PROCEEDINGS OF SCIENCE

Tensor decomposition for multiloop multileg helicity amplitudes

Lorenzo Tancredi[a,*]

[a] *Physik Department, Technische Universität München, James-Franck-Straße 1, D–85748 Garching, Germany*

E-mail: lorenzo.tancredi@tum.de

In this contribution I will present new ideas to simplify the form factor method in order to compute helicity amplitudes for multiloop multileg scattering amplitudes, directly in the so-called 't Hooft-Veltman scheme, avoiding evanescent structures and ambiguities in the scheme definitions.

Loops and Legs in Quantum Field Theory - LL2022,
25-30 April, 2022
Ettal, Germany

*Speaker

PoS(LL2022)020

1. Introduction and motivation

With the start of the Run 3 of data taking at the Large Hadron Collider (LHC) at CERN in the Summer 2022, the need of precision calculations for the production of increasingly complicated final states in hadron collisions remains one of the main priorities of the theoretical particle physics community. An important ingredient of precision calculations for hadron colliders is the so-called hard scattering cross section, which provides the scattering probability for elementary constituents (quarks and gluons in QCD) to produce the final state particles we are interested in. In turn, the hard scattering cross-section can be computed in perturbative Quantum Field Theory starting from an expansion in Feynman diagrams, which involve an increasing number of loops and external legs. In this context, the present manuscript is about a new idea to simplify one step in the calculation of the relevant Feynman diagrams, in particular for processes that involve the scattering of 4 or more elementary particles.

In fact, the last two decades have witnessed impressive developments in higher order calculations, also channeled by a steady increase in the computational resources available and by the discovery of many new computational techniques which make it possible to use those resources more efficiently, as for example the recent introduction of finite fields based techniques [1–4]. As of today, the standard chain of manipulations towards the calculation of complicated scattering amplitudes can be loosely summarised in three step:

1. Write down the integrand in terms of so-called *scalar integrals*.

2. Reduce the scalar integrals to a basis of so-called *master integrals*.

3. Compute the master integrals analytically or numerically using, for example, the *differential equations method* or other direct integration techniques.

Typically, and admittedly oversimplifying a bit, the second step is the one that is computationally the most expensive. The third, instead, is the one the usually requires conceptually non-trivial information on the analytic properties of the integrals and the special functions required for their evaluation. Somewhat naively, one could instead claim that the first step is the most straightforward, as Feynman diagrams provide a universally applicable way to obtain the integrand for any QFT, and its decomposition into scalar integrals can be performed by the so-called tensor decomposition. As we will argue here, it turns out that their naive application to $2 \to 2$ amplitudes up to three loops or $2 \to 3$ ones at two loops, has revealed various shortcomings, see for example [5].

As we will discuss, the origins of some of these problems can be traced back to the use of the tensor decomposition technique in the so-called Conventional Dimensional Regularisation scheme (CDR). In this contribution I will present a possible improvement of the “standard” technique of tensor decomposition, which allows us to keep the complexity of the decomposition under control, even in the presence of multiple final states and at high number of loops, by performing the decomposition in the so-called ’t Hooft-Veltman scheme (tHV). For alternative approaches to these issues see [6, 7] and references therein.

2. Standard tensor decomposition method

Before describing the improvement we propose, let us recall the standard technique of tensor decomposition for scattering amplitudes in dimensional regularisation. The idea is extremely simple and can be summarised in two steps

1. Use Lorentz invariance, gauge invariance (and any other symmetries available) in order to parametrise the scattering amplitude in terms of tensor structures and scalar form factors. While this step usually assumes the parametrisation to be non-perturbative (or more precisely, loop independent), we will see that this is in general not the case in the standard approach.
2. Once the decomposition is known, make use of it to define projector operators that extract the relevant form factors from the corresponding Feynman diagrams.

Let us introduce a compact notation. If $\mathcal{A}$ is the scattering amplitude under consideration, we decompose it into N tensors T_i and form factors F_j as

$$\mathcal{A} = \sum_{i=1}^{N} F_i \, T_i \tag{1}$$

where the T_i are in general tensors in Lorentz space, but we do not explicitly indicate the Lorentz indices for ease of notation. We then define N projector operators

$$\mathcal{P}_j = \sum_{k=1}^{N} c_k^{(j)} T_k^\dagger \,,$$

decomposed in terms of the adjoint of the original tensors T_i, such that

$$\mathcal{P}_j \cdot \mathcal{A} = \sum_{k=1}^{N} c_k^{(j)} \sum_{pol} T_k^\dagger \mathcal{A} = F_j \,, \tag{2}$$

where eq. (2) also defines the action of the dot operator $\mathcal{P}_j \cdot \mathcal{A}$. To express the projectors in a compact form, it is then convenient to introduce the matrix

$$M_{ij} = T_i^\dagger \cdot T_j \,, \quad \text{with} \quad \mathcal{P}_j = \sum_{k=1}^{N} \left(M^{-1}\right)_{jk} T_k^\dagger \,. \tag{3}$$

Let us see how this works with a specific example, which also reveals some of the subtleties of the standard approach. We consider the scattering of four massless quarks in QCD, in the all-incoming kinematics:

$$q(p_1) + \bar{q}(p_2) + Q(p_3) + \bar{Q}(p_4) \to 0.$$

We assume for simplicity that the two quarks q and Q are of different type. We also introduce the usual Mandelstam invariants to parametrise the kinematics

$$s_{12} = (p_1 + p_2)^2 \,, \quad s_{13} = (p_1 + p_3)^2 \,, \quad s_{23} = (p_2 + p_3)^2 \quad \text{with} \quad s_{12} + s_{13} + s_{23} = 0 \,.$$

Using Lorentz covariance and parity invariance, which we will assume to hold for QCD, we start by building all tensor structures that can contribute to this scattering process at arbitrary number of loops. As long as we work in conventional dimensional regularisation (CDR), it is easy to see that already the first assumption in our construction fails: due to the fact that the γ-algebra is not closed in d continuous space-time dimensions, one can build infinite structures of the type

$$\bar{u}(p_1)\gamma^{\mu_1}\cdots\gamma^{\mu_n}u(p_2)\bar{u}(p_3)\gamma_{\mu_1}\cdots\gamma_{\mu_n}u(p_4) \tag{4}$$

with arbitrary numbers of γ-matrices. If we insist in working in d dimensions, we must therefore already give up obtaining a decomposition that is valid at all loops.

Nevertheless, not all is lost: in practice, we are usually interested in decomposing the amplitude up to a given number of loops. Going up to two loops, for example, it is easy to see from direct inspection of the Feynman diagrams, that one can never generate tensor structures of the type in eq. (4) with more than five γ-matrices attached on each fermion line. An easy exercise shows that only the following structures are possible [8]

$$\begin{aligned}
T_1 &= \bar{u}(p_2)\gamma_{\mu_1}u(p_1)\,\bar{u}(p_4)\gamma^{\mu_1}u(p_3)\,,\\
T_2 &= \bar{u}(p_2)\not{p}_3u(p_1)\,\bar{u}(p_4)\not{p}_1u(p_3)\,,\\
T_3 &= \bar{u}(p_2)\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}u(p_1)\,\bar{u}(p_4)\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}u(p_3)\,,\\
T_4 &= \bar{u}(p_2)\gamma_{\mu_1}\not{p}_3\gamma_{\mu_3}u(p_1)\,\bar{u}(p_4)\gamma^{\mu_1}\not{p}_1\gamma^{\mu_3}u(p_3)\,,\\
T_5 &= \bar{u}(p_2)\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5}u(p_1)\,\bar{u}(p_4)\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5}u(p_3)\,,\\
T_6 &= \bar{u}(p_2)\gamma_{\mu_1}\gamma_{\mu_2}\not{p}_3\gamma_{\mu_4}\gamma_{\mu_5}u(p_1)\,\bar{u}(p_4)\gamma^{\mu_1}\gamma^{\mu_2}\not{p}_1\gamma^{\mu_4}\gamma^{\mu_5}u(p_3)\,.
\end{aligned} \tag{5}$$

We can then obtain the projector matrix as described above. The result is pretty cumbersome and as exemplification we report only its first row, which corresponds to the first projector

$$\begin{aligned}
P_1 = {} & \frac{1}{480 s_{13}s_{23}^2s_{12}^2(d-5)(d-6)(d-7)(d-3)(d-4)}\Big[\\
& + s_{13}(-9240s_{23}^2d^3 - 35040s_{13}s_{23}d^2 + 52160s_{13}^2d - 61120s_{13}^2 + 61620s_{23}^2d^2 \\
& + 164320s_{13}s_{23}d + 202496s_{23}^2 - 12720s_{13}^2d^2 - 182480s_{23}^2d + 960s_{13}^2d^3 - 259840s_{13}s_{23} \\
& + 525s_{23}^2d^4 + 2520s_{13}s_{23}d^3)T_1^\dagger \\
& - 10s_{13}(24s_{13}^2d^2 - 952s_{13}s_{23}d + 102s_{13}s_{23}d^2 - 1568s_{23}^2 + 2344s_{13}s_{23} - 264s_{23}^2d^2 \\
& + 21s_{23}^2d^3 + 256s_{13}^2 - 176s_{13}^2d + 1124s_{23}^2d)T_3^\dagger \\
& - 15(d-6)(35s_{23}^2d^3 - 55s_{13}s_{23}d^3 + 1046s_{13}s_{23}d^2 - 1872s_{13}^2d + 2432s_{13}^2 - 454s_{23}^2d^2 \\
& - 6040s_{13}s_{23}d - 2688s_{23}^2 + 368s_{13}^2d^2 + 1928s_{23}^2d - 20s_{13}^2d^3 + 11136s_{13}s_{23})T_2^\dagger \\
& + s_{13}s_{23}(-320s_{13} + 15s_{23}d^2 - 110s_{23}d + 224s_{23} + 60s_{13}d)T_5^\dagger \\
& - 5(-102s_{23}^2d + 15s_{23}^2d^2 - 1048s_{13}s_{23} + 168s_{23}^2 + 88s_{13}^2d - 128s_{13}^2 - 27s_{13}s_{23}d^2 \\
& + 326s_{13}s_{23}d - 12s_{13}^2d^2)T_6^\dagger \\
& + 30(21s_{23}^2d^3 - 37s_{13}s_{23}d^3 + 672s_{13}s_{23}d^2 - 1104s_{13}^2d + 1360s_{13}^2 - 256s_{23}^2d^2 - 3868s_{13}s_{23}d \\
& - 1344s_{23}^2 + 244s_{13}^2d^2 + 1036s_{23}^2d - 16s_{13}^2d^3 + 7328s_{13}s_{23})T_4^\dagger\Big],
\end{aligned} \tag{6}$$

and similarly complicated expressions can be obtained for the remaining projectors, see [8]. We notice immediately a clear issue in these expressions: not only are they cumbersome, but they also contain clearly unphysical poles as $d \to 4$, which are the manifestation of the fact that the structures in (5) are not linearly independent in $d = 4$. This issue becomes more serious with the increase of the number of loops and of the number of the external legs. In the next section we will see how this can be improved upon by performing the tensor decomposition in the tHV scheme [9].

3. Tensor decomposition in 't Hooft-Veltman scheme

The observation that leads us to introduce this new approach to tensor decomposition is the following: ultimately, both for formal and phenomenological applications we are often not interested in computing the form factors in CDR, but instead the so-called Helicity Amplitudes, which are often computed in the tHV scheme. In this scheme, external states are assumed to be four dimensional, while all Lorentz indices that are not contracted with external polarisation vectors or external momenta are kept in d space-time dimensions.

Let us imagine to fix the helicities of the external states on the decomposition in eq. (1). For simplicity we assume that all external states are massless and helicity is a good quantum number, but the very same argument can be used, for example, using chirality of external massive fermions. If there are E external particles and the helicity of particle i of momentum p_i is λ_i, we write

$$\mathcal{A}(\lambda_1, ..., \lambda_E) = \sum_{i=1}^{N} F_i T_i(\lambda_1, ..., \lambda_E) = \sum_{i=1}^{M \leq N} \widetilde{F}_i S_i(\lambda_1, ..., \lambda_E)\,, \tag{7}$$

where we used the fact that the only dependence on the helicities of the external states is in the tensors T_i, and in the second equality we reorganised the tensors in terms of a smaller (or at most equal) number of structures which carry information on the helicities of the external states $S_i(\lambda_1, ..., \lambda_E)$. Typically, in the massless case, a common representation for the $S_i(\lambda_1, ..., \lambda_E)$ would be in terms of spinor products, momentum twistors etc. The crucial point here is that their number $M \leq N$. Moreover, from eq. (7) it is clear that the number of independent form factors $\widetilde{F}_i$ relevant for the calculation of the helicity amplitudes, must be equal to the number of independent helicity amplitudes. Can we then perform a tensor decomposition that requires introducing from the beginning only the tensors that will be relevant to the computation of the helicity amplitudes? How we showed in [10, 11], this is indeed possible.

Let us see how this works for the case of four-quark scattering. We start off by noticing that conservation of helicity along the massless fermion lines tells us that there is a total of four helicity configurations possible. Moreover, the fact that parity invariance acts trivially for $2 \to 2$ scattering amplitudes[1], implies that only two helicity amplitudes are truly independent and we expect, therefore, that only two tensor structures should be enough. Going now back to the tensors in eq. (5), it is easy to see that the first two tensors are the only purely four dimensional ones: in fact, even if the γ-matrices are continued to d space-time dimensions, the fact that each fermion

[1]For this to be true, we need not only that the theory is parity even (QCD), but also to be dealing with the scattering of at most 4 particles. As it is well known, it is in fact not possible to build any parity odd invariant with only three independent momenta in four dimensions.

line contains at most one γ matrix guarantees that all components beyond the four dimensions are projected out by the spinor lines. On the contrary, by a simple application of dimensional splitting for the γ matrices (see for example [12]), it is easy to see that in tHV all remaining tensor structures can be written as linear combinations of the first two structures, with d dependent coefficients.

Let us then define the two independent tensors in $d = 4$ $\overline{T}_i = T_i$, $i = 1, 2$ and the matrix $M^{2\times 2}_{ij} = T^\dagger_i \cdot T_j$, which has a smooth inverse in $d = 4$

$$\left(M^{2\times 2}\right)^{-1}_{ij} = \frac{1}{d-3} X_{ij} \,, \quad X_{ij} = \frac{1}{4s_{12}^2} \begin{pmatrix} 1 & \frac{s_{12}+2s_{23}}{s_{23}(s_{12}+s_{23})} \\ \frac{s_{12}+2s_{23}}{s_{23}(s_{12}+s_{23})} & \frac{ds_{12}^2-2s_{12}^2+4s_{12}s_{23}+4s_{23}^2}{s_{23}^2(s_{12}+s_{23})^2} \end{pmatrix} . \tag{8}$$

From here, we can read out the two four-dimensional projectors

$$P^{2\times 2}_i = \sum_{j=1}^{2} \left(M^{(2\times 2)}_{ij}\right)^{-1} \overline{T}^\dagger_j . \tag{9}$$

As a next step, we would like to show that these two projectors are everything that we need, at any loop order, to obtain the helicity amplitudes in tHV. To prove that this is indeed the case, we start by a procedure of Gram-Schmidt orthogonalisation of the original tensors, namely we define

$$\overline{T}_i = T_i - \sum_{j=1}^{2} \left(P^{2\times 2}_j T_i\right) \overline{T}_j \,, \quad \text{for} \quad i \geq 3 \,.$$

As an explicit example, we find that

$$\overline{T}_3 = \left(-3d - \frac{12s_{23}}{s_{12}} - 4\right)\overline{T}_1 - \frac{24}{s_{12}}\overline{T}_2 + T_3 . \tag{10}$$

It is then easy to see that, by construction, the new tensors are identically zero when the external states are taken to be four dimensional

$$\overline{T}_i(\lambda_1, ..., \lambda_E) = 0 \,, \qquad \text{for} \quad i = 3, 4, 5, 6, ... \tag{11}$$

where the dots indicate also the extra tensors that might contribute only starting at three or more loops. This means that if we rotate our basis tensors from $T_i \to \overline{T}_i$, despite starting with in principle an infinite number of tensor structures, only the first two form factors can contribute to any of the helicity amplitudes at any number of loops

$$\mathcal{A}(\lambda_q, \lambda_{\bar{q}}, \lambda_Q, \lambda_{\bar{Q}}) = \sum_{i=1}^{\infty} F_i T_i(\lambda_q, \lambda_{\bar{q}}, \lambda_Q, \lambda_{\bar{Q}}) = \sum_{i=1}^{2} \overline{F}_i \overline{T}_i(\lambda_q, \lambda_{\bar{q}}, \lambda_Q, \lambda_{\bar{Q}}) \,. \tag{12}$$

We stress that the zeros in eq. (11) are *exact in d*, as long as the external states are confined to four space-time dimensions, as it can be seen by direct inspection of eq. (10). We reiterate that this is not a coincidence, but a simple consequence of the way the $\overline{T}_i$ have been build, namely by subtracting from them their projection along the two-dimensional space spanned by the two independent helicity amplitudes in four dimensions.

We also notice that, once more by construction, the projectors for the extra tensors $\overline{T}_i$ are all decoupled from the first two, namely if we build the matrix

$$M_{ij} = \overline{T}_i^\dagger \cdot \overline{T}_j \qquad \text{for} \quad i,j = 1,\ldots,6\,, \quad \text{such that} \quad \left(M^{-1}\right)_{ij} = \begin{pmatrix} \frac{X_{ij}}{(d-3)} & 0\cdots 0 \\ 0 & \\ \vdots & R_{ij} \\ 0 & \end{pmatrix},$$

where the entire complexity seen in eq. (6) is now hidden in the irrelevant part of the matrix R_{ij}, and the matrix X_{ij} has been defined in eq. (8). Finally, by fixing the helicities (12) we get for the two independent combinations

$$\mathcal{A}_{+-+-} = (2s_{13}F_1 - s_{13}s_{23}F_2)\,\frac{\langle 13\rangle}{\langle 24\rangle}\,, \qquad \mathcal{A}_{+--+} = (2s_{23}F_1 + s_{13}s_{23}F_2)\,\frac{\langle 14\rangle}{\langle 23\rangle}\,. \tag{13}$$

Importantly, for this construction to work, it is necessary that the independent tensors are chosen to span exactly the four-dimensional space of the physical helicity amplitudes in tHV scheme. This subtlety can be appreciated in a different example, also originally described in [11]. Let us consider the scattering of four gluons

$$g(p_1) + g(p_2) + g(p_3) + g(p_4) \to 0\,.$$

By using Lorentz and gauge invariance (in particular fixing the gauge of the gluons cyclically as $\epsilon_i \cdot p_{i+1} = 0$) we find that there are 10 tensors in d space-time dimensions [13]

$$\begin{aligned} &T_1 = \epsilon_1\cdot p_3\,\epsilon_2\cdot p_1\,\epsilon_3\cdot p_1\,\epsilon_4\cdot p_2\,, \quad T_2 = \epsilon_1\cdot p_3\,\epsilon_2\cdot p_1\,\epsilon_3\cdot\epsilon_4\,, \quad T_3 = \epsilon_1\cdot p_3\,\epsilon_3\cdot p_1\,\epsilon_2\cdot\epsilon_4\,,\\ &T_4 = \epsilon_1\cdot p_3\,\epsilon_4\cdot p_2\,\epsilon_2\cdot\epsilon_3\,, T_5 = \epsilon_2\cdot p_1\,\epsilon_3\cdot p_1\,\epsilon_1\cdot\epsilon_4\,, \quad T_6 = \epsilon_2\cdot p_1\,\epsilon_4\cdot p_2\,\epsilon_1\cdot\epsilon_3\,,\\ &T_7 = \epsilon_3\cdot p_1\,\epsilon_4\cdot p_2\,\epsilon_1\cdot\epsilon_2\,, T_8 = \epsilon_1\cdot\epsilon_2\,\epsilon_3\cdot\epsilon_4\,, \quad T_9 = \epsilon_1\cdot\epsilon_4\,\epsilon_2\cdot\epsilon_3\,, \quad T_{10} = \epsilon_1\cdot\epsilon_3\,\epsilon_2\cdot\epsilon_4\,. \end{aligned} \tag{14}$$

On the other hand, a simple counting reveals that there are only 8 independent helicity amplitudes accounting for parity invariance. The question we ask ourselves here is, how do we choose which of the 10 tensors in eq. (14) are the correct ones to span the four-dimensional space? The trick to identify the right combinations, is to notice that we can complete the vectors $p_1^\mu, p_2^\mu, p_3^\mu$ with the additional transverse four-dimensional vector $v^\mu = \epsilon^{\mu\nu\rho\sigma} p_{1,\nu} p_{2,\rho} p_{3,\sigma}$. The basis $q_i^\mu = \{p_1^\mu, p_2^\mu, p_3^\mu, v^\mu\}$ is purely four-dimensional and spans the full space, which means that $g^{\mu\nu}$ is not necessary to build the tensors, since it can be re-expressed in terms of the vectors q_j^μ,

$$g^{\mu\nu} = \sum_{ij} c_{ij} q_i^\mu q_j^\nu + O(d-4)\,. \tag{15}$$

With the usual gauge choice, we see that the four vectors allow us to build 8 possible tensor structures

$$\begin{aligned} &\widetilde{T}_1 = \epsilon_1\cdot p_3\,\epsilon_2\cdot p_1\,\epsilon_3\cdot p_1\,\epsilon_4\cdot p_2\,, \quad \widetilde{T}_2 = \epsilon_1\cdot p_3\,\epsilon_2\cdot p_1\,\epsilon_3\cdot v\,\epsilon_4\cdot v\,,\\ &\widetilde{T}_3 = \epsilon_1\cdot p_3\,\epsilon_3\cdot p_1\,\epsilon_2\cdot v\,\epsilon_4\cdot v\,, \quad \widetilde{T}_4 = \epsilon_1\cdot p_3\,\epsilon_4\cdot p_2\,\epsilon_2\cdot v\,\epsilon_3\cdot v\,,\\ &\widetilde{T}_5 = \epsilon_2\cdot p_1\,\epsilon_3\cdot p_1\,\epsilon_1\cdot v\,\epsilon_4\cdot v\,, \quad \widetilde{T}_6 = \epsilon_2\cdot p_1\,\epsilon_4\cdot p_2\,\epsilon_1\cdot v\,\epsilon_3\cdot v\,,\\ &\widetilde{T}_7 = \epsilon_3\cdot p_1\,\epsilon_4\cdot p_2\,\epsilon_1\cdot v\,\epsilon_2\cdot v\,, \quad \widetilde{T}_8 = \epsilon_1\cdot v\,\epsilon_2\cdot v\,\epsilon_3\cdot v\,\epsilon_4\cdot v\,, \end{aligned} \tag{16}$$

PoS(LL2022)020

where we used the fact that only even numbers of v^μ are allowed in a parity even theory as QCD. We could go ahead and use these 8 tensors, but that would require to manipulate expressions with multiple occurrences of the Levi-Civita tensor, whose contractions generate many terms and might be impractical to handle. This can be remedied by realising that eq. (15) can be "inverted". In particular, we can write

$$v^\mu v^\nu \sim g^{\mu\nu} + \dots, \quad v^\mu v^\nu v^\rho v^\sigma \sim (g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}) + \dots . \tag{17}$$

For our argument, the exact values of the coefficients is irrelevant. Instead, using the fact that all tensors build out of the external momenta p_i^μ already appeared in eq. (16), the equations above only tell us that we can swap the products of two v^μ with $g^{\mu\nu}$ and the product of four v^μ with the symmetric combination of $g^{\mu\nu} g^{\rho\sigma}$. In terms of the tensor structures, this implies the following choice for the tensor structures

$$\begin{aligned}
&T_1 = \epsilon_1 \cdot p_3\, \epsilon_2 \cdot p_1\, \epsilon_3 \cdot p_1\, \epsilon_4 \cdot p_2 , \quad T_2 = \epsilon_1 \cdot p_3\, \epsilon_2 \cdot p_1\, \epsilon_3 \cdot \epsilon_4 , \quad T_3 = \epsilon_1 \cdot p_3\, \epsilon_3 \cdot p_1\, \epsilon_2 \cdot \epsilon_4 , \\
&T_4 = \epsilon_1 \cdot p_3\, \epsilon_4 \cdot p_2\, \epsilon_2 \cdot \epsilon_3 , \quad T_5 = \epsilon_2 \cdot p_1\, \epsilon_3 \cdot p_1\, \epsilon_1 \cdot \epsilon_4 , \quad T_6 = \epsilon_2 \cdot p_1\, \epsilon_4 \cdot p_2\, \epsilon_1 \cdot \epsilon_3 , \\
&T_7 = \epsilon_3 \cdot p_1\, \epsilon_4 \cdot p_2\, \epsilon_1 \cdot \epsilon_2 , \quad T_8 = \epsilon_1 \cdot \epsilon_2\, \epsilon_3 \cdot \epsilon_4 + \epsilon_1 \cdot \epsilon_4\, \epsilon_2 \cdot \epsilon_3 + \epsilon_1 \cdot \epsilon_3\, \epsilon_2 \cdot \epsilon_4 .
\end{aligned} \tag{18}$$

Indeed, this choice is the correct one to span the physical space of the helicity amplitudes in tHV, which implies that no spurious poles as $d \to 4$ appear in the projectors and also that the two evanescent tensors can be neglected at every order in $(d-4)$ exactly. We refer to [11] for extra details on this construction and on its extensions to other cases. Of particular interest is the application of these ideas to the scattering of five or more particles. While, in fact, in the case of $2 \to 2$ scattering the gain in simplicity from using our method in tHV versus the standard CDR decomposition, does not look impressive from a practical point of view, it was noticed that the extension of the CDR tensor decomposition to five or more particle scattering quickly becomes unfeasible [5]. This is because the number of tensors structures increases enormously and the complexity of the corresponding projectors makes their use impractical. The method described here solves this problem very elegantly, inasmuch as the number of tensors and form factors can only grow as fast as the number of different helicity amplitudes. For the scattering of six gluons, for example, this number is bounded by $2^6 = 64$ tensor structures, versus the thousands of structures that one would obtain in the standard CDR decomposition.

4. Conclusions

In this contribution, I have shown how the standard method of tensor decomposition can be simplified when applied in the tHV scheme. In particular, one can always choose a number of tensor structures that matches the number of independent helicity amplitudes and, in doing so, one finds that the relevant projectors become much simpler than in the standard approach. While here we have only looked into simple cases involving $2 \to 2$ scattering, even more substantial simplifications can be achieved when considering the scattering of 5 or more particles. As described in detail in [10, 11], in those cases the existence of a complete basis of four dimensional momenta allows us to perform a decomposition as in (16), without having to introduce the extra vector v^μ. This new type of decomposition can also be effectively applied in the presence of massive fermions and vector bosons and in theories that are not parity invariant.

PoS(LL2022)020

Acknowledgements

I am indebted to Tiziano Peraro, with whom the ideas presented in this contribution have been originally developed. My research was supported by the Excellence Cluster ORIGINS funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy - EXC-2094 - 390783311 and by the ERC Starting Grant 949279 HighPHun.

References

[1] A. von Manteuffel and R. M. Schabinger, *A novel approach to integration by parts reduction*, *Phys. Lett.* **B744** (2015) 101–104, [arXiv:1406.4513].

[2] T. Peraro, *Scattering amplitudes over finite fields and multivariate functional reconstruction*, *JHEP* **12** (2016) 030, [arXiv:1608.01902].

[3] S. Abreu, J. Dormans, F. Febres Cordero, H. Ita, B. Page, and V. Sotnikov, *Analytic Form of the Planar Two-Loop Five-Parton Scattering Amplitudes in QCD*, *JHEP* **05** (2019) 084, [arXiv:1904.00945].

[4] G. De Laurentis and D. Maître, *Two-Loop Five-Parton Leading-Colour Finite Remainders in the Spinor-Helicity Formalism*, *JHEP* **02** (2021) 016, [arXiv:2010.14525].

[5] R. H. Boels, Q. Jin, and H. Luo, *Efficient integrand reduction for particles with spin*, arXiv:1802.06761.

[6] L. Chen, *A prescription for projectors to compute helicity amplitudes in D dimensions*, *Eur. Phys. J. C* **81** (2021), no. 5 417, [arXiv:1904.00705].

[7] J. Davies, G. Mishima, M. Steinhauser, and D. Wellmann, *$gg \to ZZ$: analytic two-loop results for the low- and high-energy regions*, *JHEP* **04** (2020) 024, [arXiv:2002.05558].

[8] E. Glover, *Two loop QCD helicity amplitudes for massless quark quark scattering*, *JHEP* **04** (2004) 021, [hep-ph/0401119].

[9] G. 't Hooft and M. J. G. Veltman, *Regularization and Renormalization of Gauge Fields*, *Nucl. Phys.* **B44** (1972) 189–213.

[10] T. Peraro and L. Tancredi, *Physical projectors for multi-leg helicity amplitudes*, *JHEP* **07** (2019) 114, [arXiv:1906.03298].

[11] T. Peraro and L. Tancredi, *Tensor decomposition for bosonic and fermionic scattering amplitudes*, *Phys. Rev. D* **103** (2021), no. 5 054042, [arXiv:2012.00820].

[12] G. Cullen, M. Koch-Janusz, and T. Reiter, *Spinney: A Form Library for Helicity Spinors*, *Comput. Phys. Commun.* **182** (2011) 2368–2387, [arXiv:1008.0803].

[13] T. Binoth, E. Glover, P. Marquard, and J. van der Bij, *Two loop corrections to light by light scattering in supersymmetric QED*, *JHEP* **05** (2002) 060, [hep-ph/0202266].

PoS(LL2022)020