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γ_5 in dimensional regularization — a no-compromise approach using the BMHV scheme

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γ_5 is notoriously difficult to define in D dimensions. The traditional BMHV scheme employs a non-anticommuting γ_5. Its key advantage is mathematical consistency and the existence of all-order proofs. Its disadvantage is the spurious breaking of gauge invariance in chiral gauge theories like the electroweak standard model. Our research programme aims to determine the special finite counterterms which are necessary to restore gauge invariance, to allow more straightforward applications of the BMHV scheme and to cross-check alternative schemes. In these proceedings we present the key concepts and methods, and we outline the calculational procedure and present results for an abelian gauge theory at the 2-loop level. An important observation is the simplicity of the results — three types of symmetry-restoring counterterms are sufficient at the 2-loop level.

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1. Introduction

The problem of γ_5 in dimensional regularization is well known. The three properties (i) anti-commutativity with γ^μ, (ii) non-zero $\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma)$, (iii) cyclicity of traces become inconsistent in $D \neq 4$ dimensions [e.g. (i) and (iii) imply that the trace in (ii) is zero].

There is a multitude of proposals how to treat this issue and how to define a D-dimensional continuation of γ_5 and many of them are routinely applied in practical computations (for a review see [1], for further original references see also [2, 3]).

A very traditional scheme is the original proposal of Ref. [4], which was later further formalized in Ref. [5] — the BMHV scheme. It is well known that this scheme has significant disadvantages in practical calculations. However, its key advantage is that full mathematical consistency and complete all-order proofs are established [5]. In our approach we aim to avoid compromises with respect to mathematical rigor. Hence this is our motivation to focus on the BMHV scheme. We accept its practical difficulties, deal with them, and aim to provide the community with results and building blocks which allow more straightforward applications of the scheme.[1]

Specifically we aim to provide the required symmetry-restoring counterterms which compensate the spurious breaking of gauge invariance caused by the non-anticommuting γ_5. Ultimately we aim for a treatment of the electroweak standard model at the multiloop level. The current status is a treatment of a general Yang-Mills theory at the 1-loop level [2] and an abelian gauge theory at the 2-loop level [3]. In these proceedings we provide an introduction to the key concepts and methods of our approach (sec. 2 and 3) and an outline of the computations and results for the abelian case (sec. 4). Sec. 5 contains a brief summary and outlook.

2. Definitions and the problem in a nutshell: breaking of Ward identity

In the BMHV scheme, formally $D = (4-2\epsilon)$-dimensional quantities k^μ of dimensional regularization can be split into their 4-dimensional and $(D-4)$-dimensional parts as

$$k^\mu = \bar{k}^\mu + \hat{k}^\mu \,. \tag{1}$$

The D-dimensional space can be viewed as a direct sum of 4-dimensional and $(D-4)$-dimensional subspaces, such that orthogonality and projection relations such as

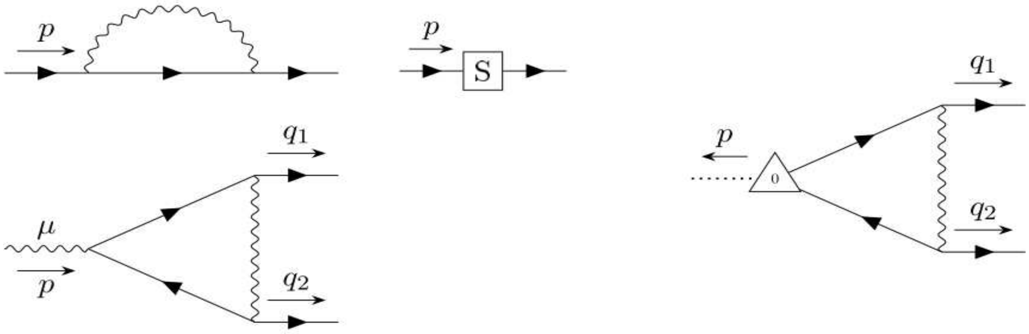
$$\bar{k}^\mu \hat{k}_\mu = 0\,, \qquad k^\mu \hat{k}_\mu = \hat{k}^\mu \hat{k}_\mu\,, \qquad k^\mu \bar{k}_\mu = \bar{k}^\mu \bar{k}_\mu \tag{2}$$

hold. The split can be done for objects such as momentum vectors, gauge fields, metric tensors, and in particular for γ^μ matrices, $\gamma^\mu = \bar{\gamma}^\mu + \hat{\gamma}^\mu$.

In the BMHV scheme, the matrix γ_5 is defined as an intrinsically 4-dimensional object. It satisfies

$$\{\gamma_5, \bar{\gamma}^\mu\} = 0\,, \qquad [\gamma_5, \hat{\gamma}^\mu] = 0\,, \tag{3}$$

[1] In addition, a better understanding of the BMHV scheme may feed back to alternative approaches to γ_5, potentially enabling consistency checks or optimizations of such approaches.



(a) (b) (c)

Figure 1: Illustration of (a) diagrams which violate a Ward identity, (b) a counterterm restoring this Ward identity, (c) a single diagram with the insertion of an operator $\widehat{\Delta}$ which directly yields the breaking of the Ward identity (a).

and thus it breaks full D-dimensional Lorentz covariance. The usual anticommutation relation holds only for the purely 4-dimensional parts of the γ^μ matrices. Importantly, this definition is consistent with the cyclicity of traces and with the relation $\gamma_5 = \frac{-i}{4!}\epsilon_{\mu\nu\rho\sigma}\bar{\gamma}^\mu\bar{\gamma}^\nu\bar{\gamma}^\rho\bar{\gamma}^\sigma$.

Let us provide a preview of the main problem caused by the definition (3), the breaking of gauge invariance in chiral gauge theories. In the abelian gauge theory defined below we expect the validity of QED-like Ward identities such as a relationship between the one-loop fermion self energy and the one-loop fermion–gauge boson three-point function, as illustrated in Fig. 1(a).

It turns out that the corresponding Ward identity is violated at the level of the BMHV regularized one-loop diagrams. The breaking has a special form, however: it is local, i.e. it has a form which can be compensated by adding a certain local, symmetry-restoring counterterm to the Lagrangian. The counterterm contributes only to the fermion self energy as illustrated in Fig. 1(b). After adding the counterterm, there is an additional contribution to the fermion self energy, and the Ward identity is fulfilled.

The precise form of the counterterm action relevant here and corresponding to the $\boxed{S}$ in Fig. 1(b) is

$$S^1_{\rm fct} = \frac{e^2}{16\pi^2}\int d^4x \left\{ \ldots + \left(\frac{5+\xi}{6}\right)(\mathcal{Y}^j_R)^2\left(\bar{\psi}_j i\bar{\partial}\!\!\!/\, P_{\rm R}\,\psi_j\right)\right\}. \tag{4}$$

The problem of the BMHV scheme is thus that Ward/Slavnov-Taylor identities are broken in intermediate steps and special, symmetry-restoring counterterms are needed. The existence of such counterterms is guaranteed provided the gauge theory in question is free of chiral anomalies and hence renormalizable. But their concrete determination is a necessary step and a complication of practical computations.

3. Goals and method: Slavnov-Taylor identities and quantum action principle

In a nutshell, our goal is to determine symmetry-restoring counterterms such as Eq. (4), for all Ward and Slavnov-Taylor identities, and at the multi-loop level.

In principle, a pedestrian way to do that might be to evaluate all Green functions entering relevant Ward and Slavnov-Taylor identities, check the validity of the identities, and ultimately evaluate possible breakings and required counterterms. There is, however, a more direct method, which can also be illustrated with the example introduced above.[2] Instead of evaluating the fermion self-energy and fermion–gauge boson three-point function, it is sufficient to evaluate the Feynman diagram in Fig. 1(c).

In this diagram, the triangle denotes the insertion of a special operator $\widehat{\Delta}$, which can be determined once and for all as will be described below. The quantum action principle guarantees that the result of this diagram corresponds directly to the *violation* of the Ward identity of Fig. 1(a). Hence, in order to compute the required counterterm (4) we only need to compute the single diagram Fig. 1(c) instead of the two diagrams Fig. 1(a). In addition, the single diagram Fig. 1(c) is simpler to compute since the operator $\widehat{\Delta}$ is evanescent, i.e. zero in 4 dimensions, and therefore only terms related to ultraviolet $1/(D-4)$ singularities can lead to non-vanishing contributions.

In general, our method is therefore to compute all relevant breakings of Ward/Slavnov-Taylor identities in terms of such Feynman diagrams based on the quantum action principle, and then to determine the required symmetry-restoring counterterms.[3]

4. Application to abelian chiral gauge theory at the 2-loop level

In this section we outline the concrete calculational procedure and the results for the 2-loop renormalization of chiral gauge theories in the BMHV scheme. We focus on an abelian gauge theory similar to the U(1) hypercharge sector of the electroweak standard model. The essential steps are:

1. Define D-dimensional regularized Lagrangian and compute the resulting symmetry breaking $\widehat{\Delta}$.
2. Determine 1-loop UV divergences and the resulting counterterm Lagrangian $\mathcal{L}_{\text{sct}}$.
3. Determine 1-loop violation of Slavnov-Taylor identity using the quantum action principle and the insertion $\widehat{\Delta}$.
4. Determine 1-loop symmetry-restoring counterterms and the resulting, finite, symmetry-restoring counterterms $\mathcal{L}_{\text{fct}}$.
5. Repeat at 2-loop order.

4.1 D-dimensional Lagrangian and its symmetry breaking

The considered U(1) gauge theory contains a set of fermion fields ψ_i whose right-handed parts are assigned "hypercharges" $\mathcal{Y}_{Ri}$ and which interact with the gauge field A^μ. In D dimensions, the

[2]For more details on both methods and literature references with sample applications see sec. 6 of [2].

[3]We remark that the method of computing potential symmetry breakings based on the quantum action principle was also used in Refs. [6–8] in the study of SUSY properties of dimensional reduction. In those references the method established that dimensional reduction *preserves* SUSY in important cases up to the 3-loop level — the corresponding diagrams involving the insertion $\widehat{\Delta}$ turned out to vanish.

fermionic part of the Lagrangian can be written as[4]

$$\mathcal{L}_{\text{fermions}} = i\overline{\psi}_i \partial\!\!\!/ \psi_i + e\mathcal{Y}_{Ri}\overline{\psi_R}_i \bar{A\!\!\!/} \psi_{Ri} \,. \tag{5}$$

Here $\psi_R = P_R\psi$ with the right-chiral projector $P_R = (1+\gamma_5)/2$. Note that the kinetic term must involve the full, D-dimensional derivative $\partial\!\!\!/$ in order to generate a regularized, D-dimensional propagator denominator in Feynman diagrams. The mismatch between $\partial\!\!\!/\psi_i$ and $\bar{A\!\!\!/}\psi_{Ri}$ causes a breaking of gauge invariance in D-dimensions.

On the level of the quantized theory, a gauge fixing is needed and gauge invariance is replaced by BRST invariance involving the Faddeev-Popov ghost field c, and the associated symmetry properties of Green functions are expressed by Ward and Slavnov-Taylor identities. All these can be summarized by the expression $\mathcal{S}(\Gamma) = 0$, where $\mathcal{S}$ is the Slavnov-Taylor operator and Γ the renormalized, finite generating functional of 1PI Green functions. The breaking of gauge invariance of the regularized Lagrangian is then equivalent to a non-zero result of the D-dimensional Slavnov-Taylor operator applied to the classical action S_0 in D dimensions,

$$\mathcal{S}_d(S_0) = \widehat{\Delta} \equiv \int \mathrm{d}^d x \, (e\mathcal{Y}_{Ri}) c \left\{ \overline{\psi}_i \left(\overleftarrow{\widehat{\partial\!\!\!/}} P_{\text{R}} + \overrightarrow{\widehat{\partial\!\!\!/}} P_{\text{L}} \right) \psi_i \right\} \,. \tag{6}$$

Here the quantity $\widehat{\Delta}$, announced in the previous sections, has been defined. It is an evanescent operator, i.e. a local field operator product which vanishes in 4 dimensions. It originates directly from the mismatch between the two terms in Eq. (5). It can be translated into the Feynman rule used already in the diagram of Fig. 1(c). $\widehat{\Delta}$ contains the essence of the difficulties and provides the basis of our method of determining the symmetry breaking at the loop level.

4.2 1-loop UV divergences

As the first step of renormalization of the theory we determine the 1-loop UV divergences by computing the $1/\epsilon$ poles of all power-counting divergent 1-loop 1PI Green functions. As a result we obtain the required set of divergent 1-loop counterterms. The corresponding counterterm action S^1_{sct} can be decomposed as

$$S^1_{\text{sct}} = S^1_{\text{sct,inv}} + S^1_{\text{sct,break}} \,, \tag{7}$$

where the first term $S^1_{\text{sct,inv}}$ originates in the familiar way from field and parameter renormalization transformations applied to the tree-level action. We suppress its result here. The second term $S^1_{\text{sct,break}}$ is specific to the BMHV scheme and results from the breaking of gauge and D-dimensional Lorentz invariance. It can be written as

$$S^1_{\text{sct,break}} = \frac{-e^2}{16\pi^2\epsilon} \frac{\mathrm{Tr}(\mathcal{Y}_R^2)}{3} \left(2(\overline{S}_{AA} - S_{AA}) + \int \mathrm{d}^d x \, \frac{1}{2} \bar{A}^\mu \widehat{\partial}^2 \bar{A}_\mu \right) \,, \tag{8}$$

where the symbol S_{AA} denotes the gauge boson kinetic term of the classical action.

We see that we need specific, non-symmetric divergent counterterms which cannot be obtained from field and parameter renormalization. These counterterms are evanescent, i.e. the field operator expressions vanish in 4 dimensions.[5]

[4] For more details and the full form of the D-dimensional classical action see Ref. [3].

[5] Note that the appearance of specific counterterms for evanescent interactions is well known also in the context of dimensional reduction/the FDH scheme; for a review and original references see sec. 2.3, 2.4 in [9].

4.3 1-loop symmetry breaking

We turn to the evaluation of the 1-loop symmetry breaking caused by the BMHV scheme. Let us first recall the ultimate structure of the 1-loop renormalized generating functional for 1PI Green functions, which is obtained as $\Gamma_{\text{renormalized}} = \text{LIM}_{D\to 4}\Gamma_{\text{DReg}}$ where LIM denotes $D \to 4$ and setting to zero all evanescent terms. The decisive object Γ_{DReg} is obtained as

$$\Gamma^{(1)}_{\text{DReg}} = \Gamma^{(1)} + S^1_{\text{sct}} + S^1_{\text{fct}} \,. \tag{9}$$

It is a sum of the generating functional for regularized 1-loop 1PI Green functions, $\Gamma^{(1)}$, and the complete 1-loop counterterm action, which in turn is decomposed into the singular counterterms described above and the finite counterterms. This finite counterterm action, $S^{(1)}_{\text{fct}}$, contains the symmetry-restoring counterterms and is the ultimate output of the computation that follows.

It is determined by the requirement that the renormalized theory satisfies the Slavnov-Taylor identity, $\mathcal{S}_d(\Gamma^{(1)}_{\text{DReg}}) = 0$ in the limit $D \to 4$. If we evaluate the Slavnov-Taylor operator on the l.h.s. at 1-loop order, the divergent parts automatically cancel, and the finite parts can be rewritten as

$$\mathcal{S}_d(\Gamma^{(1)}_{\text{DReg}}) = \mathcal{S}_d(\Gamma^{(1)})|_{\text{finite}} + \mathcal{S}_d S^1_{\text{fct}} \,. \tag{10}$$

Here the first term on the r.h.s. corresponds to the Slavnov-Taylor operator applied to the regularized 1-loop Green functions — it directly generalizes the Ward identity and the two Feynman diagrams of Fig. 1(a) discussed above. The second term will be discussed in the subsequent subsection.

This first term can now be simplified by using the regularized quantum action principle established in Ref. [5], as

$$\mathcal{S}_d(\Gamma^{(1)}) = \widehat{\Delta} \cdot \Gamma^{(1)} \,. \tag{11}$$

where $\widehat{\Delta} \cdot \Gamma^{(1)}$ denotes 1-loop regularized 1PI Green functions with one insertion of the vertex corresponding to the operator $\widehat{\Delta}$ defined in Eq. (6) — this directly generalizes the diagram of Fig. 1(c) discussed above.

As a result of this discussion, the recipe for determining the complete 1-loop symmetry breaking by the BMHV scheme is to evaluate all non-vanishing 1-loop diagrams with one insertion of $\widehat{\Delta}$. Since $\widehat{\Delta}$ is evanescent, only power-counting divergent diagrams can in principle provide a non-vanishing result in the limit $D \to 4$. There are not many such 1-loop diagrams — in fact, there are precisely four of them. Three of them are shown in Fig. 2, the fourth one vanishes provided the anomaly cancellation condition $\text{Tr}(\mathcal{Y}_R^3) = 0$ holds.

Each of these diagrams can be easily evaluated, using the Feynman rule for the ghost–fermion–fermion vertex corresponding to the $\widehat{\Delta}$ insertion. In this way, Eq. (11) and thus the first term on the r.h.s. of Eq. (10) is evaluated.

The interpretation of the three diagrams is obvious. Each of them describes the violation of a well-known QED-like Ward identity. The first diagram describes the violation of the transversality of the photon self energy, the second diagram the violation of the analogous Ward identity of the photon 4-point function. The third diagram describes the violation of the Ward identity discussed in section 2 between the fermion self energy and fermion–gauge boson three-point function.

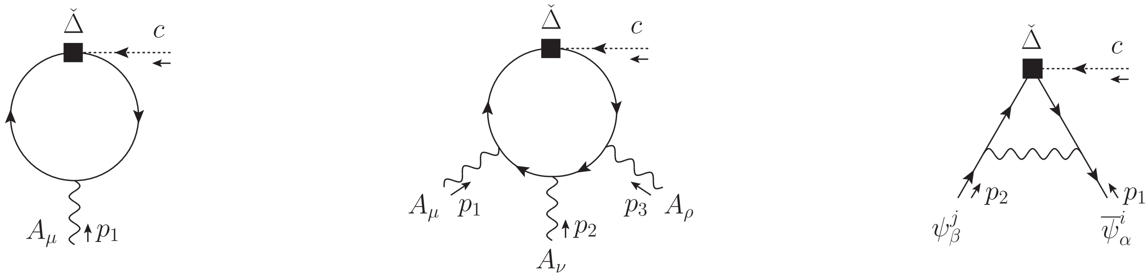


Figure 2: The three non-vanishing 1-loop diagrams with an insertion of $\widehat{\Delta}$.

4.4 1-loop symmetry-restoring counterterms

Our task is now to determine 1-loop symmetry-restoring counterterms, i.e. a counterterm action $S^{(1)}_{\rm fct}$ which is chosen such that the sum on the r.h.s. of Eq. (10) vanishes for $D \to 4$, i.e.

$$\mathcal{S}_d(\Gamma^{(1)})|_{\rm finite} + \mathcal{S}_d S^1_{\rm fct} \overset{D\to 4}{=} 0\,. \tag{12}$$

The expression $\mathcal{S}_d S^1_{\rm fct}$ denotes the linearized Slavnov-Taylor operator applied to the counterterm action, which is essentially the BRST variation of the counterterm action.

Hence we need to find a counterterm action $S^{(1)}_{\rm fct}$ whose BRST variation is the negative of the result of the three diagrams of Fig. 2. This is a straightforward algebraic exercise. The result is

$$\begin{aligned} S^1_{\rm fct} = \frac{e^2}{16\pi^2}\int d^4x \,\Bigg\{ & \frac{-{\rm Tr}(\mathcal{Y}_R^2)}{6}\bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + \frac{e^2 {\rm Tr}(\mathcal{Y}_R^4)}{12}(\bar{A}^2)^2 \\ & + \left(\frac{5+\xi}{6}\right)\sum_j (\mathcal{Y}_R^j)^2 \left(\bar{\psi}_j i\bar{\partial\!\!\!/}\, P_{\rm R}\, \psi_j\right)\Bigg\}\,. \end{aligned} \tag{13}$$

This is the full 1-loop result of the symmetry-restoring counterterms for the considered chiral abelian gauge theory in the BMHV scheme. We point out:

- The result has a very simple structure, contains only three terms and can be easily implemented as a set of additional Feynman rules.
- Each term has an obvious interpretation as a correction term to the gauge boson self energy, the quartic gauge boson interaction, and to the fermion self energy. The terms are chosen such that they guarantee the validity of the three corresponding Ward identities. The result contains and generalizes Eq. (4).
- The terms are finite and purely 4-dimensional (not evanescent). Obviously they are not gauge invariant.

4.5 2-loop UV divergences

The 2-loop computation proceeds with similar steps. Here we focus on the essential features of the 2-loop results and point out important new ingredients and difficulties.

When 1-loop counterterms are taken into account, the remaining UV divergences at the 2-loop level are local and can be cancelled by a 2-loop counterterm action $S^2_{\rm sct}$. Like at 1-loop level, this can

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be partially obtained by field and parameter renormalization, but a remainder exists. The remainder reads

$$
\begin{aligned}
S^2_{\text{sct,break}} = & -\frac{e^4}{256\pi^4\epsilon}\frac{\text{Tr}(\mathcal{Y}_R^4)}{3}\left(2(\overline{S}_{AA}-S_{AA})+\left(\frac{1}{2\epsilon}-\frac{17}{24}\right)\int \text{d}^d x\ \frac{1}{2}\bar{A}^\mu\widehat{\partial}^2\bar{A}_\mu\right) \\
& -\frac{e^4}{256\pi^4}\sum_j\frac{(\mathcal{Y}_R^j)^2}{3\epsilon}\left(\frac{5}{2}(\mathcal{Y}_R^j)^2-\frac{2}{3}\text{Tr}(\mathcal{Y}_R^2)\right)\overline{S^j_{\bar{\psi}\psi_R}}\,.
\end{aligned}
\tag{14}
$$

It contains the same kind of evanescent terms as the 1-loop result (8), but in addition there is a non-gauge invariant and non-evanescent contribution to the fermion self energy $\overline{S^j_{\bar{\psi}\psi_R}}$.

4.6 2-loop symmetry breaking

In order to determine the 2-loop symmetry breaking and the symmetry-restoring counterterms we proceed like at the 1-loop level. We write down the ultimate structure of the 2-loop renormalized generating functional (including counterterms but before carrying out $\text{LIM}_{D\to4}$),

$$
\Gamma^{(2)}_{\text{DReg}} = \Gamma^{(2)} + S^2_{\text{sct}} + S^2_{\text{fct}}\,,
\tag{15}
$$

in terms of its regularized version (including 1-loop counterterms) $\Gamma^{(2)}$ and the 2-loop singular and finite counterterm actions. If this is inserted into the Slavnov-Taylor identity at the 2-loop level, the divergent parts automatically cancel, and the following finite expressions remain,

$$
\mathcal{S}_d(\Gamma^{(2)}_{\text{DReg}}) = \mathcal{S}_d(\Gamma^{(2)})|_{\text{finite}} + \mathcal{S}_d S^2_{\text{fct}}\,.
\tag{16}
$$

The first term on the r.h.s. can be simpler evaluated using the quantum action principle,

$$
\mathcal{S}_d(\Gamma^{(2)}) = \widehat{\Delta}\cdot\Gamma^{(2)} + \Delta^1_{\text{ct}}\cdot\Gamma^{(1)}
\tag{17}
$$

where Δ^1_{ct} is defined like $\widehat{\Delta}$ but using the 1-loop counterterm action. However, the evaluation of this expression is significantly more involved than at the 1-loop level.

Instead of three non-vanishing diagrams, there are now four different types of diagrams, each with many concrete examples. They are exemplified in Fig. 3 with sample diagrams with external ghost–fermion–fermion. Nevertheless, after evaluating all these diagrams the result acquires a form like at the 1-loop level. In particular, the cancellation of non-local terms (non-polynomial in momenta) provides a strong check of the calculation.

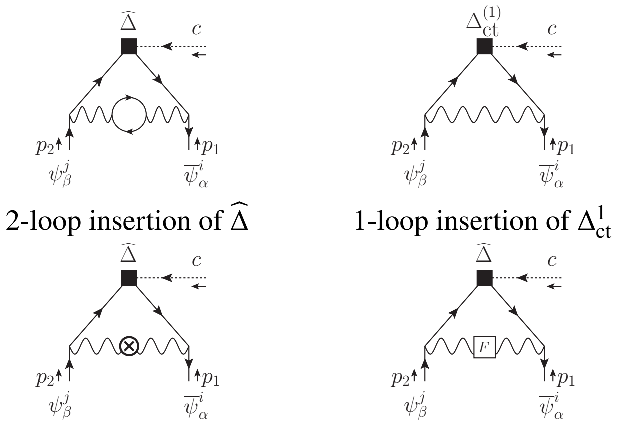
4.7 2-loop symmetry-restoring counterterms

Requiring the renormalized Slavnov-Taylor identity to hold at the 2-loop level means

$$
\mathcal{S}_d(\Gamma^{(2)})|_{\text{finite}} + \mathcal{S}_d S^2_{\text{fct}} \overset{D\to4}{=} 0\,.
\tag{18}
$$

The first term is computed in terms of the diagrams illustrated by Fig. 3, so this equation determines the desired symmetry-restoring finite counterterms $\mathcal{S}_d S^2_{\text{fct}}$. The result is

$$
\begin{aligned}
S^2_{\text{fct}} = & \frac{e^4}{(16\pi^2)^2}\int \text{d}^4 x\ \Bigg\{\text{Tr}(\mathcal{Y}_R^4)\frac{11}{48}\bar{A}_\mu\bar{\partial}^2\bar{A}^\mu + e^2\frac{\text{Tr}(\mathcal{Y}_R^6)}{8}(\bar{A}^2)^2 \\
& -\sum_j(\mathcal{Y}_R^j)^2\left(\frac{127}{36}(\mathcal{Y}_R^j)^2-\frac{1}{27}\text{Tr}(\mathcal{Y}_R^2)\right)\left(\bar{\psi}_j i\bar{\partial\!\!\!/}\, P_\text{R}\,\psi_j\right)\Bigg\}\,.
\end{aligned}
\tag{19}
$$



insertion of $\widehat{\Delta}$ into 1-loop diagram with 1-loop ct insertion

Figure 3: The four types of diagrams contributing to Eq. (17).

This is the full 2-loop result of the symmetry-restoring counterterms for the considered chiral abelian gauge theory in the BMHV scheme. We point out:

- The result is as simple as at the 1-loop level. Again it can be easily implemented as additional Feynman rules.
- It contains only the same three kinds of terms, corresponding to the gauge boson self energy, quartic gauge boson interaction and fermion self energy.
- The only difference are the prefactors, which are now of 2-loop order.

5. Summary and outlook

The BMHV scheme of dimensional regularization defines γ_5 as a purely 4-dimensional object which does not fully anticommute in D dimensions. The key advantages of this scheme are the full mathematical consistency and the existence of all-order proofs of renormalization properties such as cancellation of divergences and the quantum action principle. Its disadvantage is the spurious breaking of gauge invariance in chiral gauge theories.

Here we presented an approach to systematically determine the required symmetry-restoring counterterms which cancel this breaking of gauge invariance. The approach is based on evaluating Feynman diagrams with insertions of the breaking $\widehat{\Delta}$ and is thus simpler than explicitly evaluating required Ward/Slavnov-Taylor identities (as illustrated by Fig. 1). The insertion $\widehat{\Delta}$ corresponds to the tree-level breaking of gauge invariance and thus encapsulates the core difficulty of the scheme.

We explained the calculational procedure and the structure of contributing Feynman diagrams up to the 2-loop level. As discussed in the previous sections, a crucial observation is the simplicity of the results. It will be straightforward to take into account the obtained symmetry-restoring counterterms in practical calculations and/or to implement them in computer-algebra frameworks.

A second important observation is that the structure of the result does not change between 1-loop and 2-loop order. In general it is clear that the number of required symmetry-restoring counterterms is finite (in practice, it is small) and limited by power counting. In case of the Yang-Mills theory treated in Ref. [2] the set of symmetry-restoring counterterms comprises all two-point functions and all gauge boson self interactions, as well as a subset of the interactions between gauge bosons and matter fields.

There is no obstacle to apply the method to the full electroweak standard model and to higher loop orders, and work in these directions is in progress.

References

[1] F. Jegerlehner, Eur. Phys. J. C **18** (2001), 673-679 doi:10.1007/s100520100573 [arXiv:hep-th/0005255 [hep-th]].

[2] H. Bélusca-Maïto, A. Ilakovac, M. Mađor-Božinović and D. Stöckinger, JHEP **08** (2020) no.08, 024 doi:10.1007/JHEP08(2020)024 [arXiv:2004.14398 [hep-ph]].

[3] H. Bélusca-Maïto, A. Ilakovac, P. Kühler, M. Mađor-Božinović and D. Stöckinger, JHEP **11** (2021), 159 doi:10.1007/JHEP11(2021)159 [arXiv:2109.11042 [hep-ph]].

[4] G. 't Hooft and M. J. G. Veltman, Nucl. Phys. B **44** (1972), 189-213 doi:10.1016/0550-3213(72)90279-9

[5] P. Breitenlohner and D. Maison, Commun. Math. Phys. **52** (1977), 39 doi:10.1007/BF01609070

[6] D. Stockinger, JHEP **03** (2005), 076 doi:10.1088/1126-6708/2005/03/076 [arXiv:hep-ph/0503129 [hep-ph]].

[7] W. Hollik and D. Stockinger, Phys. Lett. B **634** (2006), 63-68 doi:10.1016/j.physletb.2006.01.030 [arXiv:hep-ph/0509298 [hep-ph]].

[8] D. Stöckinger and J. Unger, Nucl. Phys. B **935** (2018), 1-16 doi:10.1016/j.nuclphysb.2018.08.005 [arXiv:1804.05619 [hep-ph]].

[9] C. Gnendiger, A. Signer, D. Stöckinger, A. Broggio, A. L. Cherchiglia, F. Driencourt-Mangin, A. R. Fazio, B. Hiller, P. Mastrolia and T. Peraro, *et al.* Eur. Phys. J. C **77** (2017) no.7, 471 doi:10.1140/epjc/s10052-017-5023-2 [arXiv:1705.01827 [hep-ph]].