PROCEEDINGS OF SCIENCE

Absolute-mass threshold resummation for the production of four top quarks

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We study the effects of soft gluon resummation in the absolute mass threshold limit for the production of four top quarks at the LHC. We obtain predictions for the cross section at next-to-leading logarithmic (NLL) accuracy, additionally considering $O(\alpha_s)$ non-logarithmic contributions that do not vanish at threshold. The numerical results show a reduction of the theoretical error associated with the scale uncertainty by up to a factor of 2.

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1. Introduction

The production of four top quarks ($t\bar{t}t\bar{t}$) is one of the rarest Standard Model (SM) processes accessible currently at the Large Hadron Collider (LHC) providing an important test of the SM. The cross section receives potential contributions from beyond the Standard Model (BSM) processes, which could lead to an enhancement of the cross section, e.g. [1–9]. Despite its small cross section, the process has been measured at the LHC by the ATLAS [10–12] and the CMS [13, 14] collaborations. In the most recent ATLAS analysis [12] the measured total cross section shows a two standard deviation excess compared to the SM prediction, computed at next-to-leading order (NLO) in Quantum Chromodynamics (QCD) and including NLO electroweak (EW) corrections [15].

Regarding theoretical predictions, NLO QCD corrections to the total cross sections were first calculated in [16], and later in [15, 17]. A combination of NLO QCD and EW corrections has been presented in [15, 18]. The calculation of the next-to-next-to-leading order (NNLO) cross section for such a process is currently out of reach. One can, however, increase the precision of theoretical calculations by taking into account the effects of multiple soft gluon emissions. Such emissions lead to logarithmic terms of the form $\alpha_s^n[\log^m(1-\hat{\rho})/(1-\hat{\rho})]_+$ with $m \leq 2n$, $\hat{\rho} = M^2/\hat{s} = (4m_t)^2/\hat{s}$ and $\sqrt{\hat{s}}$ the partonic center-of-mass energy. These terms appear in the cross section at all orders in α_s. In this work, we aim to extend the precision of theoretical predictions for the production of four top quarks beyond the known NLO by performing soft gluon resummation up to next-to-leading logarithmic (NLL) accuracy, supplemented by non-logarithmic $O(\alpha_s)$ terms that do not vanish at threshold.

In the following, we present results for resummed cross sections in the absolute mass threshold, i.e. $\hat{\rho} \to 1$, using the Mellin-space approach in direct QCD.

2. Resummation at absolute mass threshold

The partonic resummed cross section at next-to-leading-logarithmic (NLL) accuracy in Mellin space can be written as [19, 20]

$$\hat{\sigma}^{\text{NLL}}_{ij\to t\bar{t}t\bar{t}}(N) = \text{Tr}\left[\mathbf{H}_{ij\to t\bar{t}t\bar{t}}(N)\,\mathbf{S}_{ij\to t\bar{t}t\bar{t}}(N+1)\right]\,\Delta_i(N+1)\,\Delta_j(N+1)\,, \tag{1}$$

where the hard piece $\mathbf{H}_{ij\to t\bar{t}t\bar{t}}$ accounts for the hard dynamics and the soft function $\mathbf{S}_{ij\to t\bar{t}t\bar{t}}$ gathers the contributions from soft wide-angle emission. Soft-collinear contributions from the incoming partons are accounted for in the incoming jet functions $\Delta_{i,j}$. The incoming jet functions are universal and well-known, and can be found at NLL accuracy e.g. in Ref. [21].

The evolution of the soft matrix is driven by the soft anomalous dimension (SAD) matrix $\mathbf{\Gamma}$, which can be expanded perturbatively i.e. $\mathbf{\Gamma} = (\alpha_s/\pi)\,\mathbf{\Gamma}^{(1)} + (\alpha_s/\pi)^2\,\mathbf{\Gamma}^{(2)} + \dots$. The one-loop SAD matrix $\mathbf{\Gamma}^{(1)}$ is required for predictions at NLL accuracy. Alongside the hard and the soft functions, the SAD matrix is a matrix in colour space, hence a study of the colour structure is needed. The resulting dimensions of the colour spaces for the $q\bar{q}$- and gg-initiated channels are 6 and 14 respectively. We work in a colour basis where $\mathbf{\Gamma}^{(1)}$ is diagonal in the absolute mass threshold

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limit, and denote this basis with the subscript R. The one-loop SAD matrices for $N_c = 3$ read

$$2\,\mathrm{Re}\left[\boldsymbol{\Gamma}^{(1)}_{R,q\bar{q}\to t\bar{t}t\bar{t}}\right] = \mathrm{diag}\Big(0,0,-3,-3,-3,-3\Big), \tag{2}$$

$$2\,\mathrm{Re}\left[\boldsymbol{\Gamma}^{(1)}_{R,gg\to t\bar{t}t\bar{t}}\right] = \mathrm{diag}\Big(-8,-6,-6,-4,-3,-3,-3,-3,-3,-3,-3,-3,0,0\Big). \tag{3}$$

It can be checked that the elements of the matrix correspond to the negative values of the quadratic Casimir invariants of the respective irreducible representation of the final-state colour structure.

The soft function is expressed in terms of the evolution matrices $\mathbf{U}_{ij\to t\bar{t}t\bar{t}}$ and the boundary condition $\tilde{\mathbf{S}}_{ij\to t\bar{t}t\bar{t}}$, such that $\mathbf{S}_{ij\to t\bar{t}t\bar{t}} = \bar{\mathbf{U}}_{ij\to t\bar{t}t\bar{t}}\tilde{\mathbf{S}}_{ij\to t\bar{t}t\bar{t}}\mathbf{U}_{ij\to t\bar{t}t\bar{t}}$, with

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}} = \mathrm{Pexp}\left[\int_{\mu_R}^{M/\bar{N}} \frac{dq}{q}\,\boldsymbol{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]. \tag{4}$$

In the R basis, the soft function at NLL reads

$$\mathbf{S}_{R,ij\to t\bar{t}t\bar{t}} = \tilde{\mathbf{S}}_{R,ij\to t\bar{t}t\bar{t}}\exp\left[\frac{\mathrm{Re}\left[\boldsymbol{\Gamma}^{(1)}_{R,ij\to t\bar{t}t\bar{t}}\right]}{\pi b_0}\log(1-2\lambda)\right], \tag{5}$$

with $\lambda = \alpha_s b_0 \log\bar{N}$ and $\bar{N} \equiv N e^{\gamma_E}$.

The hard and soft functions can be expanded perturbatively in powers of α_s: $\mathbf{H}_R = \mathbf{H}^{(0)}_R + \frac{\alpha_s}{\pi}\mathbf{H}^{(1)}_R + \ldots$ and $\tilde{\mathbf{S}}_R = \tilde{\mathbf{S}}^{(0)}_R + \frac{\alpha_s}{\pi}\tilde{\mathbf{S}}^{(1)}_R + \ldots$. The function $\mathbf{H}^{(1)}$ gathers one-loop virtual corrections and non-logarithmic collinear enhancements, which are not collected in the jet functions. The first-order soft function accounts for eikonal corrections to $\tilde{\mathbf{S}}^{(0)}$. To obtain predictions for the cross section at NLL accuracy, only the lowest order terms are needed. The first-order contributions enter formally at NNLL accuracy. The accuracy of the predictions, however, can be improved beyond NLL by including non-logarithmic contributions at $O(\alpha_s)$, i.e. $\mathbf{H}^{(1)}$ and $\tilde{\mathbf{S}}^{(1)}$, dropping their products such that

$$\mathrm{Tr}\left[\mathbf{H}_R\,\tilde{\mathbf{S}}_R\right] = \mathrm{Tr}\left[\mathbf{H}^{(0)}_R\,\tilde{\mathbf{S}}^{(0)}_R + \frac{\alpha_s}{\pi}\mathbf{H}^{(1)}_R\,\tilde{\mathbf{S}}^{(0)}_R + \frac{\alpha_s}{\pi}\mathbf{H}^{(0)}_R\,\tilde{\mathbf{S}}^{(1)}_R\right]. \tag{6}$$

The resulting accuracy is referred to as NLL′.

The resummation-improved predictions for the total cross section at NLO+NLL or NLO+NLL′ accuracy are obtained through matching to the full NLO fixed-order result

$$\begin{aligned}\sigma^{\mathrm{NLO+NLL}^{(\prime)}}_{t\bar{t}t\bar{t}}(\rho) = \sigma^{\mathrm{NLO}}_{t\bar{t}t\bar{t}}(\rho) + \sum_{i,j}\int_C \frac{\mathrm{d}N}{2\pi i}\,\rho^{-N}\, f_i(N+1,\mu_F^2)\, f_j(N+1,\mu_F^2)\\ \times\left[\hat{\sigma}^{\mathrm{NLL}^{(\prime)}}_{ij\to t\bar{t}t\bar{t}}(N) - \hat{\sigma}^{\mathrm{NLL}^{(\prime)}}_{ij\to t\bar{t}t\bar{t}}(N)|_{\mathrm{NLO}}\right],\end{aligned} \tag{7}$$

with $\rho = M^2/s$ the hadronic threshold variable. The last term in the second line of eq. (7) corresponds to the expansion of the resummed cross section truncated at NLO.

3. Numerical Results

We next present numerical results for absolute mass threshold soft gluon resummation applied to the production of four top quarks. We consider LHC collisions at $\sqrt{s} = 13$ TeV, and we make use of $m_t = 172.5$ GeV and LUXqed_plus_PDF4LHC15_nnlo_100 PDF set [22, 23].

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Fixed-order calculations are obtained from aMC@NLO [24, 25]. One-loop virtual contributions needed for $\mathbf{H}^{(1)}$ are also numerically extracted from the aMC@NLO code. We consider NLO cross sections including only QCD corrections, as well as a combination of QCD and EW corrections, with EW corrections up to $O(\alpha^2)$ [15].

In Figure 1 we present the scale dependence of the total cross section at several accuracies. We set the renormalization and factorization scale equal and vary them simultaneously by factors of two around the central scale $\mu_0 = 2m_t$. The NLL resummed cross section calculated at the central scale is higher by 4%, and the scale dependence is mildly reduced, compared to the NLO cross section. With the inclusion of $O(\alpha_s)$ contributions, the NLO+NLL′ cross section leads to an increase of 16% with respect to the NLO QCD result, and the NLO (QCD+EW)+NLL′ cross section is 15% higher than the full NLO (QCD+EW) cross section. The scale dependence of the cross sections with NLL′ corrections is significantly reduced.

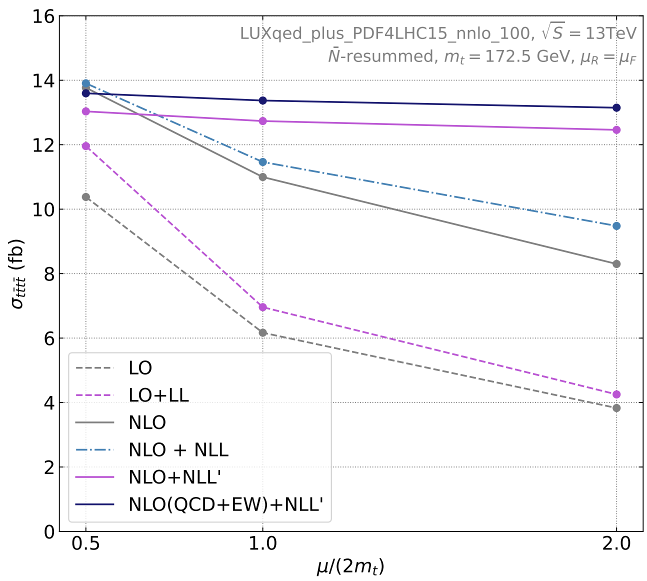

Figure 1: Scale dependence of the LO, NLO, LO+LL, NLO+NLL, NLO+NLL′ and NLO(QCD+EW)+NLL′ cross sections at $\sqrt{s}$ = 13 TeV. LO and NLO include only QCD effects, while NLO(QCD+EW) includes as well electroweak corrections. The renormalization and factorization scale are set to the same value, and are varied with respect to the central scale $\mu_0 = 2m_t$.

To see whether the stability under scale variations is an artifact of $\mu_R = \mu_F$, we also study the 7-point scale variation. In Figure 2 we show the central value of the cross section for the fixed-order calculations and the resummation-improved results. In red we present the theoretical error associated with the scale uncertainty. It can be seen that the associated scale uncertainty is highly reduced, even up to a factor of two, when including NLL′ resummation effects compared to fixed-order calculations. As an estimate of the PDF error we employ the PDF error of the NLO (QCD+EW) result, which amounts to ±6.9%, and it is calculated by scanning over all the members of the PDF set.

Our state-of-the-art prediction for the $t\bar{t}t\bar{t}$ production at $\sqrt{s}$ = 13 TeV, including soft gluon

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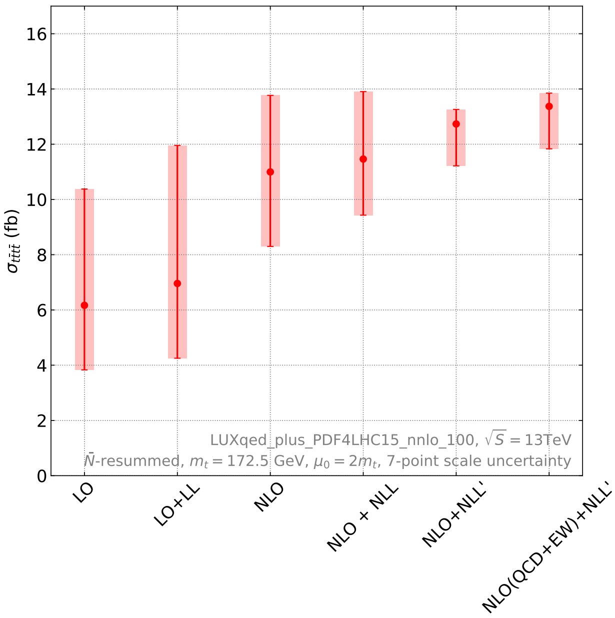

Figure 2: Total cross sections for the production of four top quarks at $\sqrt{s}$ = 13 TeV for fixed-order calculations and resummation-improved results. The associated scale uncertainty presented is computed with the 7-point scale variation method.

resummation effects, reads

$$\sigma_{t\bar{t}t\bar{t}}^{\mathrm{NLO(QCD+EW)+NLL'}} = 13.37(2)\,^{+3.6\%}_{-11.4\%}\,(\mathrm{scale})\,^{+6.9\%}_{-6.9\%}\,(\mathrm{pdf})\ \mathrm{fb}.$$

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