_\mu/2$ has been measured at the Brookhaven National Laboratory in 2001 and recently at the Fermilab Muon $g-2$ Experiment. The results deviate by 4.2 $\sigma$ from the Standard Model predictions, where the dominant source of theoretical error comes from the Hadronic Leading Order contribution $a_\mu^{HLO}$. Moreover, recent calculations performed on the lattice seem to be more consistent with the experimental value for $a_\mu^{HLO}$. MUonE is a proposed experiment at CERN whose purpose is to provide a new and independent determination of $a_\mu^{HLO}$ via elastic muon-electron scattering at low momentum transfer. To achieve a precision that is comparable to the standard timelike estimation of $a_\mu^{HLO}$, the experiment must reach an accuracy of about 10 parts per million on the differential cross section. This requires a similar level of accuracy also from the theoretical point of view: a precise calculation of the muon-electron scattering cross section with all the relevant radiative corrections as well as quantitative estimates of all possible background processes are needed. The state of the art of the theoretical calculation for $\mu e$ scattering is presented. Then, the formulation for the next-to-next-to-leading order real and virtual lepton pair contributions is described as well as their numerical impact, obtained with the Monte Carlo event generator Mesmer. These contributions are crucial to reach the precision aim of MUonE.

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## 1. Introduction

The muon anomalous magnetic moment is a fundamental quantity in particle physics and has become a stringent test for the Standard Model: there is a tension of $4.2\,\sigma$ between the combination of measurements performed by BNL and Fermilab [1], and the most complete theoretical prediction [2]. The largest source of theoretical error comes from non-perturbative strong interaction effects that constitute the Hadronic Leading Order (HLO) contribution $a_\mu^{HLO}$. Typically, $a_\mu^{HLO}$ is calculated using a dispersion relation and experimental data for the $e^+e^- \to$ hadrons cross section (*timelike* approach). Moreover, the same quantity has recently been calculated on the lattice and brings the value of $a_\mu$ closer to its experimental value [3]. Hence, a third independent calculation of $a_\mu^{HLO}$ is needed to shed some light on the discrepancy that stands between the BNL and Fermilab measurements, and the Standard Model prediction of $a_\mu$. In this respect, recently a new and different method to calculate $a_\mu^{HLO}$ has been addressed in [4]: $a_\mu^{HLO}$ can be linked to the hadronic running of the fine structure constant $\Delta\alpha_{had}$ via the relation:

$$a_\mu^{HLO} = \frac{\alpha}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{had}\left[t(x)\right]; \qquad t(x) = \frac{x^2 m_\mu^2}{x-1},$$

where $t(x)$ is a negative momentum transfer (*spacelike* approach) [5].

This new theoretical approach led to the proposal of the MUonE experiment at CERN. The MUonE experiment intends to perform a measurement on the hadronic running of the fine structure constant $\Delta\alpha_{had}$ using muon-electron scattering [6, 7], thus determining $a_\mu^{HLO}$ in the *spacelike* region. The most recent reviews on the experimental status can be found in [8, 9]. A very high experimental precision is needed (about 10 parts per million on the differential observables) in order to match the theoretical precision of $a_\mu^{HLO}$ coming from the *timelike* approach: this requires an effort on the theoretical side to include in the simulations all the relevant radiative corrections. In this respect, a fully exclusive Monte Carlo (MC) event generator, Mesmer, is under development. In this contribution, the calculation of NNLO real and virtual lepton pair corrections to $\mu e$ scattering are described, following the work in [22].

## 2. The state of the art of muon-electron scattering calculations for MUonE

MUonE requires a very precise determination of $\mu e$ scattering differential cross-section, therefore all the relevant radiative corrections need to be implemented in a fully-fledged MC program. To reach the needed 10 ppm accuracy, all NLO and NNLO QED corrections have to be included, as well as the leading logarithmic contributions due to multiple photon radiation. Many important studies have been performed to achieve this objective: a review on the theoretical work can be found in ref. [10]. In [11], all the NLO QED and EW corrections were calculated using Mesmer. More recently, in [12] the virtual NNLO photonic contributions were calculated exactly for emission from $e$ and $\mu$ leg: the two-loop QED vertex form factors were taken from [13]. Also, the one-loop corrections to real photon emission and double real photon emission have been calculated. Furthermore, the two-loop virtual diagrams with a virtual photon insertion on top of NLO boxes have been studied: of those diagrams, only the infrared (IR) part is included exactly by means of the classical Yennie-Frautschi-Suura approach [14]. All photonic NNLO effects weigh at most

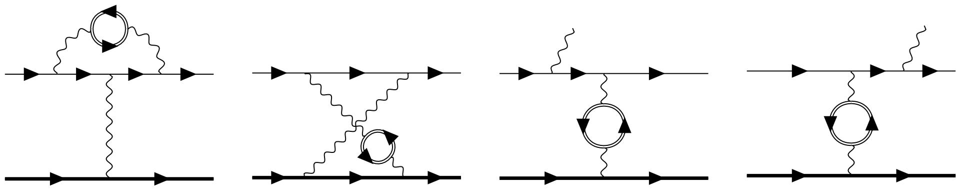

**Figure 1:** Some of the diagrams of the classes $d\sigma^{\alpha^2}_{\mathrm{virt}}$ and $d\sigma^{\alpha^2}_{\gamma}$.

some percent w.r.t. the LO at the phase space boundaries. The NNLO photonic contributions for emission from the electron leg have been calculated also in [15]. Very recently, the calculation of the two-loop amplitudes for the scattering of four fermions has been performed, keeping into account the exact mass dependence of one of the fermionic currents [16]. Another important set of two-loop contributions could come from hadronic corrections to $\mu e$ scattering: those have been calculated in [17, 18]. In recent years, possible contamination from New Physics effects have been investigated and have been shown to be below the MUonE sensitivity [19, 20]. Neutral pion production $\mu e \to \mu e \pi^0$ has been recently addressed as a possible background process for MUonE [21]: it was found that such a contribution lies below the experimental sensitivity of 10 ppm. The fixed-order NNLO radiative corrections due to the emission of virtual and real leptonic pairs have been computed in [22].

## 3. NNLO lepton pair contributions: the calculation

In this section we describe the calculation of the NNLO QED contributions that include at least one leptonic pair. Such contributions include the production of both virtual and real leptonic pairs, summing over all the possible lepton flavours, as described in [22]. The complete set of NNLO leptonic contributions to $\mu e$ scattering can be split in three pieces:

$$d\sigma^{\alpha^2}_{N_f} = d\sigma^{\alpha^2}_{\mathrm{virt}} + d\sigma^{\alpha^2}_{\gamma} + d\sigma^{\alpha^2}_{\mathrm{real}}.$$

- In $d\sigma^{\alpha^2}_{\mathrm{virt}}$ all the contributions where a virtual lepton pair is being produced without any real photon radiation are included. Some of those diagrams are shown in figure 1. Also interference terms with LO or photonic NLO diagrams lie within this class of contributions.
- The class $d\sigma^{\alpha^2}_{\gamma}$ includes all the diagrams where there is an interplay between real photon radiation and leptonic loop insertions, in particular the interference between the tree-level $\mu e \to \mu e \gamma$ and the same kind of diagrams with a leptonic vacuum polarisation (VP) inserted.
- $d\sigma^{\alpha^2}_{\mathrm{real}}$ contains all the tree level diagrams for $\mu e \to \mu e \ell^+ \ell^-$: the real lepton pair can be produced from the electron leg, from the muon leg or from the photon propagator (peripheral diagrams), as in figure 2.

The virtual and real-virtual corrections have been calculated using dispersion relation techniques, where needed: starting from the NLO diagrams, we substituted one of the virtual photons

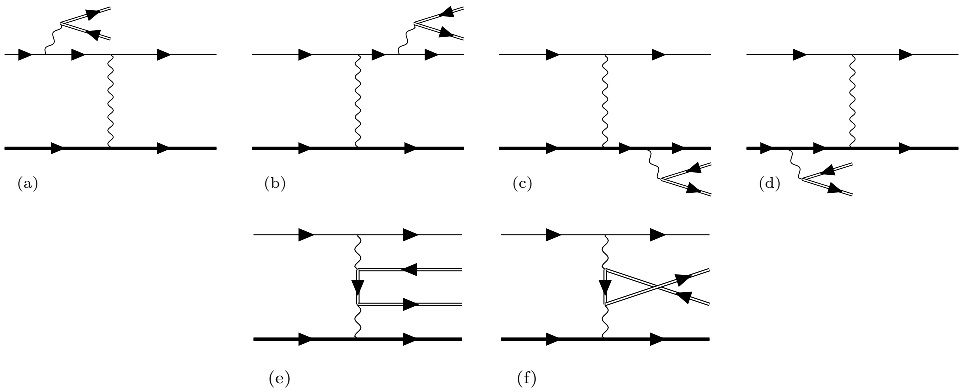


**Figure 2:** The diagrams of the class $d\sigma^{\alpha^2}_{\text{real}}$.

with a dispersion relation to account for the lepton loop:

$$\frac{-ig_{\mu\nu}}{q^2+i\varepsilon} \to -ig_{\mu\nu}\left(\frac{\alpha}{3\pi}\right)\int_{4m_\ell^2}^{\infty}\frac{dz}{z}\frac{1}{q^2-z+i\epsilon}\left(1+\frac{4m_\ell^2}{2z}\right)\sqrt{1-\frac{4m_\ell^2}{z}},$$

where $m_\ell$ is the mass of the lepton that is circulating in the loop. The integral over $z$ has been performed using Monte Carlo integration. It is important to note that IR divergences may arise from contributions of the $d\sigma^{\alpha^2}_{\gamma}$ class, due to soft and/or collinear radiation. However, these divergences are cancelled by considering the diagrams where there is a VP insertion as well as a virtual photon.

The calculation of the differential and integrated cross section of real lepton pair production $\mu e \to \mu e \ell^+\ell^-$ is a $2 \to 4$ scattering process. The phase space has been calculated starting from the general $n$-particle phase space element

$$d\Phi_n = \int \prod_{i=1}^{\infty}\frac{d^3p_i}{(2\pi)^3 2E_i}\delta^4\left(P-\sum_{i=1}^{n}p_i\right)$$

where $p_i$ are the final-state particle momenta, $E_i$ are their energies and $P$ is the initial-state total momentum. Two different parametrisations for the phase space have been used to cross-check the calculation. The QED matrix elements have been calculated using the symbolic manipulation program FORM and have been cross-checked with RECOLA [23], finding a perfect agreement.

## 4. NNLO lepton pair contributions: numerical results

In this section the numerical impact of NNLO lepton pair contributions will be presented: only the virtual contributions will be shown for the sake of brevity: the numerical impact of real lepton pair contributions, as well as the necessity of determining and including elasticity kinematical cuts on the simulated events, can be found in the original paper [22]. For the virtual contributions, the following cuts have been applied on the outgoing electron and muon:

- Acceptance cuts: $\vartheta_e, \vartheta_\mu < 100$ mrad and $E_e > 1$ GeV.
- Acoplanarity cuts: $\xi = |\pi - |\phi_e - \phi_\mu|| < \xi_c = 3.5$ mrad, where $\phi_{e,\mu}$ are the azimuthal angles.

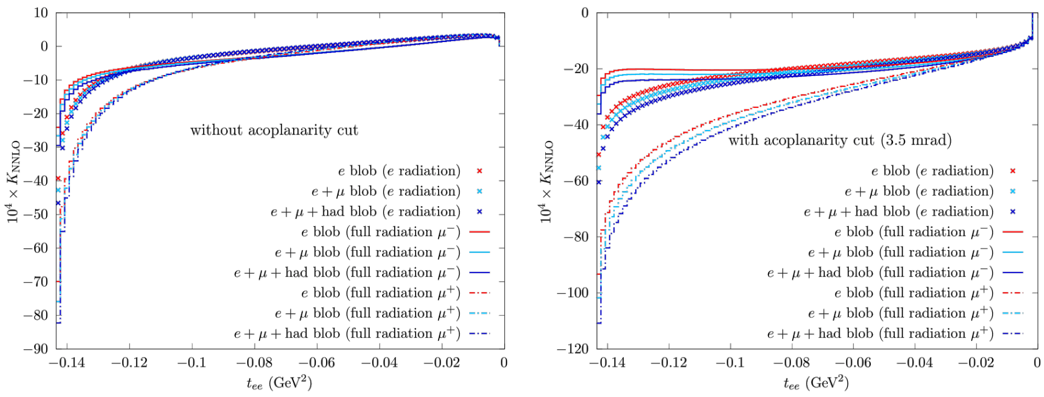


**Figure 3:** Differential $K_{NNLO}$ plotted against the momentum transfer along the electron line $t_{ee}$ for all the NNLO virtual lepton pairs contributions. On the r.h.s. the same is plotted, when also acoplanarity cuts are applied.

In figure 3 the differential *K factor*

$$K_{NNLO} = \frac{d\sigma^{\alpha^2}_{N_f}}{d\sigma_{LO}}$$

is shown as a function of the momentum transfer along the electron line $t_{ee}$ for the virtual pair corrections. In the plot, radiation from only electron or both leptonic currents is shown with all the possible fermions circulating in the loop: each of the diagrams under study accounts for at most some 0.1% at the boundaries of the phase space. The biggest contributions come from diagrams where there is radiation from the electron leg, and electrons circulate in the fermionic loop.

## 5. Conclusions

There are important efforts to develop NNLO fixed-order Monte Carlo event generators for $\mu e$ scattering. As of today, NNLO photonic corrections have been studied exactly, except for a subset that has been treated with the YFS approach. Their effects amount to some percent at phase space boundaries (that is small $\vartheta_e$ and large $|t_{ee}|$). NNLO virtual lepton pair contributions weigh some $10^{-4}$ to $10^{-3}$ w.r.t. the leading order: moreover, $e^+e^-$ emission is dominant w.r.t. $\mu^+\mu^-$. NNLO $e^+e^-$ real pair production could potentially represent a significant background: the effects of these contributions are controlled if appropriate kinematical cuts are applied. Higher-order QED corrections must be included in the Monte Carlo programs to reach the 10 ppm precision on the differential observables that is required by the MUonE experiment. This could be accomplished, for example, by matching a QED parton shower algorithm with exact NNLO matrix elements.

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