

## SM theoretical predictions for $B^0 \rightarrow \phi \ell^+ \ell^-$ decay

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Among experimental processes for testing precisely the Standard Model (SM) and searching for possible physics beyond the SM, rare bottom-hadron decays induced by the  $b \rightarrow s$  and  $b \rightarrow d$  flavor-changing neutral currents attract a lot of attention. Radiative and semileptonic  $B$ -meson decays with  $\rho^0$ ,  $\omega$ , and  $\phi$  mesons in the final state like  $B_s^0 \rightarrow \rho^0(\omega)\gamma$ ,  $B_s^0 \rightarrow \rho^0(\omega)\ell^+\ell^-$ ,  $B^0 \rightarrow \phi\gamma$ , and  $B^0 \rightarrow \phi\ell^+\ell^-$ , being mainly of pure annihilation-type topology, are of significant interest as in the SM they are extremely suppressed and New Physics effects can increase substantially their decay widths. Experimental searches of some of them were undertaken at the KEKB and LHC colliders. The upper limit on the radiative decay branching fraction,  $\mathcal{B}(B^0 \rightarrow \phi\gamma) < 1.0 \times 10^{-7}$ , obtained by the Belle collaboration in 2016, was the only one for quite some time. In 2022, the LHCb collaboration put the upper limit on its semileptonic counterpart,  $\mathcal{B}(B^0 \rightarrow \phi\mu^+\mu^-) < 3.2 \times 10^{-9}$ . Here, we consider a theory of the annihilation-type semileptonic  $B^0 \rightarrow \phi\ell^+\ell^-$  decays, where  $\ell$  is a charged lepton, and present SM theoretical predictions for their branching fractions based on the Effective Electroweak Hamiltonian approach for the  $b \rightarrow d\ell^+\ell^-$  transitions. An impact of theoretical models for the  $B$ -meson distribution amplitudes on these decays is also discussed.

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## 1. Introduction

An experimental observation of significant deviations from the SM predictions in rare decays of bottom hadrons may lead to a New Physics discovery. At present, a majority of experimental data on rare  $B$ -meson decays is in good agreement with the SM. In particular, the LHCb, CMS and ATLAS collaborations at the LHC measured the branching fraction,  $\mathcal{B}_{\text{exp}}(B_s \rightarrow \mu^+ \mu^-) = (3.01 \pm 0.35) \times 10^{-9}$  [1], of the ultra-rare annihilation-type  $B_s$ -meson decay which is consistent at  $2\sigma$  with the SM prediction,  $\mathcal{B}_{\text{th}}(B_s \rightarrow \mu^+ \mu^-) = (3.65 \pm 0.23) \times 10^{-9}$  [2]. These measurements are also accompanied by intensive experimental searches of the  $B^0 \rightarrow \mu^+ \mu^-$  decay and PDG presents the following average for its branching fraction:  $\mathcal{B}_{\text{exp}}(B \rightarrow \mu^+ \mu^-) = (0.7^{+1.3}_{-1.1}) \times 10^{-10}$  [1]. Rare semileptonic annihilation-type decays of  $B_{(s)}^0$ -mesons include  $B^0 \rightarrow \phi \ell^- \ell^+$  and  $B_s^0 \rightarrow \rho^0(\omega) \ell^- \ell^+$ , where  $\ell = e, \mu, \tau$  is a charged lepton, under the assumption that the  $\omega$ - and  $\phi$ -mesons are the states with the following quark content:  $\omega = (\bar{u}u + \bar{d}d)/\sqrt{2}$  and  $\phi = \bar{s}s$ . We accept this approximation in the theoretical analysis below. Their radiative counterparts,  $B^0 \rightarrow \phi \gamma$  and  $B_s^0 \rightarrow \rho^0(\omega) \gamma$ , under the same assumption about final mesons, have also annihilation-type topology. Due to the smallness of their branching fractions, experimental limits only on such  $B^0$ -meson decays are known at present:  $\mathcal{B}(B^0 \rightarrow \phi \gamma) < 1.0 \times 10^{-7}$  [3] and  $\mathcal{B}(B^0 \rightarrow \phi \mu^+ \mu^-) < 3.2 \times 10^{-9}$  [4].

The theoretical analysis of radiative annihilation-type  $B^0 \rightarrow \phi \gamma$  and  $B_s \rightarrow \rho^0(\omega) \gamma$  decays, including the  $\omega - \phi$  mixing effect, was undertaken in [5], where predictions for the  $B_s$ -meson indicate a significant contribution from this effect. For semileptonic annihilation-type  $B^0 \rightarrow \phi \ell^+ \ell^-$  decay we present Standard Model predictions, so far, without taking into account  $\omega - \phi$  mixing, which is postponed for another publication. We also estimate a dependence on the choice of theoretical models for the  $B$ -meson distribution amplitudes entering decay width through their first inverse moments.

## 2. The differential branching fraction of $B^0 \rightarrow \phi \ell^+ \ell^-$ decay

Calculations are done in the Effective Electroweak Hamiltonian approach [6, 7]. The effective Lagrangian density for  $b \rightarrow d$  flavor-changing neutral current (FCNC) transition is derived from the SM Lagrangian by integrating out heavy particles — the top quark,  $W$ ,  $Z$  and Higgs bosons as well as photons and gluons with energies  $\sim m_W$ . It can be written as follows:

$$\mathcal{L}_{\text{eff}}(x) = \mathcal{L}_{\text{QED}}(x) + \mathcal{L}_{\text{QCD}}(x) - \mathcal{H}_{\text{weak}}^{b \rightarrow d}(x), \quad (1)$$

where  $\mathcal{L}_{\text{QED}}(x)$  and  $\mathcal{L}_{\text{QCD}}(x)$  are usual QED and QCD Lagrangians for particles left in the theory. The FCNC Hamiltonian,  $\mathcal{H}_{\text{weak}}^{b \rightarrow d}$ , describes the  $b \rightarrow d$  transition:

$$\mathcal{H}_{\text{weak}}^{b \rightarrow d} = -\frac{4G_F}{\sqrt{2}} \sum_{p=u,c} V_{pd}^* V_{pb} \sum_j C_j(\mu) \mathcal{P}_j(\mu) + \text{h. c.}, \quad (2)$$

where  $G_F$  is the Fermi constant,  $C_j(\mu)$  are Wilson coefficients,  $\mathcal{P}_j(\mu)$  are local  $b \rightarrow d$  transition operators, and  $V_{q_1 q_2}$  are the Kabibbo-Kobayashi-Maskawa matrix elements. The standard  $\mathcal{P}_j(\mu)$  basis includes 10 operators [7]. The leading-order contribution to the  $B^0 \rightarrow \phi \ell^+ \ell^-$  decay amplitude is given by the penguin operators  $\mathcal{P}_3$  and  $\mathcal{P}_5$ . This differs from the analysis of the  $B^0 \rightarrow \rho^0 \ell^+ \ell^-$

decay [8, 9], in which contributions from the annihilation diagrams, despite being subdominant, are determined both tree and penguin operators.

In the  $B^0 \rightarrow \phi \ell^+ \ell^-$  decay, the current with the  $b$ - and  $d$ -quarks determines the initial  $B$ -meson, and the other, constructed from the  $s$ -quarks, is related with the final  $\phi$  meson. As a result, the total decay amplitude is factorized as the product of the  $\phi$  meson decay constant,  $f_\phi$ , and the  $B$ -meson wave function defined on the light cone in the limit of a massless  $d$ -antiquark and infinitely heavy  $b$ -quark. This wave function is determined by two  $B$ -meson distribution amplitudes (DAs),  $\varphi_+^B(t)$  and  $\varphi_-^B(t)$ , through the transition matrix element from the meson state to the vacuum [10]:

$$\langle 0 | q_\alpha(z) E(0, z) h_{\nu, \beta}(0) | \bar{B}(v) \rangle = -\frac{if_B m_B}{4} \left[ (1 + \hat{v}) \left\{ \tilde{\varphi}_+^B(t) - [\tilde{\varphi}_+^B(t) - \tilde{\varphi}_-^B(t)] \frac{\hat{z}}{2t} \right\} \gamma_5 \right]_{\beta\alpha}, \quad (3)$$

where  $m_B$  and  $f_B$  are the  $B$ -meson mass and decay constant, respectively,  $t = (vz)$  is the proper time of the  $B$ -meson,  $E(0, z)$  is the Wilson line, which ensures the gauge invariance of the  $B$ -meson interpolation current,  $v^\mu = (1, 0, 0, 0)$  is  $B$ -meson four-velocity in its rest frame,  $z^\mu$  is a light-like separation between quarks. The decay amplitudes include Fourier transforms of the DAs [8, 10], and subleading DA,  $\phi_-^B(\omega)$ , is related to the leading one,  $\phi_+^B(\omega)$ , by the Wandzura-Wilczek relation [10]:

$$\tilde{\varphi}_\pm^B(t) = \int_0^\infty d\omega e^{-i\omega t} \phi_\pm^B(\omega), \quad \phi_-^B(\omega) = \int_\omega^\infty \frac{\phi_+^B(\omega')}{\omega'} d\omega'. \quad (4)$$

After the leading DA is specified, the subleading one can be calculated in this approximation.

The differential branching fraction of the  $B^0 \rightarrow \phi \ell^+ \ell^-$  decay can be written as follows:

$$\begin{aligned} \frac{d\mathcal{B}}{dq^2} &= \tau_B \frac{G_F^2 |V_{td}^* V_{tb}|^2 \alpha^2}{216\pi} m_B f_B^2 f_\phi^2 Q_d^2 \lambda^3 \left( 1, \frac{m_\phi}{m_B}, \frac{\sqrt{q^2}}{m_B} \right) \\ &\times |C_3 + 4C_5|^2 \left[ |\lambda_{B,-}^{-1}(q^2)|^2 + \frac{m_\phi^2}{q^2 (1 - q^2/m_B^2)^2} |\lambda_{B,+}^{-1}(q^2)|^2 \right], \end{aligned} \quad (5)$$

where  $\tau_B$  is the  $B$ -meson lifetime,  $m_\phi$  is the  $\phi$  meson mass,  $\alpha$  is the fine structure constant,  $Q_d = -1/3$  is the relative charge of the  $d$ -quark,  $\lambda(a, b, c)$  is the kinematical function [1]. The differential branching fraction (5) depends also on the first inverse moments (FIMs),  $\lambda_{B,\pm}^{-1}(q^2)$ , of the  $B$ -meson DAs,  $\phi_\pm^B(\omega)$ , which are non-perturbative quantities [9]:

$$\lambda_{B,\pm}^{-1}(q^2) = \int_0^\infty \frac{\phi_\pm^B(\omega) d\omega}{\omega - q^2/m_B - i\epsilon}, \quad (6)$$

where  $q^2$  is the four-momentum squared of the lepton pair. Their momentum-dependence is determined by a choice of DA theoretical models. From the number of models suggested [10–15], we consider two — the Exponential model, known as Grozin and Neubert (GN) model, [10] and Linear model, also called as KKQT model [11]. In numerical analysis, the first inverse moments based on them are used to illustrate a variation due to a choice of the DA theoretical model.

### 3. Numerical analysis of $B^0 \rightarrow \phi \ell^+ \ell^-$ decay

Partially integrated branching fractions are measured at experiments and theoretical predictions are desirable to work out in a similar form:

$$\Delta\mathcal{B}(q_{\min}^2 < q^2 < q_{\max}^2) = \int_{q_{\min}^2}^{q_{\max}^2} \frac{d\mathcal{B}}{dq^2} dq^2. \quad (7)$$

We get estimates for the branching fraction in the interval  $q^2 \in [1 \text{ GeV}^2, 8 \text{ GeV}^2]$ , relevant for the  $e^+e^-$  and  $\mu^+\mu^-$  production, for the Exponential (GN) and Linear (KKQT) models:

$$\begin{aligned} \Delta\mathcal{B}^{\text{GN}}(1 \text{ GeV}^2 < q^2 < 8 \text{ GeV}^2) &= (2.1_{-1.0}^{+1.6}) \times 10^{-13}, \\ \Delta\mathcal{B}^{\text{KKQT}}(1 \text{ GeV}^2 < q^2 < 8 \text{ GeV}^2) &= (3.9_{-1.8}^{+2.9}) \times 10^{-13}. \end{aligned} \quad (8)$$

The results obtained demonstrate a strong dependence on the choice of the DA model. The difference in the model predictions is of the order of the factorization scale uncertainty. Near the lower bound of the interval, one can expect a long-distance contribution from the  $\phi$  meson. Note that the  $B^0 \rightarrow \phi\phi$  decay is under experimental searches and the upper limit on its branching fraction,  $\mathcal{B} < 2.7 \times 10^{-8}$ , is known at present [1]. Keeping in mind that the  $\phi$  meson decay into the muon pair is quite suppressed,  $\mathcal{B}(\phi \rightarrow \mu^+\mu^-) = (2.85 \pm 0.19) \times 10^{-4}$  [1], the  $\phi$  meson contribution can be a few times  $10^{-12}$ , but with shifting the lower cut it can be safely excluded. Here, we neglected such a long-distance effect and its impact on the spectrum will be analyzed elsewhere.

The estimation of the total branching fraction due to the perturbative contribution is as follows:

$$\mathcal{B}_{\text{th}}(B^0 \rightarrow \phi \ell^+ \ell^-) \sim 10^{-12}, \quad (9)$$

which is three orders of magnitude lower than the upper limit obtained by the LHCb Collaboration [4].

### 4. Summary and outlook

The theoretical analysis of the  $B^0 \rightarrow \phi \ell^+ \ell^-$  decay in the leading order within the Effective Electroweak Hamiltonian approach is presented. The branching fraction is perturbatively calculated in the region of small  $q^2$  and its variation caused by a choice of the theoretical model of the  $B$ -meson DAs is demonstrated explicitly. Theoretical prediction for the total branching fraction,  $\mathcal{B}_{\text{th}}(B^0 \rightarrow \phi \ell^+ \ell^-) \sim 10^{-12}$ , agrees with the experimental limit,  $\mathcal{B}_{\text{exp}}(B^0 \rightarrow \phi \mu^+ \mu^-) < 3.2 \times 10^{-9}$ , by the LHCb Collaboration [4]. A more precise prediction for the total branching fraction which covers the entire kinematically allowed region is under derivation. Since the  $\omega$ - $\phi$  mixing affects the  $B^0 \rightarrow \phi\gamma$  branching fraction, it is necessary to take into account this effect on the branching fraction of the decay considered as well as to include into a consideration long-distance contributions.

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