PROCEEDINGS OF SCIENCE

Real corrections to Higgs boson pair production at NNLO in the large top quark mass limit

Joshua Davies[a,*]

[a] *Department of Physics and Astronomy, University of Sussex Brighton BN1 9QH, UK*

E-mail: j.o.davies@sussex.ac.uk

In this talk I will discuss the computation of the top quark mass dependence of NNLO double Higgs boson production, including both the real and virtual contributions. The dependence is computed in the large top quark mass limit, which gives a good approximation of the exact cross section below the top quark production threshold.

Loops and Legs in Quantum Field Theory - LL2022,
25-30 April, 2022
Ettal, Germany

*Speaker

https://pos.sissa.it/

PoS(LL2022)007

1. Introduction

Since the discovery of the Higgs boson [1, 2], an important goal of the Large Hadron Collider (LHC) at CERN is the characterization of the scalar sector of the Standard Model (SM) of particle physics. It contains both cubic and quartic Higgs boson self interactions, whose strengths are governed by the parameter λ. Within the SM λ is determined by other, known, parameters; in particular, $\lambda = m_H^2/(2v^2) \approx 0.13$, where m_H is the Higgs boson mass and v its vacuum expectation value. However, to verify if the scalar sector of the SM is indeed realized in nature it is necessary to obtain an independent measurement of λ.

The most promising approach to make such a measurement is to study the production of a pair of Higgs bosons, a process which is sensitive to the value of λ already at its leading order (LO). The dominant production channel for this process at a hadron collider is by gluon fusion; the LO cross section for this process has been known for a long time [3, 4], and at next-to-leading order (NLO) it is very well studied in the literature. Approximate results exist in the limit of a large top quark mass (in its infinite limit [5] and including $1/m_t$ corrections [6, 7]), in the high-energy limit [8–10], for small Higgs boson transverse momentum [11] and around the top quark threshold [12]. Exact results for the real radiation were first obtained in [13], and the complete, exact NLO results are known from the numerical evaluations of Refs. [14–16].

At next-to-next-to-leading order (NNLO) comparatively little is known, however the computation of these corrections is well motivated due to the large uncertainties due to the renormalization-scale variation and renormalization scheme used for the top quark mass at NLO, studied in Ref. [17]. While a full NNLO calculation is beyond current techniques, approximate results are already available in the limit of a large top quark mass. The cross section is available in the infinite-m_t limit [18–20] and $1/m_t$ corrections are available for the virtual corrections [21, 22]. In Ref. [23] an approximation is constructed which combines the exact NLO corrections with the infinite-m_t virtual corrections and an exact computation of the double-real radiation.

In these proceedings we report on the computation of $1/m_t$ corrections to the NNLO real-radiation contribution, to compliment the expansion to $1/m_t^8$ of the virtual contribution in Ref. [22]. These results have been published in Refs. [24–27]. These proceedings are organised as follows: in Section 2 we discuss the ingredients needed to compute the total $gg \to HH$ cross section, in Section 3 we discuss the expansion of the diagrams in the limit of a large top quark mass, and in Section 4 we present results.

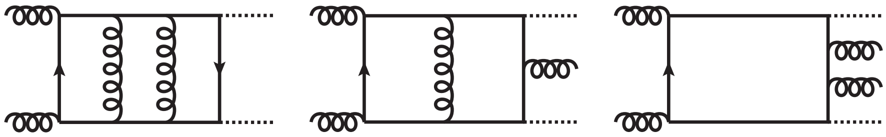
2. HH Cross Section at NNLO

The contributions to the NNLO (in QCD) total HH cross section can be separated into three pieces:

$$\sigma_{ij}^{(2)} = \sigma_{ij,\mathrm{virt}}^{(2)} + \sigma_{ij,\mathrm{real}}^{(2)} + \sigma_{ij,\mathrm{coll}}^{(2)}, \tag{1}$$

where the labels ij denote the production channel being considered. For example, we consider the channels $ij = gg, gq, q\bar{q}, qq'$, although gg is the dominant one at the LHC.

The so-called "virtual" contributions, shown in Fig. 1a, consist of three-loop 2→2 diagrams and the "real" contributions, shown in Figs. 1b and 1c, consist of two-loop 2→3 diagrams ("real-virtual") and one-loop 2→4 diagrams ("double-real"). Each term on the right-hand side of Eq. (1)



(a) Virtual **(b)** Real-virtual **(c)** Double-real

Figure 1: Types of diagram contributing to the NNLO $gg \to HH$ total cross section.

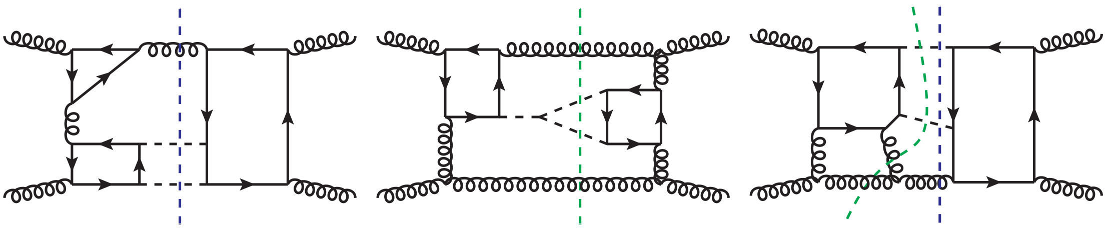


Figure 2: Five-loop forward scattering amplitudes with cuts contributing to the real-virtual (blue dashed lines) and double-real (green dashed lines) contributions at NNLO.

is individually infra-red divergent. A finite result is only obtained once the virtual, real and collinear counterterm contributions are added together. The double-real contributions can easily be computed, numerically, in an exact manner however in this work we also compute them in the large top quark mass limit, such that the infra-red poles cancel explicitly.

The total cross section can be computed by making use of the optical theorem, which relates the phase-space integrated squared amplitudes to the imaginary part of forward scattering amplitudes. In this framework, the real-radiation diagrams of Fig. 1 become the five-loop forward scattering diagrams shown in Fig. 2. Five-loop diagrams with two different internal mass scales are too complicated to compute exactly, however they can be investigated in the large top quark mass limit. In the next section, we discuss how this expansion can be performed for such diagrams. We do not compute the virtual contribution using the optical theorem, but simply use the results of Ref. [22].

3. Large Mass Expansion

The large value of the top quark's mass, compared to the lighter quarks, means that when considering amplitudes which involve Yukawa couplings it suffices to consider only the top quark running in the loops. Furthermore, we can perform an asymptotic expansion in the limit where m_t is larger than any other scales involved in the problem. For $gg \to HH$ for example, this limit means that the ratios m_H^2/m_t^2, s/m_t^2 and t/m_t^2 are all small, where s, t are the usual Mandelstam variables.

This asymptotic expansion can be realised by the method of "expansion by subgraph", which is implemented in the software package `exp` [28, 29]. The procedure is as follows:

1. Identify all subgraphs of the Feynman graph which contain m_t, the heavy scale,

2. Expand each subgraph in its light scales: these may be external momenta, masses or loop momenta belonging to the remaining lines of the graph.

PoS(LL2022)007

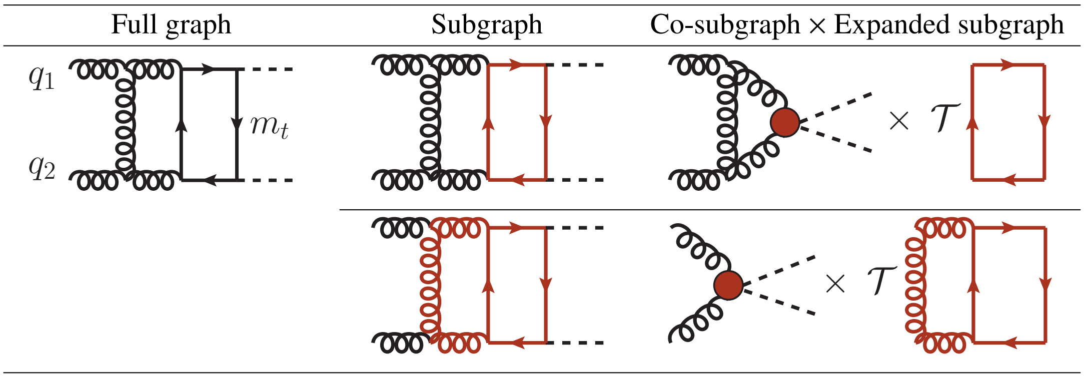


Figure 3: The first column shows the full Feynman graph. We can identify two subgraphs which contain the scale m_t, the one- and two-loop subgraphs highlighted in orange in the second column. The expansion of these subgraphs, denoted by the operator $\mathcal{T}$, yields one- and two-loop vacuum integrals multiplied by a one- and zero-loop massless "co-subgraph", respectively.

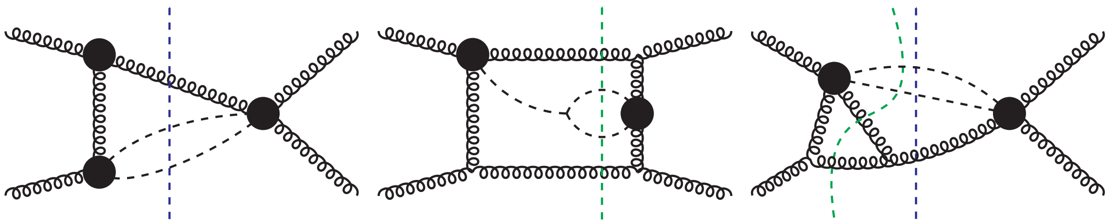
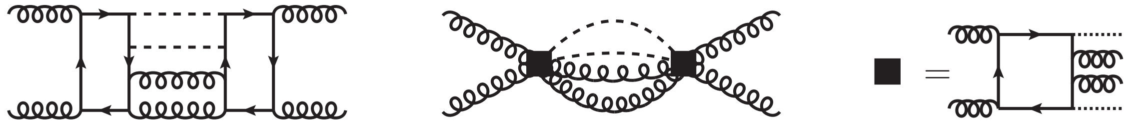


Figure 4: The large top quark mass expansion of the five-loop forward scattering diagrams of Fig. 2. The blue and green dashed lines represent three- and four-particle cuts, as before. The black blobs represent the subgraph expansions in terms of vacuum integrals.

The result of these steps is the factorization of the graph into massive vacuum integrals which depend on the scale m_t and the so-called "co-subgraph" which is independent of m_t. Fig. 3 shows this schematically, for a two-loop $gg \to HH$ diagram. It is this factorization into simpler integrals that makes it possible to compute the five-loop forward-scattering diagrams of Fig. 2; a representation of the expansion of these integrals is given in Fig. 4. The vacuum integrals can be computed with the `FORM` [30] package `MATAD` [31], and the massless "phase-space" integrals are computed with `FIRE` [32], `LiteRed` [33] and `LIMIT` [34] using the method of differential equations; we compute them both exactly, in terms of Goncharov Polylogarithms (GPLs), and as a series expansion around the Higgs pair production threshold, $\delta = 1 - 4m_H^2/s \approx 0$, which is sufficiently precise below the top quark pair threshold and is more straightforward to combine with the virtual and collinear counterterm contributions.

While this large mass expansion procedure is conceptually straightforward, it quickly becomes extremely computationally challenging if one wants to compute more than the leading term in the $1/m_t$ expansion. For this reason we introduce "building blocks", subgraphs which appear in common in many Feynman diagrams, and pre-compute their $1/m_t$ expansion. These building blocks can then be inserted into diagrams which contain effective vertices as placeholders for the building blocks. The advantages of this method are twofold: firstly, pre-expanding the building



(a) The full diagram. There are 3,600 diagrams of this type, which each would have to be expanded.

(b) The building-block diagram, in terms of two effective “four-gluon-two-Higgs” vertices. There is only one such diagram.

(c) A representative diagram of the building block with four external gluons and two external Higgs bosons.

Figure 5: The building-block method, applied to a five-loop forward scattering diagram which contributes to the NNLO double-real contribution.

block removes the duplicated effort spent on the expansion of the same subgraphs which appear in many different Feynman diagrams. Secondly, generating the Feynman diagrams in terms of the effective vertices leads to a huge reduction in the number of diagrams which have to be considered. An example of the building-block method is shown in Fig. 5.

Overall, six building blocks need to be computed, with two, three or four external gluons and one or two external Higgs bosons. The gg channel is given by around 160,000 “full” diagrams, but only 4,612 “effective vertex” diagrams into which the building blocks can be substituted. Without this simplification, an expansion beyond the leading order in $1/m_t$ would not have been possible with the computing resources available. The leading-order term is computed with both methods as a check. For both methods, careful `FORM` programming is required to avoid large numbers of terms in the intermediate expressions.

4. Results

Combining the expansion of the real radiation contribution discussed in Section 3 with the virtual and collinear counterterm contributions, we finally arrive at a finite expression for the total cross section, given by Eq. (1). The cross sections have the following form,

$$\sigma_{gg}^{(2),n_h^2} = \frac{a_s^4 G_F^2 m_H^2}{\pi} \left\{ \delta^{3/2}\left(\frac{19}{6192}+\frac{n_l}{1296}\right)+O(\delta^{5/2})+\rho\left[\sqrt{\delta}\left(\frac{133}{552960}+\frac{7n_l}{103680}\right)+O(\delta^{3/2})\right]+O(\rho^2)\right., \tag{2}$$

where n_h^2 denotes the diagrams which contain two closed top quark loops, n_l is the number of light quark flavours, $a_s = \alpha_s^{(5)}/\pi$, $\rho = m_H^2/m_t^2$ and $\delta = 1 - 4m_H^2/s$. The structure is a series in both δ and in ρ and $\log\rho$, although the $\log\rho$ terms do not appear in the low-order terms displayed here.

In Fig. 6 we show the total cross section for the gluon-fusion channel (i.e. Eq. (2)), expanded up to ρ^3 and δ^{30}. We observe that reasonable convergence of the ρ expansion is achieved for values of $\sqrt{s}$ below about 320 GeV. Above this value the sub-leading expansion terms start to become larger, and above the top quark pair threshold at $\sqrt{s} = 2m_t \approx 346$ GeV (denoted by the vertical black line) there is no convergence. The sub-leading expansion terms provide, approximately, a 100% correction to the infinite-m_t limit (ρ^0).

In Fig. 7 we show, also for the gluon-fusion channel, the LO, NLO and NNLO total cross sections. The LO curve is exact, and at NLO we show both the ρ^0 and ρ^3 approximations. We observe that below the top quark pair production threshold, the NLO contribution provides a 100%

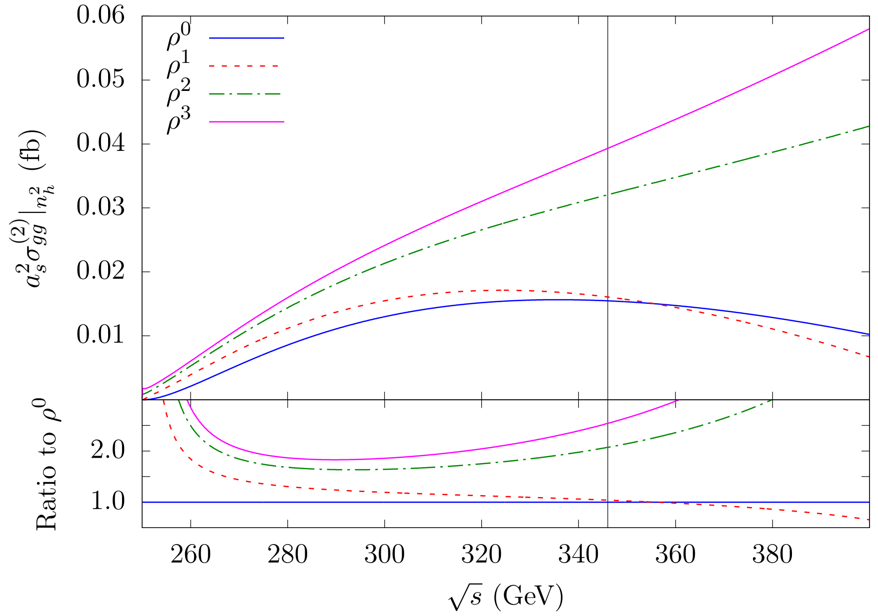


Figure 6: The total cross section for the n_h^2 part of the gluon-fusion channel, for various expansion depths in ρ. The lower panel shows the ratio of the expansions including the sub-leading terms, to the leading term.

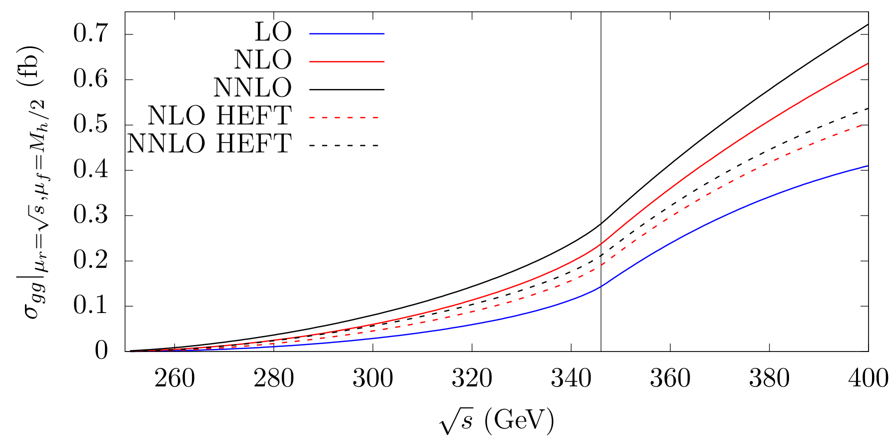


Figure 7: The total cross section for the gluon-fusion channel, at different orders in the strong coupling constant. The dashed curves show the infinite-m_t approximation (ρ^0) while the solid curves show the ρ^3 exapansion. The LO curve is exact.

correction to the LO curve, while the NNLO contribution provides an additional 30% correction to the NLO curve.

5. Conclusion

In these proceedings we have discussed a computation of the total cross section for Higgs pair production at NNLO in QCD. The expansion in the limit of a large top quark mass provides a good approximation of the total cross section in the region where $\sqrt{s} < 2m_t$. We find that the sub-leading $1/m_t$ terms provide a large correction compared to the infinite-m_t limit. The results of this calculation also provide useful input for methods which combine expansions in different kinematic regions, to create an approximation with a wider region of validity.

Acknowledgements

The work of JD was supported by the Science and Technology Facilities Council (STFC) under the Consolidated Grant ST/T00102X/1.

References

[1] ATLAS collaboration, *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*, Phys. Lett. B **716** (2012) 1 [1207.7214].

[2] CMS collaboration, *Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC*, Phys. Lett. B **716** (2012) 30 [1207.7235].

[3] E.W.N. Glover and J.J. van der Bij, *HIGGS BOSON PAIR PRODUCTION VIA GLUON FUSION*, Nucl. Phys. B **309** (1988) 282.

[4] T. Plehn, M. Spira and P.M. Zerwas, *Pair production of neutral Higgs particles in gluon-gluon collisions*, Nucl. Phys. B **479** (1996) 46 [hep-ph/9603205].

[5] S. Dawson, S. Dittmaier and M. Spira, *Neutral Higgs boson pair production at hadron colliders: QCD corrections*, Phys. Rev. D **58** (1998) 115012 [hep-ph/9805244].

[6] J. Grigo, J. Hoff, K. Melnikov and M. Steinhauser, *On the Higgs boson pair production at the LHC*, Nucl. Phys. B **875** (2013) 1 [1305.7340].

[7] G. Degrassi, P.P. Giardino and R. Gröber, *On the two-loop virtual QCD corrections to Higgs boson pair production in the Standard Model*, Eur. Phys. J. C **76** (2016) 411 [1603.00385].

[8] J. Davies, G. Mishima, M. Steinhauser and D. Wellmann, *Double-Higgs boson production in the high-energy limit: planar master integrals*, JHEP **03** (2018) 048 [1801.09696].

[9] J. Davies, G. Mishima, M. Steinhauser and D. Wellmann, *Double Higgs boson production at NLO in the high-energy limit: complete analytic results*, JHEP **01** (2019) 176 [1811.05489].

[10] J. Davies, G. Heinrich, S.P. Jones, M. Kerner, G. Mishima, M. Steinhauser et al., *Double Higgs boson production at NLO: combining the exact numerical result and high-energy expansion*, JHEP **11** (2019) 024 [1907.06408].

[11] R. Bonciani, G. Degrassi, P.P. Giardino and R. Gröber, *Analytical Method for Next-to-Leading-Order QCD Corrections to Double-Higgs Production*, Phys. Rev. Lett. **121** (2018) 162003 [1806.11564].

[12] R. Gröber, A. Maier and T. Rauh, *Reconstruction of top-quark mass effects in Higgs pair production and other gluon-fusion processes*, *JHEP* **03** (2018) 020 [1709.07799].

[13] F. Maltoni, E. Vryonidou and M. Zaro, *Top-quark mass effects in double and triple Higgs production in gluon-gluon fusion at NLO*, *JHEP* **11** (2014) 079 [1408.6542].

[14] S. Borowka, N. Greiner, G. Heinrich, S.P. Jones, M. Kerner, J. Schlenk et al., *Higgs Boson Pair Production in Gluon Fusion at Next-to-Leading Order with Full Top-Quark Mass Dependence*, *Phys. Rev. Lett.* **117** (2016) 012001 [1604.06447].

[15] S. Borowka, N. Greiner, G. Heinrich, S.P. Jones, M. Kerner, J. Schlenk et al., *Full top quark mass dependence in Higgs boson pair production at NLO*, *JHEP* **10** (2016) 107 [1608.04798].

[16] J. Baglio, F. Campanario, S. Glaus, M. Mühlleitner, M. Spira and J. Streicher, *Gluon fusion into Higgs pairs at NLO QCD and the top mass scheme*, *Eur. Phys. J. C* **79** (2019) 459 [1811.05692].

[17] J. Baglio, F. Campanario, S. Glaus, M. Mühlleitner, J. Ronca and M. Spira, $gg \to HH$ *: Combined uncertainties*, *Phys. Rev. D* **103** (2021) 056002 [2008.11626].

[18] D. de Florian and J. Mazzitelli, *Higgs Boson Pair Production at Next-to-Next-to-Leading Order in QCD*, *Phys. Rev. Lett.* **111** (2013) 201801 [1309.6594].

[19] D. de Florian and J. Mazzitelli, *Two-loop virtual corrections to Higgs pair production*, *Phys. Lett. B* **724** (2013) 306 [1305.5206].

[20] J. Grigo, K. Melnikov and M. Steinhauser, *Virtual corrections to Higgs boson pair production in the large top quark mass limit*, *Nucl. Phys. B* **888** (2014) 17 [1408.2422].

[21] J. Grigo, J. Hoff and M. Steinhauser, *Higgs boson pair production: top quark mass effects at NLO and NNLO*, *Nucl. Phys. B* **900** (2015) 412 [1508.00909].

[22] J. Davies and M. Steinhauser, *Three-loop form factors for Higgs boson pair production in the large top mass limit*, *JHEP* **10** (2019) 166 [1909.01361].

[23] M. Grazzini, G. Heinrich, S. Jones, S. Kallweit, M. Kerner, J.M. Lindert et al., *Higgs boson pair production at NNLO with top quark mass effects*, *JHEP* **05** (2018) 059 [1803.02463].

[24] J. Davies, F. Herren, G. Mishima and M. Steinhauser, *Real-virtual corrections to Higgs boson pair production at NNLO: three closed top quark loops*, *JHEP* **05** (2019) 157 [1904.11998].

[25] J. Davies, F. Herren, G. Mishima and M. Steinhauser, *NNLO real corrections to* $gg \to HH$ *in the large-*m_t *limit*, *PoS* **RADCOR2019** (2019) 022 [1912.01646].

[26] J. Davies, F. Herren, G. Mishima and M. Steinhauser, *Real corrections to Higgs boson pair production at NNLO in the large top quark mass limit*, *JHEP* **01** (2022) 049 [2110.03697].

[27] J. Davies, *Top quark mass corrections to single and double Higgs boson production in gluon fusion*, SciPost Phys. Proc. **7** (2022) 031 [2110.06603].

[28] R. Harlander, T. Seidensticker and M. Steinhauser, *Complete corrections of Order alpha alpha-s to the decay of the Z boson into bottom quarks*, Phys. Lett. B **426** (1998) 125 [hep-ph/9712228].

[29] T. Seidensticker, *Automatic application of successive asymptotic expansions of Feynman diagrams*, in *6th International Workshop on New Computing Techniques in Physics Research: Software Engineering, Artificial Intelligence Neural Nets, Genetic Algorithms, Symbolic Algebra, Automatic Calculation*, 5, 1999 [hep-ph/9905298].

[30] B. Ruijl, T. Ueda and J. Vermaseren, *FORM version 4.2*, 1707.06453.

[31] M. Steinhauser, *MATAD: A Program package for the computation of MAssive TADpoles*, Comput. Phys. Commun. **134** (2001) 335 [hep-ph/0009029].

[32] A.V. Smirnov and F.S. Chuharev, *FIRE6: Feynman Integral REduction with Modular Arithmetic*, Comput. Phys. Commun. **247** (2020) 106877 [1901.07808].

[33] R.N. Lee, *LiteRed 1.4: a powerful tool for reduction of multiloop integrals*, J. Phys. Conf. Ser. **523** (2014) 012059 [1310.1145].

[34] F. Herren, *Precision Calculations for Higgs Boson Physics at the LHC - Four-Loop Corrections to Gluon-Fusion Processes and Higgs Boson Pair-Production at NNLO*, Ph.D. thesis, KIT, Karlsruhe, 2020. 10.5445/IR/1000125521.