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Investigating vector boson scattering: A fully gauge-invariant study

Bernd Riederer* **and Axel Maas**

Institute of Physics, NAWI Graz, University of Graz,
Universitätsplatz 5, Graz, Austria

E-mail: bernd.riederer@uni-graz.at, axel.maas@uni-graz.at

Vector boson scattering (VBS) plays a central role in the search for new physics at collider experiments such as ATLAS and CMS at the LHC. Usually predictions for this kind of processes are obtained using perturbative approaches in fixed gauges. Here we present a fully gauge-invariant study of VBS in the scalar-channel involving a SM-like Higgs with finite extent. To this end, we combine results obtained in a reduced SM setup from (augmented) perturbation theory with those from non-perturbative lattice simulations.

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*Speaker

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1. Introduction

Vector boson scattering (VBS) has gained a lot of interest in the recent years, especially as a possible path to search for physics beyond the standard model (BSM) [1, 2]. Within the standard model VBS can be very well described using only the electroweak sector in combination with the Brout-Englert-Higgs (BEH) effect. In a standard perturbative framework scattering quantities, like differential or integrated cross-sections as well as phase-shifts, can be reliably obtained within this subset of the SM [2]. Modifications to VBS from next-to-leading-order (NLO) effects of additional SM-processes, like QCD, influence these quantities by $\sim$10 % at the LHC, see e. g. [3].

In perturbation theory (PT) the elementary fields of the theory are considered as the physical degrees of freedom. This cannot be the case [4], since they are charged under the weak gauge group and thus gauge-dependent and unphysical [4]. Instead, it is necessary to choose a completely gauge-invariant approach to obtain physical scattering quantities. As it turns out this requires using composite objects (i.e. bound states) as the elementary degrees of freedom, see [5] for a review.

Here we focus on the fully gauge-invariant description of VBS for a reduced standard model setup. We introduce the necessary framework for the gauge-invariant description in section 2, the Fröhlich-Morchio-Strocchi-mechanism (FMS) [4] and augmented perturbation theory (APT) [5–7]. From this the expected modifications of the scattering observables due to the finite extent of the involved particles are obtained. Additionally, we also show results from lattice simulations. The approaches are then collected in section 3 and compared. This gives us finally a clear picture of VBS. For a more detailed discussion see [8].

2. A Higgs with finite extent

A fully gauge-invariant approach to scattering processes requires using composite objects as the elementary degrees of freedom. This lies in the nature of the BEH construction itself [5]. To illustrate the issue consider a scalar field ϕ in the fundamental representation of the gauge group, in the electroweak sector $SU(2)_W$. This field is further restricted by a potential of the form

$$V(\phi) = \lambda\left(\phi^\dagger\phi - f^2\right)^2, \tag{1}$$

which is invariant under the gauge transformation $G(x)$, with $\phi(x) \to G(x)\phi(x)$. The potential has a non-trivial minimum at $\phi^\dagger\phi = f^2$. For the usual perturbative procedure of the BEH effect [5, 9] one needs to select a particular minimum of the potential by fixing the gauge, e.g. 't Hooft gauge, followed by a shift of the field according to $\phi \to v + \eta$, where $|v| = f$ is the vacuum expectation value (VEV). This results in mass terms at tree-level for the gauge-bosons and is commonly called "*spontaneous gauge-symmetry breaking*".

Consequently, the elementary fields are used to obtain cross-sections by calculating the corresponding matrix elements, see e.g. [9]. The problem with this approach is that the shift in the BEH effect requires gauge-fixing, which indeed is the only possibility to do so. So in fact these elementary fields are gauge-dependent, which renders these states unphysical [4]. This problem is perturbatively circumvented by applying a Becchi-Rouet-Stora-Tyutin (BRST) construction [9] to identify the physical degrees of freedom. Beyond perturbation theory this construction does not hold anymore due to the existence of Gribov copies even at arbitrary weak coupling [5, 10].

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The only remaining way out is therefore to use inherently gauge-invariant (i.e. composite) operators as the physical degrees of freedom. For the process of VBS the two most relevant operators with the correct spin and parity quantum numbers J^P of the Higgs- and gauge-bosons are

$$O^H = O^{0^+} = \phi^\dagger \phi \qquad\qquad O^{W\,a}_\mu = O^{1^-\,a}_\mu = \phi^\dagger D^a_\mu \phi\,, \tag{2}$$

with D^a_μ the covariant derivative. Since the physical degrees of freedom are described by composite operators we see that the BEH effect requires them to be bound states, and thus having a non-zero radius. They are not point-like objects anymore as in standard PT.

To obtain scattering quantities for these operators it seems now unavoidable to use nonperturbative methods, like it is done in QCD. However, when considering instead the usual approach to the BEH effect this raises the question of why it agrees so strikingly well with experimental results [9] while neglecting the inherently nonperturbative structure. This suggests some correspondence between the usual perturbative treatment and the fully gauge-invariant approach.

Consider the weak sector of the SM, which contains exactly these building blocks. Our previous arguments suggest that instead of elementary fields the operators of eq. (2) should be used as asymptotic states to obtain correlators, which needs nonperturbative methods in principle. However, due to the BEH-effect it is possible to augment perturbation theory by an additional step that preserves gauge-invariance, while still allowing perturbative access to the quantities of interest. The full procedure will therefore be called augmented perturbation theory (APT) and consists of two steps: the FMS-expansion [4, 5] followed by usual PT.

As an example we take the propagator of a physical Higgs-boson given by $\left\langle O^H(x)^\dagger O^H(y)\right\rangle$. The first step of APT is to insert the usual BEH split in a convenient gauge. This leaves us with a sum of (individually gauge-dependent) correlation functions, e.g.

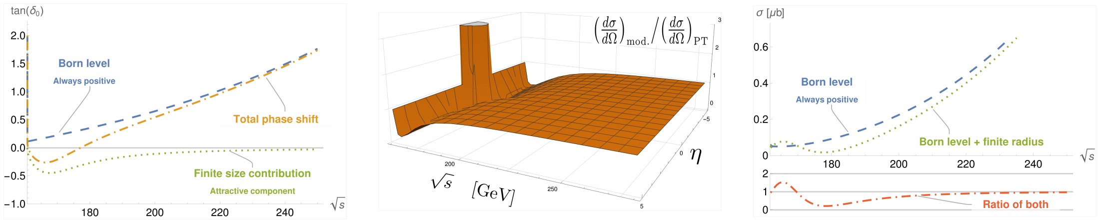
$$\left\langle \left[\phi^\dagger\phi\right](x)^\dagger \left[\phi^\dagger\phi\right](y)\right\rangle = \left\langle [v\eta](x)^\dagger [v\eta](y)\right\rangle + \left\langle \left[\eta^\dagger\eta\right](x)^\dagger [v\eta](y) + (x)\leftrightarrow(y)\right\rangle + \left\langle \left[\eta^\dagger\eta\right](x)^\dagger \left[\eta^\dagger\eta\right](y)\right\rangle . \tag{3}$$

In a second step a double expansion in v and the other coupling constants can be made. At leading order in v, the propagator of the composite operator O^H therefore coincides to all orders in all other couplings with the elementary Higgs propagator $\left\langle \eta(x)^\dagger \eta(y)\right\rangle$, and especially has the same mass and width [7]. This procedure can be applied to any correlation function, and also to matrix elements as will be seen in section 3. Due to the finite extent of the observable particles, modifications to scattering quantities from off-shell contributions are expected compared to usual PT.

So far we have motivated that the gauge-invariant description of weakly interacting particles in the SM should give them some finite radius. Although, this will not change masses and decay-widths of the particles involved in VBS it still may modify cross-sections. In addition, some BSM models, like composite Higgs, directly introduce a finite extent to the particles involved in the VBS process. Therefore, regardless of the previous motivations, it is worthwhile to consider here what modifications are to be expected from a non-vanishing radius of the involved physical particles.

In principle to get measurable predictions it is needed to obtain (differential) cross-sections from fundamental theory. Therefore, experiment and theory can be connected via the equation

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s}|\mathcal{M}|^2 \tag{4}$$



(a) phase shift **(b)** differential cross-section **(c)** integrated cross-section

Figure 1: Expected modifications to VBS for scattering properties in the elastic region due to a Higgs with finite extent. Obtained for a reduced SM-setup with a SM-like Higgs and scattering length $a_0^{-1} \approx -40\,\text{GeV}$.

with $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$ the differential cross-section, $\sqrt{s}$ the center of mass energy and $\mathcal{M}$ the transition matrix. This matrix can be obtained for any theory and process from the possible exchange diagrams and corresponding Feynman rules up to arbitrary order in (A)PT [5, 6, 9]. Here we are explicitly interested in VBS in the scalar channel which additionally requires a partial wave analysis. This can be achieved by deconstructing the matrix according to

$$\mathcal{M} = 16\pi \sum_J (2J+1) f_J P_J(\cos\theta), \tag{5}$$

$$f_J = \frac{1}{32\pi(2J+1)} \int_{-1}^{1} \mathcal{M}\, P_J(\cos\theta)\,\mathrm{d}(\cos\theta) = e^{\mathrm{i}\,\delta_J}\sin(\delta_J) \quad \text{and} \quad \tan(\delta_J) = \frac{f_J}{1+\mathrm{i}\, f_J}, \tag{6}$$

with f_J the partial transition amplitude, P_J the Legendre polynomials and δ_J the phase shift. For VBS at the here investigated Born level it is additionally necessary to perform a reunitarization [8, 11], requiring to replace the initial f_J by $1/(\mathrm{Re}(1/f_J) - \mathrm{i})$.

From eqs. (5) and (6) it can be seen that the phase shift δ_J in the respective partial wave fully characterizes the scattering process. Independent of the perturbative level, the finite extent of the particles is therefore going to modify the phase shift and the transition amplitude by

$$\tan(\delta_J) \longrightarrow \tan(\delta_J) - \tan(\Delta\delta_J) \quad \text{and} \quad f_J \longrightarrow f_J - \frac{\tan(\Delta\delta_J)}{[\tan(\delta_J)+\mathrm{i}][\tan(\Delta\delta_J) - \tan(\delta_J) - \mathrm{i}]} = f_J - \Delta f_J \tag{7}$$

respectively. The transition matrix is consequently also split into $\mathcal{M} \rightarrow \mathcal{M}_{f_J} - \mathcal{M}_{\Delta f_J}$. The ratio of the modified differential cross-section to the one obtained from (A)PT changes thus to

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{mod.}} \Bigg/ \left(\frac{d\sigma}{d\Omega}\right)_{\text{PT}} = \frac{\left|\mathcal{M}_{f_J} - \mathcal{M}_{\Delta f_J}\right|^2}{\left|\mathcal{M}_{f_J}\right|^2} = \left|\frac{\left(\mathcal{M}_{f_J} - \mathcal{M}_{\Delta f_J}\right)^2}{\mathcal{M}_{f_J}^2}\right| = \left|1 - \frac{2\mathcal{M}_{\Delta f_J}}{\mathcal{M}_{f_J}}\left(1 - \frac{\mathcal{M}_{\Delta f_J}}{2\mathcal{M}_{f_J}}\right)\right|. \tag{8}$$

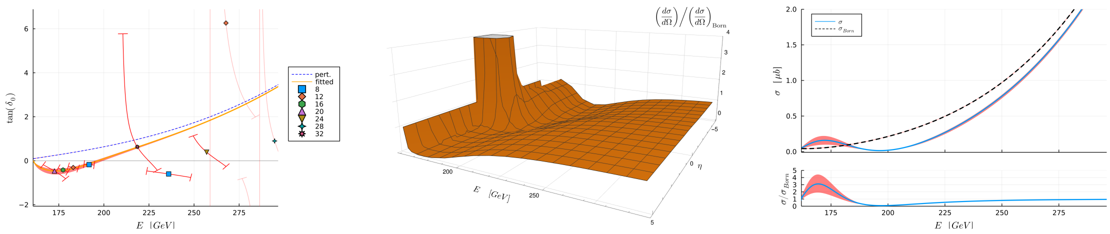
Finally, the influence of the finite extent close to the elastic threshold can be parameterized conveniently by introducing the scattering length a_0

$$\tan(\Delta\delta_J(s)) \approx -a_0\sqrt{s - 4m_W^2}\,, \tag{9}$$

which is negative for a particle with finite extent and $a_0 \geq 0$ for point-like objects.

In fig. 1 we show the expected modifications of the different quantities for the VBS process with a finite-sized Higgs particle. The perturbative values have been obtained at Born-level for a reduced SM-setup with $m_W = m_Z = 80.375\,\text{GeV}$ and $m_H = 125\,\text{GeV}$. Modifications appear only close to the threshold region and become negligible for higher momenta. Therefore, the differential and integrated cross-section in figs. 1b and 1c respectively, show the typical profile for probing particles with some finite extent. We see suppressed forward- and backward scattering and enhanced scattering around zero rapidity η for small center-of-mass energies. This gives a qualitative picture.

3. Nonperturbative results



(a) phase shift **(b)** differential cross-section **(c)** integrated cross-section

Figure 2: Phase shift from lattice simulations and corresponding differential/integrated cross-section as a function of energy and pseudo-rapidity η. The differential cross-section is normalized to the APT prediction.

We study now the same process using lattice simulations, see [8] for details, to compare it with the analytic expectations obtained in section 2 and to check the reliability of the FMS-approach. The characteristics of this theory required us to simulate at very coarse lattices and relatively large weak-coupling compared to the SM.

The main obstacle when comparing results from (A)PT with lattice simulations is that all possible initial states on the lattice will mix. In the case of VBS this means that the scattering matrix needs to be constructed from all possible two 1^- states in an s-wave with zero total momentum and net-zero weak/custodial charge, i.e. $W^\pm W^\mp \to W^\pm W^\mp$, $W^\pm W^\mp \leftrightarrow ZZ$[1] and $ZZ \to ZZ$. This results in a sum over all 81 possible full 4-point vertices of the vector-operator from (2) and yields a perturbative expression [8] for the transition matrix that can be compared to the lattice results.

In the simulation, we obtained a mass for the Higgs boson of $m_H = 148^{+6}_{-20}\text{GeV}$. From the considerations in section 2 we expect that this state is described by a particle with non-vanishing extent, and would result in a negative phase shift close to the threshold. In fig. 2a we see this expected behavior with the data lying significantly and consistently below zero close to the threshold. In addition, the naive perturbative prediction does not agree with the data at all here. Therefore, we used the method as described in section 2 to include the finite extent of the Higgs-boson and obtained a scattering length of roughly $-40\,\text{GeV}$. Remarkably this result is in agreement with a previous investigation of the weak radius for the physical vector bosons [12], although using completely different techniques. Figures 2b and 2c show the corresponding differential and integrated cross-section. Here we see the picture that we have predicted in section 2 for a Higgs with a finite radius. Therefore, lattice simulations indeed support that in the scalar channel a bound state appears, described e. g. by eq. (2), rather than the elementary field itself. It is thus possible to study this from deviations of VBS cross-sections at experiments, at least in principle.

[1]Note that the Z is degenerate with the $W^\pm$-bosons in the reduced SM-setup we are using.

4. Conclusions and outlook

We have presented a fully gauge-invariant study of the VBS process. To get a conclusive picture we tackled the problem with two different approaches once using augmented perturbation theory (APT) and once using lattice simulations. The gauge-invariant approach requires using manifestly gauge-invariant operators as initial and final states rather than elementary fields, as is usually done in PT. However, this additionally leads to the asymptotic states being bound-states with a non-vanishing extent. Therefore, this allows us to simultaneously probe the theoretical foundations of the standard model, as well as deriving a description for VBS with any kind of bound-states involved, like e.g. a composite Higgs from BSM. We showed that the finite extent modifies VBS close to threshold as expected for bound-state scattering and can thus be used as a possible avenue for experiments to search for new physics. In the non-perturbative approach we have seen a significant deviation of the data from the usual perturbative prediction which further supports the previous statements. By including the finite extent of the Higgs boson we were able to compensate the discrepancy and find again the expected behavior for VBS.

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