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Zero-jettiness beam functions at N^{3}LO

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The zero-jettiness beam functions describe collinear emissions from initial state legs and appear in the factorisation theorem for cross sections in the limit of small zero-jettiness. They are an important building block for slicing schemes for colour-singlet production at hadron colliders. We report on our ongoing calculation of this quantity at next-to-next-to-next-to-leading order (N^{3}LO) in QCD, highlighting in particular the aspects of partial fraction relations and the calculation of master integrals.

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1. Introduction

With the LHC collecting more and more data, there has been a continuous push for higher precision, both in experimental measurements and in theoretical predictions. Over the last few years, first N^{3}LO QCD predictions for hadron collider observables have started to appear – first for inclusive quantities [1–9], and more recently also for differential distributions [10–18]. A first obvious target for such high-precision calculations are colour-singlet production processes, e.g., Higgs boson production or Drell-Yan processes. They serve as standard candles for the Standard Model (SM), but they are also crucial for many important measurements, including determinations of SM parameters, such as the strong coupling constant α_s, parton distribution functions, the weak boson masses etc.

Differential calculations beyond leading order require a prescription for how to deal with infrared (IR) singularities, which appear due to massless particles becoming soft or collinear to each other. They are typically regulated in dimensional regularisation and appear as poles in the dimensional regulator ϵ. For appropriate (IR-safe) observables, these poles cancel upon combining real and virtual contributions. However, since they appear in real emission contributions only after integrating over the phase space of unresolved particles, this poses a problem for numerical calculations, which are typically required for differential predictions. It is therefore necessary to define a scheme to extract and cancel these singularities before the numerical phase-space integration can be performed. There are many different schemes to deal with IR singularities available in the literature. Many of them fall in two broad categories: slicing schemes and subtraction schemes. At N^{3}LO, calculations based on slicing schemes seem more achievable at the moment due to the possibility to build on existing NNLO calculations for processes with higher multiplicities.

The idea of phase-space slicing is to subdivide the phase space into a fully-unresolved and a resolved region. To achieve this, one often uses an observable which is sensitive to the fully-unresolved region and then defines a threshold of this observable that separates the two regions. Below the threshold lies the fully-unresolved region which contains the most complicated singularity structure. However, if the slicing variables is chosen well, the cross section in this region significantly simplifies thanks to, e.g., factorisation theorems. This approximation captures the most singular behaviour and the integration over the unresolved phase space can then be performed analytically, thereby exposing the poles in ϵ explicitly. Above the threshold, the cross section is screened from the fully unresolved configuration due to the threshold and therefore corresponds to the same process at one order lower in perturbation theory but with an additional hard parton. For colour-singlet production at N^{3}LO, for example, the region above the threshold can equivalently be interpreted as colour-singlet plus one jet production at NNLO, where the threshold of the slicing observable determines the cuts on the jet. The IR singularities in this region can be dealt with using available NNLO subtraction schemes. If the threshold is chosen small enough, the sum of the two phase-space regions should become independent of the value of the threshold parameter, in spite of the approximation made below the threshold.

There are several possible slicing variables. The two most common ones for colour-singlet production are the transverse momentum q_T of the colour-singlet state [19] and zero-jettiness τ

[20]. The latter is defined as [21]

$$\tau = \sum_j \min_{i\in\{1,2\}} \frac{p_i \cdot k_j}{Q_j}\,, \tag{1}$$

where j labels final state partons with momenta k_j and the momenta $p_i, i \in \{1,2\}$, are the momenta of the incoming partons. The Q_i are normalisation scales. The definition ensures that the zero-jettiness variable τ vanishes exactly when all emission momenta k_j become soft or collinear to one of the initial state partons, i.e. when the configuration approaches the fully-unresolved limit.

For a zero-jettiness slicing scheme, we are interested in finding an approximation of the cross section in the fully-unresolved region. In the limit $\tau \to 0$, there exists a factorisation theorem [21–23], which was proven in soft-collinear effective theory (SCET), that the cross-section factorises as follows,

$$\lim_{\tau\to 0} \sigma = B \otimes B \otimes S \otimes H \otimes \sigma_{\mathrm{LO}} + O(\tau)\,. \tag{2}$$

Here, H is the hard function, which describes corrections to the hard process. The soft function S describes the effects of soft, non-collinear gluons or $q\bar{q}$ pairs. It is known to NNLO and the calculation of the N^{3}LO contribution is currently underway. Finally, there are the beam functions B, which describe collinear emissions off initial state partons. First N^{3}LO results for the beam functions in singular limits were published in Ref. [24] and in the large $N_c \sim n_f$ limit in Refs. [25–27]. The full result was published in Ref. [28]. Here we report on the ongoing independent calculation of these beam functions at N^{3}LO, which is certainly warranted given the complexity of this problem.

2. Beam functions for zero-jettiness

The beam functions for zero-jettiness are non-perturbative objects which depend on the longitudinal momentum fraction x and the transverse virtuality $t = -((p^*)^2 - k_\perp^2)$, where p^* is the momentum of the parton entering the hard process and $k_\perp$ is the transverse component of the sum of momenta of all emitted partons. They are related to traditional collinear PDFs $f_i(x,\mu)$ via a convolution,

$$B_i(t,x,\mu) = \int_0^1 \mathrm{d}z \sum_{j\in\{q,\bar{q},g\}} I_{ij}(t,z,\mu) f_j\left(\frac{x}{z},\mu\right) + O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{t}\right), \tag{3}$$

where $i \in \{q,\bar{q},g\}$ is a flavour index, and $I_{ij}(t,z,\mu)$ are perturbatively calculable matching coefficients. At leading order the matching coefficient reads $I_{ij}^{\mathrm{LO}}(t,z,\mu) = \delta(1-z)\delta(t)\delta_{ij}$, which implies that the leading order beam functions correspond to $B_i^{\mathrm{LO}}(t,x,\mu) = f_i(x,\mu)\delta(t)$. The goal is to compute the matching coefficients at N^{3}LO in QCD.

In order to understand how to calculate the matching coefficients, it is useful to recall that both the zero-jettiness beam function and the PDFs are defined as matrix elements between proton states, $|P(p)\rangle$, of certain operators in SCET, which we write schematically as (the exact definition of the operators O_i and Q_i is not important for the following discussion)

$$B_i \sim \langle P(p)|O_i(t,xp,\mu)|P(p)\rangle\,, \qquad f_i \sim \langle P(p)|Q_j(x'p,\mu)|P(p)\rangle\,. \tag{4}$$

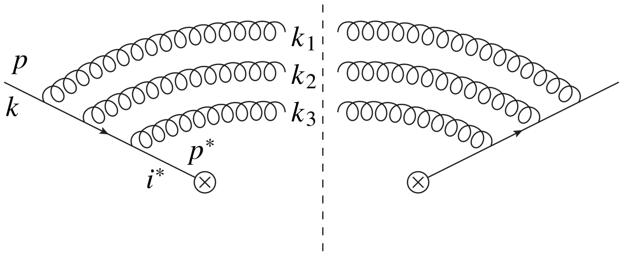

Figure 1: Diagram from the triple real contribution of the partonic beam function B_{ik}. The external parton carries momentum p and flavour k and the off-shell parton entering the hard process carries momentum p^* and flavour i^*. We label the momenta of the emitted partons by k_i.

The matching relation, mentioned above, arises from an operator product expansion, which is of course valid for arbitrary external states. Thus, the matching coefficients are independent of the external states which means that we can choose massless partons as external states to simplify the calculation. To this end, we define partonic beam functions B_{ik} and partonic PDFs f_{jk} by replacing the external states, i.e.,

$$B_{ik} \sim \langle p_k(p)|O_i(t, xp, \mu)|p_k(p)\rangle \,, \qquad\qquad f_{jk} \sim \langle p_k(p)|Q_j(x'p, \mu)|p_k(p)\rangle \,. \tag{5}$$

The partonic beam functions and PDFs carry an additional flavour index k, which corresponds to the flavour of the external parton state $|p_k(p)\rangle$. Since the partonic PDFs are expressible in terms of the DGLAP splitting functions, the matching coefficients can be determined by calculating the partonic beam functions B_{ij} in perturbative QCD.

Our calculation is based on the observation made in Ref. [29], that the partonic beam functions can be calculated from the collinear limits of QCD amplitudes via

$$B_{ik}^{\text{bare}} \sim \sum_{n_R} \int \prod_{i=1}^{n_R} \frac{\mathrm{d}^d k_i}{(2\pi)^{d-1}} \delta_+(k_i^2)\, \delta\left(2p\cdot k_{1\ldots n_R} - \frac{t}{z}\right) \delta\left(\frac{2\bar{p}\cdot k_{1\ldots n_R}}{s} - (1-z)\right) \frac{\hat{C}_p |M(p,\bar{p},\{k_i\})|^2}{|M_0(zp,\bar{p})|^2} \,, \tag{6}$$

where n_R is the number of additional partons radiated, $k_{1\ldots n_R} = k_1 + \cdots + k_{n_R}$ is the sum of their momenta and $\bar{p} = (p^0, -\vec{p})$ is used as the reference vector which is back-to-back with the incoming momentum p. The operator $\hat{C}_p$ extracts the collinear $k_i \parallel p$ limit from the squared matrix element $|M(p,\bar{p},\{k_i\})|^2$. Since we divide by the corresponding reduced matrix element $|M_0(zp,\bar{p})|^2$, we essentially obtain the splitting function from the ratio $\hat{C}_p|M(p,\bar{p},\{k_i\})|^2/|M_0(zp,\bar{p})|^2$ and we have to integrate over the phase space of the emission momenta, which is constrained by the two delta functions that introduce the observables t and z. We treat the delta functions using reverse unitarity [30] by expressing them as cut propagators. To construct the splitting functions, we follow Ref. [31] and generate diagrams for an on-shell parton with momentum p and flavour k going to an off-shell parton with momentum p^* and flavour i and additional emissions of n_R partons. The off-shell parton is then connected to a suitable projector which extracts the collinear behaviour. For example, for quarks the Feynman rule for the operator reads

$$p^* \;\rightarrow\!\otimes \quad \otimes\!\rightarrow \quad \rightarrow \frac{\bar{p}_\mu \gamma^\mu}{4N_c\, p^*\cdot\bar{p}} \,. \tag{7}$$

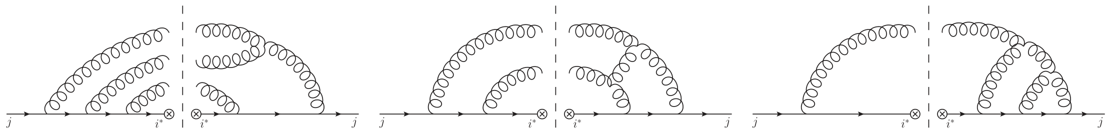

Figure 2: Example diagrams for the triple-real (RRR), double-real-single-virtual (RRV) and single-real-double-virtual (RVV) contributions.

For the rest of the diagrams we use standard QCD Feynman rules, with the only further peculiarity that we have to work in axial gauge, where the gluon propagator reads

$$k \quad \rightarrow \frac{i\left(-g^{\mu\nu} + \frac{k^\mu \bar{p}^\nu + k^\nu \bar{p}^\mu}{k\cdot\bar{p}}\right)}{k^2 + i0^+} . \tag{8}$$

This ensures that the leading collinear behaviour can be obtained by considering only emissions off a single leg. The second term in the numerator in Eq. (8) introduces linear propagators.

3. Calculation

In general, the calculation of the bare beam function follows a mostly standard chain of calculational steps. We generate diagrams for the process $k \to i^* +$ emissions, interfere them with the appropriate, complex conjugated diagrams and perform the Dirac and colour algebra. We then apply partial fractioning to be able to map the scalar integrals to integral families, which we further reduce to master integrals using integration-by-parts identities. Afterwards, we use partial fractioning once more on the master integrals, in order to find relations between master integrals from different integral families and thereby reduce the overall number of master integrals. To solve the master integrals we use the method of differential equations and we fix the integration constants in the limit $z \to 1$. Finally, we apply renormalisation and IR subtractions in order to translate the bare partonic beam functions to partonic beam functions, from which we can finally extract the matching coefficients.

We organise the calculation according to the number of virtual loop integrations that the different contributions require. There are triple-real (RRR), double-real-single-virtual (RRV) and single-real-double-virtual (RVV) contributions, but there are no contributions from triple-virtual diagrams to the beam functions. Examples for diagrams appearing in these three contributions are shown in Fig. 2. The calculation of the RRR and RRV contributions follows the steps described above. This amounts to having to calculate about 450 three-loop master integrals. The solutions to these master integrals depend on iterated integrals over an alphabet containing both linear letters,

$$f_a(z) = \frac{1}{z-a}, \qquad \text{where } a \in \left\{0, \pm 1, \pm 2, \pm 4, \pm 2i, \exp\left(\pm i\pi\frac{2}{3}\right)\right\} , \tag{9}$$

and square-root valued letters

$$\left\{\frac{1}{\sqrt{z(4-z)}}, \frac{1}{\sqrt{z(4+z)}}, \frac{1}{\sqrt{4+z^2}}, \frac{1}{z\sqrt{4+z^2}}\right\} . \tag{10}$$

The RVV contribution, on the other hand, we calculate using the results for the two-loop splitting functions published in the literature [32, 33].

In the following, we would like to highlight two particular aspects of the calculation: the use of partial fractioning and the calculation of the master integrals. We have to employ partial fractioning due to linear relations between propagators, which are, for example, introduced by the delta functions that describe the observables. An example from the RRV contribution for such a linear relation is

$$2(k_1 + k_2) \cdot \bar{p} = s(1 - z)\,, \tag{11}$$

which gives rise to partial fractioning identities such as

$$\frac{1}{(k_1 \cdot \bar{p})(k_2 \cdot \bar{p})} = \frac{2}{s(1-z)} \left[\frac{1}{k_1 \cdot \bar{p}} + \frac{1}{k_2 \cdot \bar{p}} \right] . \tag{12}$$

In order to map the scalar integrals to integral families, all propagators appearing in the integrals have to be linearly independent. Therefore, we have to apply partial fractioning to map integrals with linearly dependent propagators to sums of integrals with linearly independent propagators. Similar to linear relations induced by the delta functions, each gluon propagator in the axial gauge introduces a linear propagator $(k \cdot \bar{p})^{-1}$ which also leads to linearly dependent propagators: Since there are three linearly independent scalar products involving $\bar{p}$ and since every integral has to contain at least the cut propagator corresponding to $\delta(2k_{1\ldots n_R} \cdot \bar{p} - s(1-z))$ it is easy so see that many of the integrals arising from diagrams with more than two gluons require partial fractioning in order to be mapped to integral families. To systematically apply the partial fractioning identities, we use Gröbner bases, as described in Refs. [34, 35] (see also Refs. [36–38] for related work).

In addition, we also apply partial fractioning to derive relations between master integrals from different integral families. An example for such a relation from the RRR contribution is

$$\begin{aligned}
&\overbrace{\int \frac{\mathrm{d}\Phi_B}{(k_1 - p)^2 (k_{13} - p)^2 (k_3 \cdot \bar{p})(k_{13} \cdot \bar{p})}}^{=I^{\mathrm{T29}}_{3391}} \\
&\quad = \underbrace{\int \frac{\mathrm{d}\Phi_B}{(k_1 - p)^2 (k_{12} - p)^2 (k_1 \cdot \bar{p})(k_2 \cdot \bar{p})}}_{=I^{\mathrm{T1}}_{3263}} - \underbrace{\int \frac{\mathrm{d}\Phi_B}{(k_1 - p)^2 (k_{13} - p)^2 (k_1 \cdot \bar{p})(k_{13} \cdot \bar{p})}}_{=I^{\mathrm{T28}}_{3291}} ,
\end{aligned} \tag{13}$$

where $\mathrm{d}\Phi_B$ is the phase-space measure shown in Eq. (6) and I^f_s represents an integral from integral family f and sector s. The relation arises from the linear dependence of the linear propagators in the three integrals, which is immediately clear at the integrand level after relabelling $k_2 \leftrightarrow k_3$. Since each integral family can only contain linearly independent propagators, the three master integrals must belong to different integral families. Applying these relations significantly reduces the number of master integrals that have to be calculated. We construct these relations by first generating a list of seed integrals from all sectors of all integral families, computing a Gröbner basis for the set of all appearing propagators and applying it to all seed integrals. Subsequently, we apply IBP relations

PoS(LL2022)027

to reduce all appearing integrals to master integrals and solve the resulting linear system. This approach is rather brute force and leaves open the question whether all possible relations are found in this way. The relation in Eq. (13) already requires the shift $k_2 \leftrightarrow k_3$, which shows that in each term of the partial fractioning relation, there exists a freedom to use symmetries of the Feynman integrals such as, for example, loop momentum shifts. It stands to reason that there might be partial fractioning relations which cannot be found by the procedure described above since the construction of the Gröbner basis is based on one fixed choice for the momentum space representation of the propagators. It would certainly be interesting to investigate in the future if this construction can be further refined.

The second aspect we would like to discuss in some more detail concerns the calculation of the master integrals. In principle, the integrals depend on three kinematic variables: the centre of mass energy squared s, the transverse virtuality t and the longitudinal momentum fraction z. By rescaling $k_i = \tilde{k}_i\sqrt{t}$, $p = \tilde{p}\sqrt{t}$ and $\bar{p} = \tilde{\bar{p}}s/\sqrt{t}$, the dependence on s and t can be scaled out, e.g.,

$$I^{\mathrm{RRV1}}_{n_1,\dots,n_8}(s,t,z) = s^{-1-n_7-n_8}t^{3-3\epsilon-(n_1+\dots+n_6)}I^{\mathrm{RRV1}}_{n_1,\dots,n_8}(z)\,, \tag{14}$$

and the only remaining non-trivial dependence is that on z. Thus, we derive differential equations in z for the master integrals and fix the boundary conditions by computing the integrals in the limit $z \to 1$. To calculate the integrals in this limit, we use a number of different techniques, including deriving constraints between different integration constants from the analytic structure of the integrals, direct calculation, auxiliary differential equations and mapping the limit of RRR integrals to threshold integrals that appear in Higgs production ($gg \to H$).

Let us briefly sketch the main ideas behind the last approach. To calculate the soft region of beam function master integral $I(s,t,z)$ in the limit $z \to 1$ we first introduce a new integral $\mathcal{B}(s,t,z)$, in which the propagators appearing in $I(s,t,z)$ are replaced by their eikonal approximations (but the integration measure is the same). In the limit $z \to 1$ the two integrals agree, $\lim_{z\to1} I(s,t,z) = \lim_{z\to1}\mathcal{B}(s,t,z)$. Since the integrand of $\mathcal{B}(s,t,z)$ is homogeneous in the integration momenta, also the dependence on z can be scaled out,

$$\mathcal{B}(s,t,z) = s^{e_s}t^{e_t}z^{-e_t}(1-z)^{e_{\bar{z}}}\tilde{\mathcal{B}}\,, \tag{15}$$

where e_s, e_t and $e_{\bar{z}}$ are exponents that can be read off from the integrand and integration measure and $\tilde{\mathcal{B}}$ is a numerical constant. Next, we compare the phase-space measure for the beam function to the phase-space measure for Higgs production in the threshold limit. The important aspect are the delta functions that appear. For the beam function they read

$$\mathcal{B}(s,t,z) \sim \delta\left(2k_{123}\cdot p - \frac{t}{z}\right)\delta\left(2k_{123}\cdot\bar{p} - s(1-z)\right)\,, \tag{16}$$

while the phase-space measure for Higgs production in the threshold limit contains

$$\mathcal{H}(s) \sim \delta\left(2k_{123}\cdot(p+\bar{p}) - s\right)\,. \tag{17}$$

The central idea is now to choose a special kinematic point, $t = sz^2$, and to integrate over $z \in [0,1]$,

$$\int_0^1 \mathrm{d}z\,\mathcal{B}(s,sz^2,z) = \int_0^1 \mathrm{d}z\,\delta(2k_{123}\cdot p - sz)\delta(2k_{123}\cdot\bar{p} - s(1-z)) = \frac{1}{s}\mathcal{H}(s)\,. \tag{18}$$

PoS(LL2022)027

Combining this with the information from Eq. (15), we find

$$\frac{1}{s}\mathcal{H}(s) = s^{e_s+e_t} B(e_t+1, e_{\bar{z}}+1)\tilde{\mathcal{B}}\,, \tag{19}$$

where $B(a,b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$ is the Euler Beta function. This shows that the numerical constant $\tilde{\mathcal{B}}$ which determines the limit $z \to 1$ of the beam function integrals is expressible in terms of Higgs production threshold integrals $\mathcal{H}(s)$. This connection is helpful because it allows to further relate the boundary constants by using IBP relations for the Higgs threshold integrals and because there are results available for these integrals in the literature [39–43]. We discovered that the integrals which we calculated for the RRR contribution [26] covers 9 of the 10 master integrals computed for Ref. [39], including the most complicated one from that reference, $\mathcal{F}_9$. Similarly, for the RRV contribution, our boundary constants cover several of the soft region master integrals from Ref. [40], including the most complicated one, $\mathcal{M}_{13}$. We note that this implies that it should be possible to use our results to independently redo the calculation of the N^{3}LO QCD corrections to the process $gg \to H$ in the threshold region.

4. Results

Overall there are five independent matching coefficients: $I_{q_i q_j}$, I_{qg}, I_{gq}, I_{gg} and $I_{q_i \bar{q}_j}$. Matching coefficients for other possible flavour combinations can be derived from the given list using crossing symmetries. We have published first N^{3}LO results for $I_{q_i q_j}$ in the large $N_c \sim n_f$ limit in Ref. [27]. The full result for all five independent matching coefficients was published in Ref. [28] by Ebert, Mistlberger and Vita. Based on the calculation described in this article, we have obtained results for the matching coefficients $I_{q_i q_j}$, I_{qg} and I_{gq} while the calculation of I_{gg} and $I_{q_i \bar{q}_j}$ is still in progress. For the available results, the matching coefficients agree with the results published in Ref. [28]. As a by-product, the pole cancellation for the matching coefficients also cross-checks the three-loop DGLAP splitting functions [44–50]. We also observe a simplification of the alphabet of the iterated integrals which appear in the final results for the matching coefficients compared to the alphabet observed in the master integrals. In particular, only one of the square roots remains. The remaining letters read

$$\left\{\frac{1}{z}, \frac{1}{z-1}, \frac{1}{z+1}, \frac{1}{z-2}, \frac{1}{\sqrt{z(4-z)}}\right\}. \tag{20}$$

The full results of this calculation will be presented in a forthcoming publication.

5. Conclusions

We calculate the beam functions for zero-jettiness at N^{3}LO via phase-space integrals over splitting functions in QCD. In this ongoing calculation, we have completed the matching coefficients $I_{q_i q_j}$, I_{qg} and I_{gq}, confirming the results published in Ref. [28]. The calculation of the final matching coefficients $I_{q_i \bar{q}_j}$ and I_{gg} is currently underway. In parallel, there is an effort to calculate also the soft function for zero-jettiness at N^{3}LO. First results have recently been published in Refs. [51–53]. It will be interesting to put the beam functions to use in slicing calculations for colour-singlet production, as well as in resummation applications in the future.

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