

Branching probabilities in transverse momentum dependent Monte Carlo algorithms

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We discuss a recently proposed branching algorithm which incorporates transverse momentum dependent (TMD) parton splitting probabilities, and can be used for Monte Carlo event generators based on TMD distributions.

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The parton branching formulation [1] for transverse momentum dependent (TMD) distributions [2, 3] has allowed one to explore the impact of TMD evolution on a variety of collider processes (for instance, deep inelastic scattering (DIS) [4], Drell-Yan (DY) lepton pair production [5], di-jets [6], DY + jets [7, 8]), and investigate the role of transverse momentum recoils [9, 10] and soft-gluon angular ordering [11–13] in Monte Carlo parton showers.

In the above studies, the branching probabilities are given by splitting functions computed in the collinear approximation [14–16]. It has long been known, however, that in regions of phase space sensitive to infrared phenomena [17, 18] contributions beyond the collinear approximation can be relevant. In this article, based on the work [19], we discuss TMD splitting functions defined from high-energy factorization [20]. These splitting functions take into account finite transverse momentum tails in the branching probabilities, which become important when the gluons exchanged in the initial-state (spacelike) parton decay chain carry small longitudinal momentum fractions x , and may be treated as soft. In the following we describe the results of constructing a Monte Carlo branching algorithm [19, 21] which relies on the implementation of TMD splitting functions in the framework [1].

The starting point is the branching kinematics for the space-like parton shower [22], including angular ordering of soft emissions. We describe the parton splitting process at each vertex through the off-shell TMD splitting functions computed in Refs. [20, 23–26], based on high-energy factorization [27]. These splitting functions are positive definite, and interpolate consistently between the collinear limit [14–16] and the high-energy limit [28, 29].

In Ref. [19] we construct corresponding TMD Sudakov form factors, which are obtained from the angular average \bar{P}_{ba} of the TMD splitting functions as

$$\Delta_a(\mu^2, k_\perp^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz z \bar{P}_{ba}^R(z, k_\perp^2, \mu'^2)\right), \quad (1)$$

where a, b are flavor indices, μ is the evolution scale, z is the longitudinal momentum fraction, k_\perp is the transverse momentum, and z_M is the soft-gluon resolution scale, possibly dependent on μ [13] according to the angular ordering. Using the unitarity picture as in [1], and requiring four-momentum conservation in the parton decay chain, we obtain branching equations for the TMD parton distributions $\tilde{\mathcal{A}}_a$ [3] of the form

$$\begin{aligned} \tilde{\mathcal{A}}_a(x, k_\perp^2, \mu^2) &= \Delta_a(\mu^2, k_\perp^2) \tilde{\mathcal{A}}_a(x, k_\perp^2, \mu_0^2) + \\ &\sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2} \int_x^1 dz K_{ab}(z, \mu'_\perp, k_\perp, \mu, z_M) \tilde{\mathcal{A}}_b\left(\frac{x}{z}, (k_\perp + (1-z)\mu'_\perp)^2, \mu'^2\right). \end{aligned} \quad (2)$$

where the kernel K_{ab} contains the TMD splitting functions, Sudakov form factors and phase space constraints.

With the TMD splittings and form factors, in Eq. (2) we aim at a combined treatment of small- x and Sudakov contributions to parton evolution. This is relevant to describe the exclusive structure of jet final states at high energies, see for instance [30–33]. Other approaches to the treatment of Sudakov and small- x effects have recently been investigated, see e.g. the study [34]. The distinctive feature of the approach in Ref. [19] is that it works at the level of unintegrated, k_\perp -dependent splitting functions which factorize in the high-energy limit and control the summation of small- x

logarithmic contributions to the evolution. These splitting functions are then used in the branching algorithm, where they are integrated to construct the new Sudakov factors. As a result, the TMD distributions fulfill integral relations expressing the momentum sum rules.

To illustrate the effects of the branching evolution, in Fig. 1 we solve Eq. (2) by numerical Monte Carlo techniques for given boundary conditions, which we take to be the TMD parameterizations [4] at $\mu_0 = 1.4$ GeV. (Other parameterizations available e.g. in the library [2] could also be used for the purpose of this illustration.) The solid magenta curves in the top panels of Fig. 1 show the x dependence of the gluon and down-quark TMD distributions evolved to $\mu = 100$ GeV and integrated over k_\perp^2 , while the solid magenta curves in the bottom panels show the k_\perp dependence of the gluon and down-quark distributions at $\mu = 100$ GeV for a fixed value of x . For comparison, in Fig. 1 we also plot the results which are obtained with the same distributions at scale μ_0 but without including any k_\perp dependence in the splitting kernels, that is, with the purely collinear splitting kernels (dashed red curves), and the results which are obtained by including the k_\perp dependence of splitting functions in resolvable emissions only (dotted blue curves). In contrast to the full result and the result with purely collinear kernels, the model with the k_\perp dependent splitting functions in resolvable emissions only does not satisfy momentum sum rules, which leads to a significant departure from the full result. We see that the influence of the TMD splitting kernels on evolution is significant especially for low x , gives rise to a change in the k_\perp and x shapes of the distributions and does not disappear completely after integration over k_\perp .

The numerical implementation of an approach which both includes the TMD splitting functions and satisfies the momentum sum rules is one of the main achievements of this work. The construction of a full event generator which uses this approach, e.g. by extending the methods of [35] to the small- x phase space [36], will be the subject of future work. Such an event generator could be compared with existing small- x Monte Carlo generators, e.g. [37–45]. Also, fits of TMDs to experimental data based on the new evolution equation, using the xFitter platform [46, 47], are in progress.

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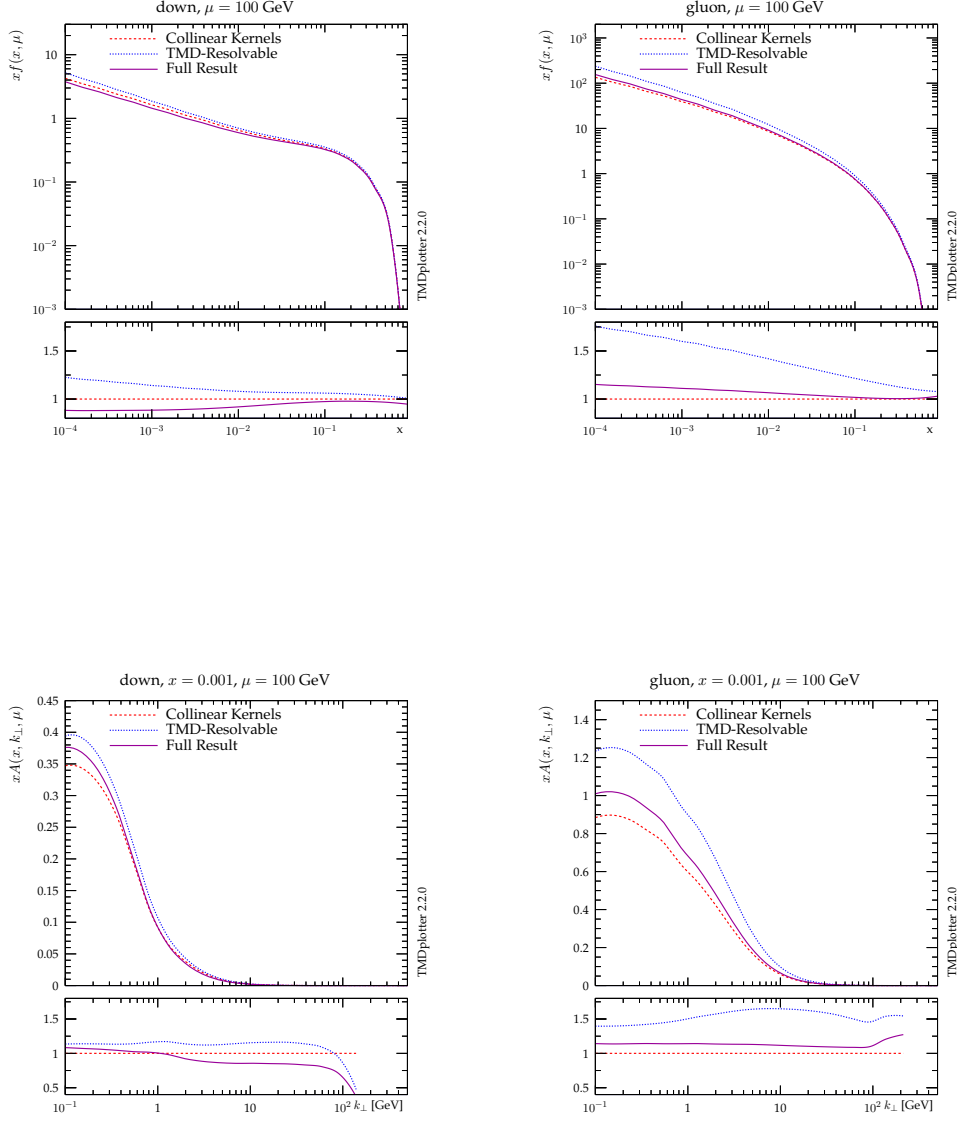


Figure 1: TMD (bottom) and integrated (top) distributions evolved by using the new branching equations with k_{\perp} -dependent splitting kernels (solid magenta curves), compared with the contribution from k_{\perp} -dependent splittings in resolvable emissions only (dotted blue curves) and the result of evolution with purely collinear splitting kernels (dashed red curves).

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