PROCEEDINGS OF SCIENCE

Studying light flavour resonances with polarised photon beams

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The study of exotic mesons such as gluonic hybrids gives a greater insight into how quarks and gluons bind to form such states and hence increase our understanding of the fundamental strong force. Furthermore, the double pion photoproduction is known as a ideal tool for the investigation of nucleon resonances, especially the exotic meson states. Hereby, to study the interference of meson resonance production and meson-baryon rescattering effects, we focus on the reaction $\gamma p \to \pi^+\pi^- p$. Aiming at the description of the latest data collected at CLAS12 and GlueX experiments, we used Deck model with a virtual pion exchange to generalize the moment extraction formalism with a linearly polarized photons. We compute the moments of the $\pi^+ \pi^-$ angular distribution with $E_\gamma = 8.5\, GeV^2$ for $L = 0, 1, 2, 3, 4$ in the helicity frame i.e the rest frame of the $\pi\pi$ with the direction opposite to the recoil nucleon defining the z axis.

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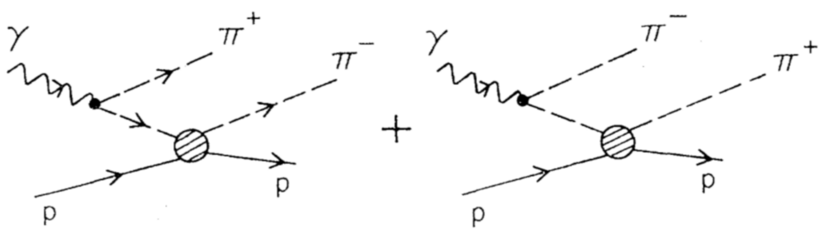

Figure 1: Feynman diagrams for diffraction dissociation of $\gamma p \to \pi^+\pi^- p$.

1. Introduction

According to the quark model, quarks can be organized as triplets to form Baryons like protons and neutron. They also can be arranged as quark-antiquark pair to form mesons.
All mesons participate in both the weak and strong interactions, charged ones also participate in the electromagnetic interaction. They are classified according to their quark content, total angular momentum J, parity P, and various other properties such as C-parity and G-parity. However J^{PC} quantum numbers like 0^{+-} , 1^{-+}, 2^{+-} , are not accessible in $q\bar{q}$ systems, this firm establishment of such spin-exotic states was a direct hint for the existence of objects with either gluonic excitations or more than two quarks involved i.e. tetraquarks and pentaquarks.
Mesons with gluonic excitations are known as hybrids which can be photoproduced through $\gamma + p \to \eta\pi + p$ at JLab. Due to the fact that there are a lot of experimental data for the double pion photoproduction, we presented a full description of the reaction $\gamma p \to \pi^+\pi^- p$.

2. Description of the model

We want to describe the two pion photoproduction via the so-called Deck Mechanism [1] where the incoming photon scatters diffractively from the target nucleon with the dominance of one pion exchange. Consider the process $\gamma(q,\lambda_\gamma) + p(p_1,\lambda_1) \to \pi^+(k_1) + \pi^-(k_2) + p(p_2,\lambda_2)$, with q denoting the photon four-momentum, p_1, p_2 the initial and final proton momenta and k_1, k_2 denoting the pions momenta. Where λ_γ, λ_1, λ_2 are the polarization of the photon, incoming and outgoing proton respectively.

We may write the amplitude for the Deck model showed in Fig.1 as:

$$M = e\left[\frac{\epsilon_\lambda.\boldsymbol{k_1}}{\boldsymbol{q}.\boldsymbol{k_1}}T^- - \frac{\epsilon_\lambda.\boldsymbol{k_2}}{\boldsymbol{q}.\boldsymbol{k_2}}T^+ + \frac{\epsilon_\lambda.(\boldsymbol{p_1}+\boldsymbol{p_2})}{\boldsymbol{q}.(\boldsymbol{p_1}+\boldsymbol{p_2})}(T^+ - T^-)\right], \tag{1}$$

where T^+ and T^- are the pion-nucleon elastic scattering amplitudes[1], and ϵ_λ is the photon polarization vector with $\lambda = \pm 1$.

[1] T^+ is the amplitude of a positive pion exchange between the photon and the proton i.e. the right side of Fig.1 while T^- corresponds for a negative pion exchange i.e. the left side of Fig.1

2.1 Kinematics

The explicit expressions for the four-vectors and photon polarization vector in the s-channel helicity frame which we denote with a subscript H and the Gottfried-Jackson frame, which we denote with a subscript GJ. Both frames share three kinematic invariants:

$$s = (p_1 + q)^2 \tag{2}$$
$$t = (p_1 - p_2)^2 \tag{3}$$
$$s_{\pi\pi} = (k_1 + k_2)^2. \tag{4}$$

Where s the total c.m. energy squared, $t$the four-momentum transfer and $s_{\pi\pi}$ the invariant mass of the $\pi\pi$ system squared, with the final two kinematic variables being two angles defined in the specific frames.

2.1.1 Helicity Frame:

In this frame, we may use the three aforementioned invariants (s, t and $s_{\pi\pi}$) as well as the two angles $\Omega_{\mathrm{H}} = (\theta_{\mathrm{H}}, \phi_{\mathrm{H}})$ of the π^+ in the $(\pi^+\pi^-)$ rest frame. This implies that the $\pi^+\pi^-$ pair decays back-to-back, ie $\mathbf{k}_1^{\mathrm{H}} = -\mathbf{k}_2^{\mathrm{H}}$. This allows one to compute the energies of the particles in this frame:

$$E_1 = \frac{s + t - m_p^2}{2\sqrt{s_{\pi\pi}}} \tag{5}$$
$$E_2 = \frac{s - s_{\pi\pi} - m_p^2}{2\sqrt{s_{\pi\pi}}} \tag{6}$$
$$E_q = q = \frac{s_{\pi\pi} - t}{2\sqrt{s_{\pi\pi}}} \tag{7}$$
$$E_{k_1} = E_{k_2} = \frac{\sqrt{s_{\pi\pi}}}{2}. \tag{8}$$

In this frame, the direction of the recoiling proton (p_2) defines the negative z-axis. Hereby, our momenta three vectors can be written as:

$$\boldsymbol{p_1}^H = |\vec{p}_1|(\sin\theta_1, 0, \cos\theta_1) \tag{9}$$
$$\boldsymbol{p_2}^H = |\vec{p}_2|(0, 0, -1) \tag{10}$$
$$\boldsymbol{k_1}^H = |\vec{k}_1|(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) = -\vec{\boldsymbol{k}}_{\boldsymbol{2}}{}^H \tag{11}$$
$$\boldsymbol{q}^H = |\vec{q}|(-\sin\theta_q, 0, \cos\theta_q) \tag{12}$$

2.1.2 Gottfried-Jackson Frame:

In this frame the photon is directed parallel to the z-axis, so that one can express the three momenta vectors as follows:

$$\boldsymbol{p_1}^{GJ} = |\vec{p}_1|(-\sin\theta_1, 0, \cos\theta_1) \tag{13}$$
$$\boldsymbol{p_2}^{GJ} = |\vec{p}_2|(-\sin\theta_2, 0, \cos\theta_2) \tag{14}$$
$$\boldsymbol{k_1}^{GJ} = |\vec{k}_1|(\sin\theta_{GJ}\cos\phi_{GJ}, \sin\theta_{GJ}\sin\phi_{GJ}, \cos\theta_{GJ}) = -\vec{\boldsymbol{k}}_{\boldsymbol{2}}{}^{GJ} \tag{15}$$
$$\boldsymbol{q}^{GJ} = |\vec{q}|(0, 0, 1). \tag{16}$$

as well as the corresponding energies:

$$E_1 = \frac{s+t-m_p^2}{2\sqrt{s_{\pi\pi}}} \tag{17}$$

$$E_2 = \frac{s-s_{\pi\pi}-m_p^2}{2\sqrt{s_{\pi\pi}}} \tag{18}$$

$$E_q = q = \frac{s_{\pi\pi}-t}{2\sqrt{s_{\pi\pi}}} \tag{19}$$

$$E_{k_1} = E_{k_2} = \frac{\sqrt{s_{\pi\pi}}}{2}, \tag{20}$$

2.2 Pion proton Scattering Amplitude

In our work we assume that the intermediate pion is virtual and space-like,so the classical $\pi^\pm N$ scattering amplitude i.e. $T_\lambda = \bar{u}_\lambda(p_2)\left[A + \frac{1}{2}\gamma_\mu(k_2+k_1)^\mu B\right]u_\lambda(p_1)$ will not hold. Hereby, the scattering amplitude for our case takes the following form:

$$T_\lambda^+ = \bar{u}_\lambda(p_2)\left[A^+ + \frac{1}{2}\gamma_\mu(q-k_2+k_1)^\mu B^+\right]u_\lambda(p_1), \tag{21}$$

$$T_\lambda^- = \bar{u}_\lambda(p_2)\left[A^- + \frac{1}{2}\gamma_\mu(q-k_1+k_2)^\mu B^-\right]u_\lambda(p_1). \tag{22}$$

Where A and B are scalar functions in the nucleon resonance region which can be parametrized using a partial wave expansion as follows:

$$\frac{1}{4\pi}A = \frac{\sqrt{s}+m_p}{Z_1^+Z_2^+}f_1 - \frac{\sqrt{s}-m_p}{Z_1^-Z_2^-}f_2, \tag{23}$$

$$\frac{1}{4\pi}B = \frac{1}{Z_1^+Z_2^+}f_1 - \frac{1}{Z_1^-Z_2^-}f_2, \tag{24}$$

Where f_1 and f_2 are called the reduced helicity amplitudes, E_i are the nucleon momenta and $Z_i^\pm = \sqrt{E_i^{\mathrm{CM}} \pm m_p}$. The partial wave decomposition:

$$f_1 = \frac{1}{\sqrt{|\mathbf{p}_1^{\mathrm{CM}}||\mathbf{p}_2^{\mathrm{CM}}|}}\sum_{l=0}^{\infty} f_{l+}(s)P'_{l+1}(z) - \frac{1}{\sqrt{|\mathbf{p}_1^{\mathrm{CM}}||\mathbf{p}_2^{\mathrm{CM}}|}}\sum_{l=2}^{\infty} f_{l-}(s)P'_{l-1}(z) \tag{25}$$

$$f_2 = \frac{1}{\sqrt{|\mathbf{p}_1^{\mathrm{CM}}||\mathbf{p}_2^{\mathrm{CM}}|}}\sum_{l=1}^{\infty}\left[f_{l-}(s) - f_{l+}(s)\right]P'_l(z) \tag{26}$$

where $P'_l(z) = \frac{d}{dz}P_l(z)$ are the first derivatives of the Legendre polynomials, and $z = \cos(\theta)$.

3. Moments of Angular Distribution

We seek to validate our theoretical model by comparison to the CLAS dataset. Thus we follow the conventions for angular moments given in Ref. [2]as:[2]

$$H^0(L,M) = \frac{1}{2\pi}\int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi \int_0^{2\pi} d\Phi I(\Omega,\Phi) d^L_{M0}(\theta)\cos M\phi \tag{27}$$

$$H^1(L,M) = \frac{1}{\pi P_\gamma}\int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi \int_0^{2\pi} d\Phi I(\Omega,\Phi) d^L_{M0}(\theta)\cos M\phi \cos 2\Phi. \tag{28}$$

The intensity vector is defined as:

$$I^0(\Omega) = \frac{\kappa}{2}\sum_{\lambda,\lambda_1,\lambda_2} \mathcal{M}_{\lambda;\lambda_1\lambda_2}(\Omega)\mathcal{M}^*_{\lambda;\lambda_1\lambda_2}(\Omega) \tag{29}$$

$$I^1(\Omega) = \frac{\kappa}{2}\sum_{\lambda,\lambda_1,\lambda_2} \mathcal{M}_{-\lambda;\lambda_1\lambda_2}(\Omega)\mathcal{M}^*_{\lambda;\lambda_1\lambda_2}(\Omega) \tag{30}$$

$$I^2(\Omega) = i\frac{\kappa}{2}\sum_{\lambda,\lambda_1,\lambda_2} \lambda\mathcal{M}_{-\lambda;\lambda_1\lambda_2}(\Omega)\mathcal{M}^*_{\lambda;\lambda_1\lambda_2}(\Omega) \tag{31}$$

where $0 < P_\gamma < 1$ is the degree of linear polarization of the photon beam and the phase space factor κ i.e.:

$$\kappa = \frac{1}{(2\pi)^3}\frac{1}{4\pi}\frac{1}{2\pi}\frac{\lambda^{1/2}(s_{\pi\pi}, m_\pi^2, m_\pi^2)}{16\sqrt{s_{12}}(s-m_p^2)^2}\frac{1}{2} \tag{32}$$

We have computed these moments at $E_\gamma = 8.5GeV^2$ in both frames for different LM, however we will show only a sample of these moments i.e. Figs.2,3. In both figures, the Helicity frame results

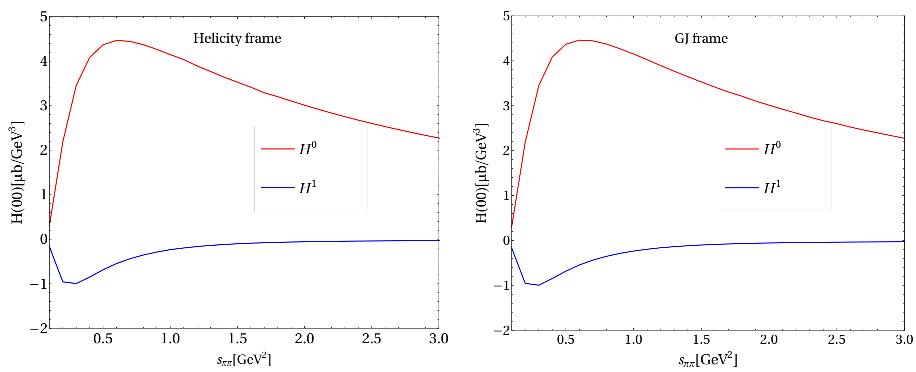

Figure 2: $\pi^+\pi^-$ Moments H^0 (red) and H^1 (blue) for $LM = 00$ at $E_\gamma = 8.5GeV^2$ in both frames.

are on the left while the Gottfried-Jackson frame results are on the right. One can notice that both moments H^0 and H^1 are exactly the same in both frames for the (00) combination in Fig.2. This is definitely an expected results since these moments are proportional to the cross section which means that they are Lorentz invariant and hence frame independent and that is not the case for other combinations as shown in Fig.3.

[2]These moments are related to the Y_{LM} defined in Ref. [3] via $Y_{LM} = 2\pi\sqrt{2L+1}H^0_{LM}$.

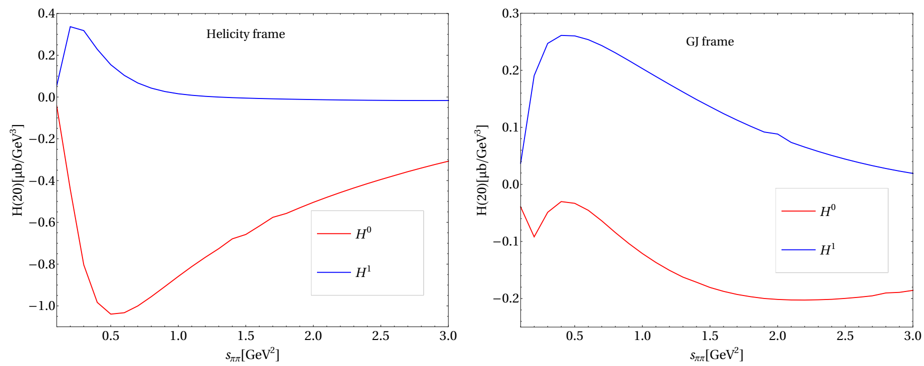

Figure 3: $\pi^+\pi^-$ Moments H^0 (red) and H^1 (blue) for $LM = 20$ at $E_\gamma = 8.5GeV^2$ in both frames.

4. Summary

We have described the model used in studying double pion photoproduction process and computed the Moments of $\pi^+\pi^-$ angular distributions for different LM in two frames. In the mean time we are working on obtaining the prediction for the P-wave projected differential cross section to our model which will be ready soon. So stay tuned!

5. Acknowledgement

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References

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