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Two-loop QED corrections to the di-muon production in electron-positron annihilation, and related processes

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This contribution aims at elucidating the method we employed in the calculation of double-virtual interferences of di-muon production via electron-positron annihilation at Next-to-Next-to-Leading Order (NNLO) in Quantum Electrodynamics (QED), and heavy-quark pair production via light-quark annihilation at NNLO Quantum Chromodynamics (QCD).

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1. Introduction

The Muon g-2 collaboration at Fermilab has recently measured the anomalous magnetic moment of the muon, showing compatibility with the earlier findings obtained at Brookhaven National Lab [3–5]. The anomalous magnetic moment of the muon, $(g-2)_\mu$, shows a $4.2\,\sigma$ deviation from the Standard Model (SM) prediction [6]. Recently, a novel experiment, MUonE [7, 8], has been proposed at CERN, with the goal of measuring the running of the effective electromagnetic coupling at low momentum transfer in the space-like region. To extract the running of the effective electromagnetic coupling from the experimental data, the pure perturbative contribution to the e-μ elastic scattering cross section in Quantum Electrodynamics (QED) must be controlled at least up Next-to-Next-to-Leading Order (NNLO) in the fine-structure constant [9].

There are two processes that can be related to the e-μ elastic scattering: its cross-related QED di-muon production via electron-positron annihilation and its analogous in Quantum Chromodynamics (QCD) $t\bar{t}$ production via quark-antiquark annihilation. The latter process was known numerically and partial analytical results were available [10–16].

In the following, we describe the method we employed to obtain the fully analytical expression for the two-loop correction to $e^+e^- \to \mu^+\mu^-$ at NNLO QED and the $q\bar{q} \to t\bar{t}$ at NNLO QCD. Plots showing the finite part of such corrections will be shown. This proceeding resumes the results of Refs. [1] and [2].

2. Four-fermion scattering process

We consider the process

$$f(p_1) + \overline{f}(p_2) \to F(p_3) + \overline{F}(p_4). \tag{1}$$

where $p_1^2 = p_2^2 = 0$ and $p_3^2 = p_4^2 = M^2$. The kinematic invariants are define as $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_2 - p_3)^2$, where $s + t + u = 2M^2$ because of momentum conservation. In particular, we consider the case when $f = e^-, F = \mu^-$ in QED and $f = q, F = t$ in QCD.

The scattering amplitude $\mathcal{A}$ for this process admits a perturbative expansion with respect to the coupling constant α, such that

$$\mathcal{A} = 4\pi\alpha \left[\mathcal{A}^{(0)} + \left(\frac{\alpha}{\pi}\right) \mathcal{A}^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 \mathcal{A}^{(2)} + O(\alpha^3) \right]. \tag{2}$$

In this context, α can be either the electromagnetic coupling or the strong coupling (for energies such that QCD can be treated perturbatively).

We are interested in the following interference terms:

$$\mathcal{M}^{(0)} = \frac{1}{4} \sum_{\substack{\text{colours} \\ \text{spins}}} |\mathcal{A}^{(0)}|^2, \qquad \mathcal{M}^{(n)} = \frac{1}{4} \sum_{\substack{\text{colours} \\ \text{spins}}} 2\mathrm{Re}\left(\mathcal{A}^{(0)*} \mathcal{A}^{(n)}\right), \tag{3}$$

where $\mathcal{A}^{(0)}$ is the Born term. These interferences can be decomposed into gauge invariant contributions w.r.t. the number of light and heavy flavours (n_l and n_h respectively) and the number of colours N_c as follows:

$$
\begin{aligned}
\mathcal{M}^{(1)} &= A^{(1)} + n_l B_l^{(1)} + n_h C_h^{(1)} \\
\mathcal{M}^{(2)} &= A^{(2)} + n_l B_l^{(2)} + n_h C_h^{(2)} + n_l^2 D_l^{(2)} + n_l n_h E_{hl}^{(2)} + n_h^2 F_h^{(2)},
\end{aligned} \tag{4}
$$

for the QED process, and

$$
\begin{aligned}
\mathcal{M}^{(1)} &= 2(N_c^2 - 1)\left(N_c A^{(1)} + \frac{B^{(1)}}{N_c} + n_l C_l^{(1)} + n_h C_h^{(1)} \right) \\
\mathcal{M}^{(2)} &= 2(N_c^2 - 1)\left(A^{(2)} N_c^2 + B^{(2)} + \frac{C^{(2)}}{N_c^2} + n_l D_l^{(2)} N_c + n_h D_h^{(2)} N_c + n_l \frac{E_l^{(2)}}{N_c} + n_h \frac{E_h^{(2)}}{N_c} \right. \\
&\qquad \left. + n_l^2 F_l^{(2)} + n_l n_h F_{hl}^{(2)} + n_h^2 F_h^{(2)} \right),
\end{aligned} \tag{5}
$$

for the QCD one.

3. Evaluating the amplitude

The generation of the bare amplitude $\mathcal{A}_{\rm b}^{(n)}$ is performed with FeynArts [17] and FeynCalc [18]. We identify 6 one-loop and 69 two-loop diagrams for the QED process and 10 one-loop and 218 two-loop diagrams for the QCD one. In general, the amplitudes $\mathcal{A}_{\rm b}^{(n)}$ are Ultraviolet (UV) and Infrared (IR) divergent in four space-time dimensions; in order to regularise such divergences, dimensional regularisation is introduced, promoting the space-time dimension $d = 4 - 2\epsilon$ to be a free parameter. The physical limit $d \to 4$ (i.e. $\epsilon \to 0$) can be taken after the renormalisation procedure.

After computing the explicit interferences of the loop diagrams with the Born amplitude, we perform the spin-colour averaged sum and the Dirac algebra of the numerators, and $\mathcal{M}_{\rm b}^{(n)}$ takes the following expression:

$$
\mathcal{M}_{\rm b}^{(n)} = \int \prod_{i=1}^{n} \frac{d^d k_i}{(2\pi)^d} \sum_G \frac{\mathcal{N}_G(\mathbf{k}, \mathbf{p}, M^2)}{\prod_{\sigma \in G} D_\sigma(\mathbf{k}, \mathbf{p}, M^2)}. \tag{6}
$$

Integrands in Eq. (6) are rational polynomial of the masses and the scalar product between momenta. We apply the so-called Adaptive Integrand Decomposition (AID) [19]. Its key ingredient is the splitting of the d-dimensional space-time into the sub-space generated by the external momenta and its transverse one: $d = d_{\parallel} + d_{\perp}$. AID is completely automated in the Mathematica package AIDA [20], which carries out the integrand decomposition down to a combination of Feynman Integrals whose numerator depend on irreducible scalar products only.

Since integra*ls* have more relations than integra*nds*, Integration-by-Parts identities (IBPs) [21, 22] drastically reduce the number of Feynman integrals down to a minimal set of Master Integrals (MIs) $\mathbf{I}^{(n)}$, such that

$$
\mathcal{M}_{\rm b}^{(n)} = \mathbb{C}^{(n)} \cdot \mathbf{I}^{(n)}, \tag{7}
$$

where $\mathbb{C}^{(n)}$ is a vector of rational function depending on s, t, M^2 and the dimension d. IBPs are generated and applied on the integrals of Eq. (6) by means of public code Reduze [23].

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The complete set of MIs $\mathbf{I}^{(n)}$ has been computed analytically in [24, 25] by using the differential equation method via Magnus Exponential. $\mathbf{I}^{(n)}$ admits a Laurent expansion around $\epsilon = 0$, where its coefficient are expressed in terms of Generalized PolyLogarithms (GPLs), iteratively defined as

$$G(w_n, \ldots, w_1; \tau) = \int_0^\tau \frac{dt}{t - w_n} G(w_{n-1}, \ldots, w_1; t), \qquad G(w_1; \tau) = \log\left(1 - \frac{\tau}{w_1}\right), \tag{8}$$

where $w_i = w_i(s, t, M^2)$ are called *letters*, and depend only on the physical scales of the process. Notice that, for both QED and QCD processes, the set of MIs is completely analogous.

The interferences $\mathcal{M}_{\mathrm{b}}^{(n)}$ are still UV divergent. Renormalising the bare fermion fields ψ_b and the bare mass M_b in the on-shell scheme, and the bare coupling α_{b} in the $\overline{\mathrm{MS}}$ scheme

$$\psi_{\mathrm{b}} = \sqrt{Z_\psi}\psi, \qquad M_{\mathrm{b}} = \sqrt{Z_M} M, \qquad \alpha_{\mathrm{b}} = Z_\alpha^{\overline{\mathrm{MS}}} \alpha, \qquad Z_i = 1 + \sum_{j=1} \left(\frac{\alpha}{\pi}\right)^j \delta Z_i^{(j)}, \tag{9}$$

one obtains UV finite quantities $\mathcal{A}^{(n)}$ and interferences $\mathcal{M}^{(n)}$ by considering the renormalised amplitude $\mathcal{A}$, such that:

$$\begin{aligned} &\mathcal{A} = Z_f Z_F \mathcal{A}(\alpha_{\mathrm{b}}(\alpha), M_{\mathrm{b}}(\alpha)), \qquad \mathcal{A}^{(n)} = \mathcal{A}_{\mathrm{b}}^{(n)} + \delta\mathcal{A}^{(n)}, \\ &\mathcal{M}^{(n)} = \mathcal{M}_{\mathrm{b}}^{(n)} + \delta\mathcal{M}^{(n)}, \qquad \delta\mathcal{M}^{(n)} = \frac{1}{4} \sum_{\substack{\text{colours} \\ \text{spins}}} \mathrm{Re}(\mathcal{A}_{\mathrm{b}}^{(0)*} \delta\mathcal{A}_{\mathrm{b}}^{(n)}), \end{aligned} \tag{10}$$

where

$$\begin{aligned} \delta\mathcal{A}^{(0)} &= 0, \qquad \delta\mathcal{A}^{(1)} = (\delta Z_\alpha^{(1)} + \delta Z_F^{(1)}) \mathcal{A}_{\mathrm{b}}^{(0)}, \\ \delta\mathcal{A}^{(2)} &= (2\delta Z_\alpha^{(1)} + \delta Z_F^{(1)}) \mathcal{A}_{\mathrm{b}}^{(1)} + (\delta Z_\alpha^{(2)} + \delta Z_F^{(2)} + \delta Z_f^{(2)} + \delta Z_\alpha^{(1)} \delta Z_F^{(2)}) \mathcal{A}_{\mathrm{b}}^{(0)} \\ &\quad + \delta Z_M^{(1)} \mathcal{A}_{\mathrm{b}}^{(1,\mathrm{massCT})}, \end{aligned} \tag{11}$$

Explicit expressions for the $\delta Z_i^{(j)}$ can be found in Refs. [1] and [2].

4. Finite terms

The analytical expansion of the renormalised interferences $\mathcal{M}^{(1)}$ and $\mathcal{M}^{(2)}$ are expressed as

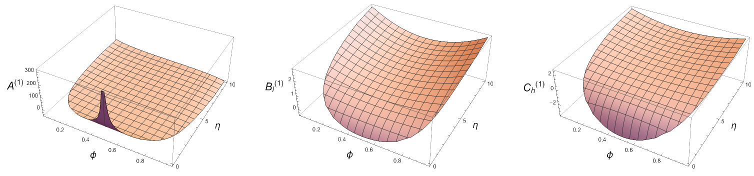
$$\mathcal{M}^{(1)} = \sum_{k=-2}^{1} \mathcal{M}_k^{(1)} \epsilon^k + O(\epsilon^2), \quad \mathcal{M}^{(2)} = \sum_{k=-4}^{0} \mathcal{M}_k^{(2)} \epsilon^k + O(\epsilon). \tag{12}$$

In Figures 1 and 2 we present the plot of their finite part $\mathcal{M}_0^{(1)}$ and $\mathcal{M}_0^{(2)}$ in terms of the variables

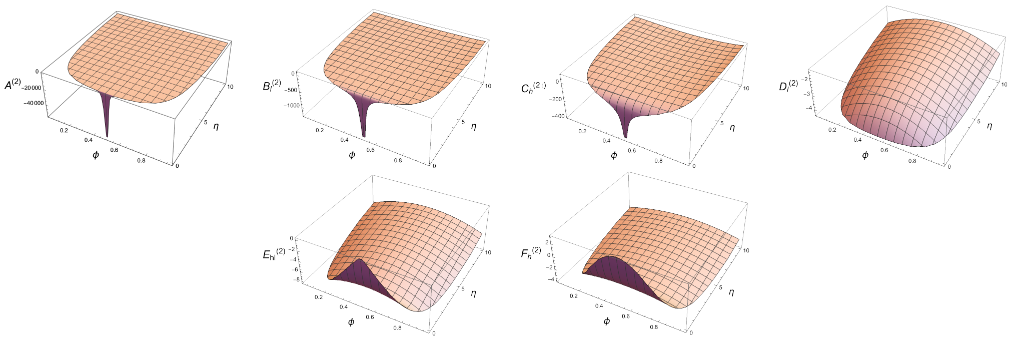
$$\eta = \frac{s}{4M^2} - 1, \qquad \phi = -\frac{t - M^2}{s}. \tag{13}$$

5. Conclusions

We have presented the method we employed in the calculation of the two-loop NNLO QED di-muon production via electron-positron annihilation and the two-loop NNLO QCD $t\bar{t}$-production via quark-antiquark annihilation. We provided plots of the finite part of the NNLO contributions considered. In particular, the latter contributions have been computed on a grid of 1600 points, available by the ancillary files of [2].

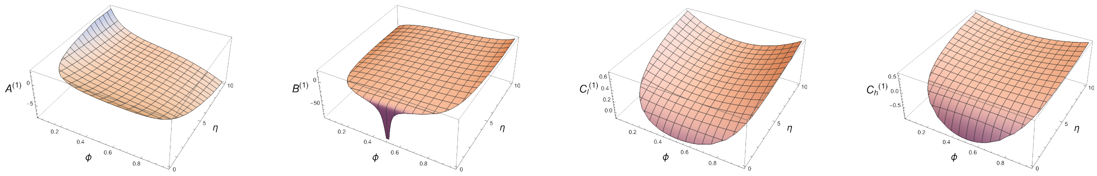


(a)

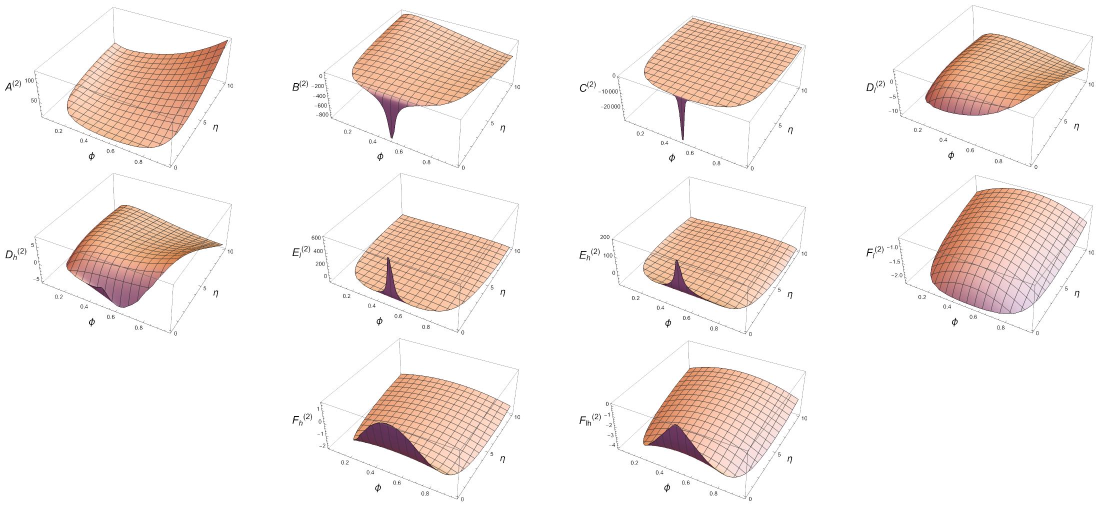


(b)

Figure 1: Three-dimensional plots of the coefficients (finite part) appearing in the decomposition of the renormalized one- (1a) and two-loop (1b) amplitude of $e^+e^- \to \mu^+\mu^-$, defined in Eq. (4).



(a)



(b)

Figure 2: Three-dimensional plots of the coefficients (finite part) appearing in the decomposition of the renormalized one- (2a) and two-loop (2b) amplitude of $q\bar{q} \to t\bar{t}$, defined in Eq. (5).

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