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$\alpha_s(m_\tau)$ from an improved vector isovector spectral function

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We discuss a new determination of the strong coupling at the τ-mass scale, $\alpha_s(m_\tau^2)$, based on a new vector isovector spectral function, which combines ALEPH and OPAL spectra for the dominant 2π and 4π decay channels with estimates for several sub-leading contributions obtained using CVC from recently measured electroproduction cross-sections, as well as results for $\tau \to K^- K^0 \nu_\tau$ obtained by BaBar. This fully inclusive vector isovector spectral function is entirely based on experimental data, and no longer relies on Monte Carlo input for any of the sub-leading modes. We obtain $\alpha_s(m_\tau) = 0.3077 \pm 0.0075$, corresponding to $\alpha_s(m_Z) = 0.1171 \pm 0.0010$.

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PoS(ICHEP2022)797

1. Improved vector isovector spectral function

The determination of the QCD coupling, α_s, from weighted integrals over the spectral functions measured from inclusive hadronic tau decay data remains, as of today, one of the most precise α_s determinations from experimental data. We report here on the work of Ref. [1] where an updated determination of $\alpha_s(m_\tau)$ has been performed, from an improved vector isovector spectral function that combines information from tau decay data with results implied by recently measured electroproduction cross sections using conserved vector-current (CVC) relations.

Until recently, the determinations of α_s from τ decay data relied on ALEPH [2–4] or OPAL spectral functions [5, 6]. ALEPH's data set have smaller uncertainties, but the two extractions were in good agreement, which motivated the use of a weigthed average between the results obtained from the two data sets as the final result [2, 6]. Ideally, however, as is well stablished in the study of the hadronic vacuum polarization contribution to the muon $g-2$, one should first rigorously combine all available experimental data into a single spectral function that should then be the basis for a final analysis. Undertaking such a combination, which was one of the questions addressed in Ref. [1], provides an opportunity to revisit the description of sub-leading, or "residual", hadronic decay modes. The original ALEPH and OPAL spectral functions were based on detailed experimental measurements of the spectra for the dominant decay modes, which are $\pi\pi^0$, $\pi 2\pi^0$, 3π, $\pi 3\pi^0$, and $3\pi\pi^0$. These channels are, however, not sufficient to obtain fully inclusive spectral functions which led the LEP experiments to add additional residual modes, such as $\tau \to \omega(\to \text{non}\, 3\pi)\pi^-\nu_\tau$, $\tau \to 6\pi\nu_\tau$, and $\tau \to K^-K^0\nu_\tau$, using Monte Carlo (MC) simulations. The experimental knowledge about many of these residual modes have changed in the last few years. Several new experimental results have appeared, and it becomes possible to improve upon the MC description of the residual modes that was used by ALEPH and OPAL. The spectrum of $\tau \to K^-K^0\nu_\tau$ was measured by BaBar [7] and, what is more, essentially all remaining residual contributions can now be obtained, using CVC, from exclusive-channel cross sections in $e^+e^- \to$ hadrons obtained by the CMD-3, BaBar, and SND collaborations.[1] Isospin-breaking corrections to the residual mode contributions thus obtained are certainly negligible, since their total contribution amounts to only a few percent of the fully inclusive spectral function. Of course, since we rely on electroproduction data, only an improved vector spectral function can be obtained through this procedure.

The program outlined above was carried out in Ref. [1]. The ALEPH and OPAL data for the sum of dominant vector decay modes, namely $\pi\pi^0$, $\pi 3\pi^0$, and $3\pi\pi^0$ channels, were combined using the algorithm of Ref. [8]. (The combination channel by channel cannot be done — taking into account all correlations — due to the 100% correlated errors of the 4π channels.) To this combined 2π+4π spectral function, we added the residual contributions from $\pi^-\omega(\to \text{non-}3\pi)$, K^-K^0, $\eta\pi^-\pi^0$, $K\bar{K}\pi$, $3\pi^-2\pi^+\pi^0$, $2\pi^-\pi^+3\pi^0$, $(3\pi)^-\omega(\to \text{non-}3\pi)$, $K\bar{K}\pi\pi$, $\pi^-5\pi^0$, and $\eta\omega\pi + \eta 4\pi$. This results in a new, almost fully inclusive, vector isovector spectral function entirely based on experimental data and with smaller errors especially towards the end point of the spectrum (see Fig. 1). The new spectral function covers 99.95% of the total inclusive vector isovector branching fraction.

2. New result for $\alpha_s(m_\tau)$

We review here the results obtained from the QCD Finite-Energy-Sum-Rule (FESR) analysis of Ref. [1]. The analysis follows the Duality Violation (DV) model strategy, introduced in Ref. [5]

[1] For a full reference list of the 14 papers that enter this analysis we refer to Ref. [1].

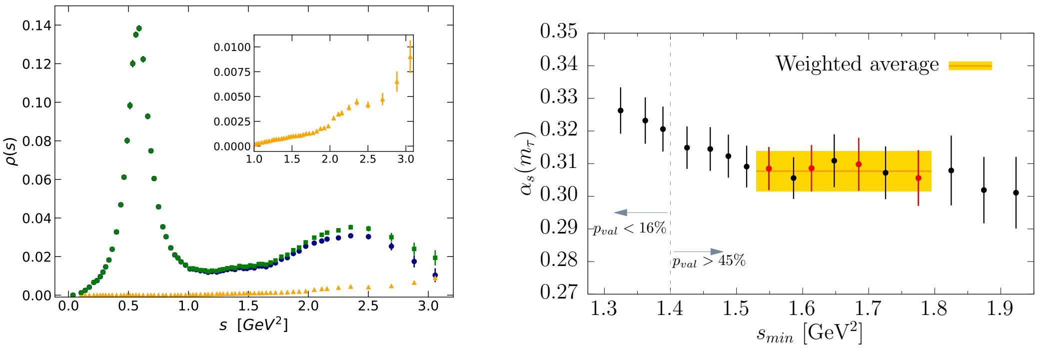

Figure 1: (Left-hand panel) New vector isovector spectral function (in green) obtained from the addition of the combined ALEPH and OPAL dominant modes (blue) with the residual modes, in yellow. (Right-hand panel) Values of $\alpha_s(m_\tau)$ as a function of $s_{\rm min}$, the minimum value of s_0 included in the respective fit. Results in red are used in our final weighted average, shown as a yellow band. Figures extracted from Ref. [1].

and used previously in the analysis of the ALEPH and OPAL data in Refs. [2] and [6], respectively.

The QCD analysis employs FESRs where the experimental side is given by weighted integrals over the spectral function, $\rho(s)$, obtained here from the discretized integration of the data shown in Fig. 1. On the theory side, one has an integral over the relevant correlation function $\Pi(z)$ along a closed contour in the complex plane, such that the strong coupling is kept within the perturbative domain. The sum rules are calculated from threshold up to a given value of s_0 and the weight function $w(s)$ must be analytic:

$$\frac{1}{s_0}\int_0^{s_0} ds\, w(s)\,\rho(s) = -\frac{1}{2\pi i\, s_0}\oint_{|z|=s_0} dz\, w(z)\,\Pi(z). \tag{1}$$

On the theory side, the QCD contributions can be divided into a perturbative part, which is known up to $O(\alpha_s^4)$, and non-perturbative contributions arising from the operator product expansion (OPE) and the DVs. The choice of weight functions $w(s)$ that enter the analysis play an important role in emphasizing or deemphasizing the different contributions. In particular, the choice of weight functions that can suppress DVs, called *pinched*, namely those that have a zero of the type $(s-s_0)^n$, has been advocated by some authors [3, 4]. Higher pinching, however, is associated with sensitivity to higher- dimension condensates in the OPE, which introduces several new, unknown, parameters in the theoretical description. In this situation, an arbitrary truncation of the OPE becomes mandatory — we refer to this strategy as the truncated-OPE strategy — and generates an often underestimated theoretical error associated with it [9, 10]. Here, instead, we follow the DV strategy, which completely avoids the arbitrary truncation of the OPE and employs at least one weight function that is not pinched. In this strategy, the DVs are no longer suppressed and must be explicitly included in the theoretical description. This is done with the parametrization of Refs. [11, 12], which is based on widely accepted assumptions about the QCD spectrum. For sufficiently large values of s the duality violation contribution to the spectral function can be parametrized as

$$\rho_{\rm DVs}(s) = \frac{1}{\pi}\Pi_{\rm DV}(s) = e^{-\delta-\gamma s}\sin(\alpha+\beta s) \tag{2}$$

This introduces four additional parameters (δ, γ, α, and β) into the theory description, but they can

PoS(ICHEP2022)797

all be extracted from χ^2 fits to data, where one fits, simultaneously, sum rules of the type of Eq. (1) with multiple values of s_0 in the range $1.55\ \text{GeV}^2 \lesssim s_0 \leq m_\tau^2$.

In the results we report here, the perturbative contribution is calculated within the so-called Fixed Order Perturbation Theory (FOPT), where the expansion is strictly performed in powers of $\alpha_s(s_0)$ [13]. An alternate renormalization-scale setting, commonly used in this context, is the Contour Improved Perturbation Theory (CIPT), which resums the running of the coupling along the integration contour in the complex plane [14, 15]. For a long time, the difference between the α_s values obtained with the two prescriptions has been an important source of theoretical uncertainty. Very recently, however, it became clear that CIPT is incompatible with the standard form of the OPE [16, 17]. In other words, the choice of renormalization scale in the perturbative part should be accompanied by appropriately defined versions of the OPE condensate contributions. It has also become clear that the observed discrepancy between CIPT and FOPT series — termed asymptotic separation [16, 17] — is strongly dominated by the contribution from the leading IR renormalon, associated with the gluon condensate. Motivated by this observation, a new scheme for the gluon condensate that leads to compatible predictions for FOPT and CIPT has only very recently been proposed [18, 19], and was not yet employed in a complete α_s analysis. In this situation, since the results from CIPT, up to now, have always been obtained with an incoherent treatment of the OPE, they should not be used in quoting final values for α_s and FOPT should be preferred.

In Fig.1 (right-hand panel) we show results for $\alpha_s(m_\tau)$ obtained from several fits employing sum rules in different intervals $s_{\min} \leq s_0 \leq m_\tau^2$ using the unpinched weight $w(x) = 1$. The results show stability as a function of $s_{\min}$ in a window $1.5\ \text{GeV}^2 \lesssim s \lesssim 1.8\ \text{GeV}^2$. For larger values of $s_{\min}$ the results remain fully compatible but the errors are larger due to the smaller number of experimental data points available in the fit. For $s_{\min} \lesssim 1.4\ \text{GeV}^2$ one observes a drift in $\alpha_s(m_\tau)$ values, accompanied by worse p-values for the fits, indicating the break-down of the theoretical description at low energies. Our final result is based on a weighted average — taking into account all correlations — of the results shown in red in the right-hand panel of Fig.1, leading to $\alpha_s(m_\tau) = 0.3077 \pm 0.0075$ (in the $\overline{\text{MS}}$ scheme and with $n_f = 3$) which corresponds to $\alpha_s(m_Z) = 0.1171 \pm 0.0010$, in $\overline{\text{MS}}$ and with $n_f = 5$. The uncertainty includes a component from the experimental data, an uncertainty from the truncation of perturbation theory, as well as as an estimate of the DV-model dependency obtained from fits to moments of different weight functions $w_i(x)$, with different sensitivities to the DV contribution.

3. Stability of the DV parametrization

Recently, in a invited contribution to the JHEP anniversary issue, the work of Ref. [20] has called into question the stability of the DV parametrization employed in our work. An exercise performed in that work that is justified on theoretical grounds [12] and is worth considering is the inclusion of an $1/s$-suppressed correction to the parametrization of Eq. (2). Here we perform a similar study using the new, more precise, vector-isovector spectral function of Ref. [1], multiplying the parametrization of Eq. (2) by a correction $(1 + c/s)$, where c is a free parameter. We performed different fits varying the value of c in a large range, including negative values of c for which Ref. [20] finds particularly large shifts in α_s values using the ALEPH data set. Our results are shown in Fig. 2 as a function of c for fits with $s_{\min} = 1.55\ \text{GeV}^2$ and $w(x) = 1$ (as done in [20]). It is clear that

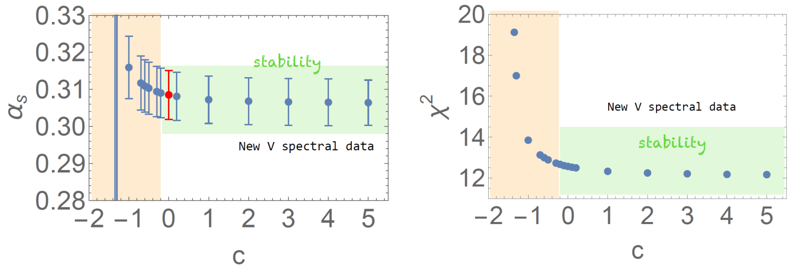

Figure 2: $\alpha_s(m_\tau)$ and χ^2 as a function of c (in GeV^2) from fits to the new vector spectral function [1] with $s_{\rm min} = 1.55$ GeV^2 and $w(x) = 1$ (see Sec. 3). Results for α_s are stable in the region where fits are acceptable.

the negative c values, associated with larger shifts in α_s, are immediately excluded by a very rapid worsening of the χ^2 of the fit (right-hand panel of Fig. 2). In the region where the fits remain acceptable, the results are extremely stable. We can safely conclude that our results pass this test and no additional uncertainty related to this type of modification to the DV ansatz is needed.

A more detailed discussion of some of the shortcomings of the criticisms of Ref. [20] can be found in [21]. A complete response to the claims of [20] will be presented in a future publication.

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