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Threshold resummation of quark-gluon partonic channels at next-to-leading power

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We discuss recent progress concerning the resummation of large logarithms at next-to-leading power (NLP) in scattering processes near threshold. We begin by briefly reviewing the diagrammatic and SCET approach, which are used to derive factorization theorems for physical observables in this kinematic limit. Then, we focus on the quark-gluon channel in deep inelastic scattering and Drell-Yan. We show that the use of consistency conditions for the cancellation of leading poles in the hadronic cross section can be used to achieve the resummation of large leading logarithms (LLs) at NLP, both within diagrammatic and SCET methods. In this context it is also possible to investigate the problem of endpoint divergences appearing at NLP in SCET, and relate its solution to the concept of re-factorization.

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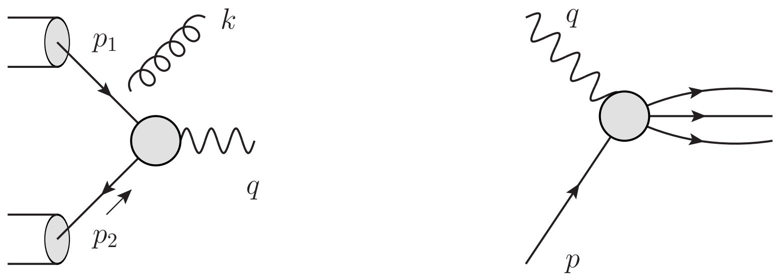

$(p_1 + p_2)^2 \equiv \hat{s}, \quad z \equiv \frac{q^2}{\hat{s}} \to 1$ $\qquad$ $Q^2 = -q^2, \quad x \equiv \frac{Q^2}{2p\cdot q} \to 1$

Figure 1: Kinematic definition of Drell-Yan and deep inelastic scattering near threshold.

1. Particle scattering near threshold

In this talk we discuss scattering processes near threshold, focusing on Drell-Yan (DY) and deep inelastic scattering (DIS), shown respectively in the left and right diagrams of figure 1. In this limit the partonic cross section is organized as a power expansion

$$\Delta_{ab}(\xi) \sim \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \left[c_n \delta(1-\xi) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-\xi)}{1-\xi} \right]_+ + d_{nm} \ln^m(1-\xi) \right) + \ldots \right], \tag{1}$$

where $\xi = z$ for DY, and $\xi = x$ for DIS. The terms proportional to c_{nm} and c_n contribute at leading power (LP) in $1-\xi$, while the terms d_{nm} contribute at next-to-leading power (NLP). The large logarithms spoil the convergence of the perturbative expansion, and need to be resummed. The summation of LP logarithms has been known for a long time, since the seminal papers [1–5]. Later, LP threshold resummation has been reinterpreted and clarified using a wide variety of methods, including the use of Wilson lines [6,7], the renormalization group [8], the connection to factorization theorems [9], and soft collinear effective theory [10–12]. The state-of-the-art for resummation at LP is next-to-next-to-next-to-leading logarithmic (N^3LL) accuracy for color singlet final states, and next-to-next-to-leading logarithmic (NNLL) accuracy for processes involving colored particles in the final state.

More recently, the quest for precision physics has led physicists to consider the summation of large logarithms at NLP, i.e. those multiplying the coefficients d_{nm} in eq. (1). This task is much more involved compared to the summation of large logarithms at LP and has been subject of intense work in the past few years, see [13–51] and references therein.

2. Factorization at next-to-leading power

At LP, large logarithms are related to the emission of soft radiation, which is described in terms of uncorrelated eikonal gluons. It is easy to show that the uncorrelated emission of eikonal gluon exponentiate. Together with the factorization of the phase space in Mellin (or Laplace) space, this leads to the resummation of large logarithms. At NLP, soft radiation becomes sensitive to the nature of the emitting particles, and begins to resolve the hard scattering kernels. In details, one has to take into account emission of soft radiation sentitive to the spin of the emitting particle (fig. 2 (a)) and

PoS(LL2022)057

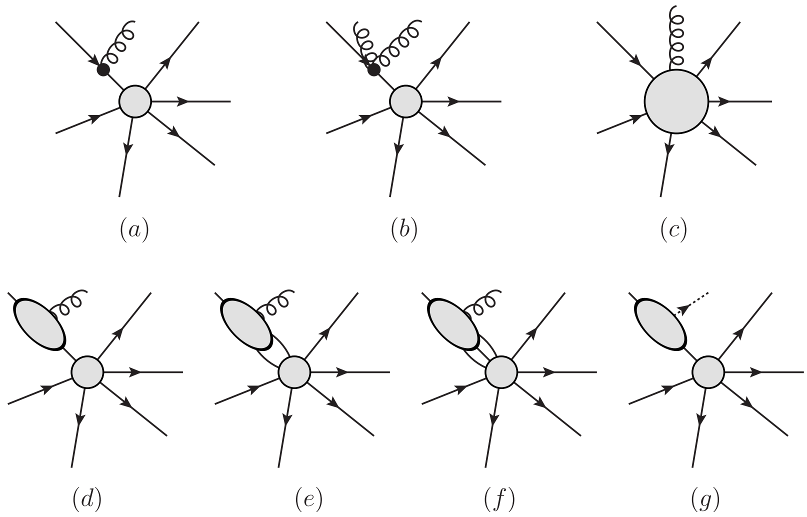

Figure 2: Diagrammatic description of soft and collinear radiation contributing at NLP, for scattering processes near threshold, as discussed in the main text.

multiple soft emissions (b); emission of soft radiation which resolves the hard interaction, (fig. 2 (c)), which has been discussed first in [52, 53]; emission of soft radiation from clusters of collinear virtual particles, (fig. 2 (d), (e), (f)). The first of such configurations, now known as "radiative jets", has been discussed in [13]. Last, one needs to take into account the emission of soft quarks, represented diagrammatically in fig. 2 (g).

The starting point of any resummation program requires to be able to properly define these configurations in a quantum field theory framework. To this end, two main methods have been considered. One, referred to as diagrammatic- or direct-QCD approach, attempts to describe the configurations in figs. 2 (a)–(g) in terms of matrix elements built of QCD fields and Wilson lines. For instance, the radiative jet of fig. 2 (d) is defined as [13, 19, 20]

$$J_{\mu,a}\,(p,n,k)\,u(p) \;=\; \int d^d y\; \mathrm{e}^{-\mathrm{i}(p-k)\cdot y}\; \big\langle 0 \big| \Phi_n(\infty,y)\, \psi(y)\, j_{|mu,a}(0) \big| p \big\rangle\;, \qquad (2)$$

where $j_a^\mu(x)$ is given by the non-abelian current

$$j_a^\mu(x) \;=\; g\,\Big\{ -\overline{\psi}(x)\,\gamma^\mu\,\mathbf{T}_a\,\psi(x) \;+\; f_a{}^{bc}\Big[F_c^{\mu\nu}(x)\,A_{\nu\,b}(x) + \partial_\nu\Big(A_b^\mu(x) A_c^\nu(x)\Big)\Big]\Big\}\;, \qquad (3)$$

and $\Phi_n(\infty, y)$ is a Wilson line from ∞ to y. In general, these radiative jets need to satisfy Ward identities. Preliminary results concerning the virtual collinear configurations in figs. 2 (e)–(g) have been presented in [54] for QED, and in [22] for Yukawa theory.

The second method consists of an effective field theory approach: soft and collinear modes in a given process are split into separate fields. The resulting theory is known as soft-collinear effective field theory (SCET) [55–58]. In this respect, the approach provides a systematic power counting, such that the diagrammatic configurations in fig. 2 are automatically described in terms of short-distance coefficients times soft and collinear matrix elements, built from time-ordered products of SCET operators and power-suppressed Lagrangian insertions [18, 23, 27, 30]. For instance, within this approach the radiative jet of eq. (2) is described in terms of a matching coefficient, referred to as collinear function [36]: in position space

$$i \int d^4z\, \mathbf{T}\Big[\{\psi_c(tn_+)\}\, \mathcal{L}_c^{(2)}(z)\Big] = 2\pi \int du \int dz_-\, \tilde{J}(t,u;z_-)\, \chi_c^{\rm PDF}(un_+)\,, \tag{4}$$

where $\mathcal{L}_c^{(2)}$ refers to only the collinear pieces of the SCET Lagrangian insertion $\mathcal{L}^{(2)}$, where the index (2) denotes suppression by two powers of the power-counting parameter $\lambda \sim \sqrt{1-\xi}$. In a similar way, one defines matching coefficients for all the diagrammatic structures in figs. 2, up to any subleading power in λ. We refer the reader to [36] for further details.

3. Endpoint divergences in SCET

The derivation of factorization theorems as discussed in the previous section gives one the tools to address the summation of large logarithms. Let's start by focusing on the SCET approach, and consider for instance the DY invariant mass distribution

$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b\, f_{a/A}(x_a) f_{b/B}(x_b)\, (1-\epsilon)\, z\, \Delta_{ab}(z)\,. \tag{5}$$

Near threshold, the leading $ab = q\bar{q}$ production channel receives two NLP corrections, one of kinematic origin due to expansion of the LP phase space, and one of dynamical origin, due to the power expansion of the partonic matrix element. The latter factorizes as

$$\begin{aligned}
\Delta_{\rm NLP}^{dyn}(z) \;=&\; -\frac{2}{(1-\epsilon)}\, Q \left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\gamma_\perp^\rho\right]_{\beta\gamma} \\
&\times \int d(n_+p)\, C^{A0}(n_+p, x_b n_- p_B)\, C^{*A0}\left(x_a n_+ p_A,\, x_b n_- p_B\right) \\
&\times \sum_{i=1}^{5} \int \{d\omega_j\}\, J_{i,\gamma\beta}\left(n_+p, x_a n_+ p_A; \{\omega_j\}\right)\, S_i(\Omega; \{\omega_j\}) + \text{h.c.}\,,
\end{aligned} \tag{6}$$

where $\Omega = Q(1-z)$. The cross section factorizes into the short-distance coefficients C^{A0}, times the sum over terms containing the convolution between a set of collinear and soft functions. Such convolutions are ubiquitous in a non-local effective field theory such as SCET: convolution parameters represent the small component of collinear momenta, which have the same scaling as the corresponding soft momentum components, and are thus still present in the low-energy theory. In this respect, the convolution in eq. (6) is expected, in principle even at LP. The reason that one does not have to deal with convolutions in scattering processes near threshold – at LP – is that at this power order they are trival, see [36] for a more exhaustive discussion. At NLP, however, these convolutions become non-trivial, and present an additional problem: they are often divergent in $d = 4$. In case of DY, one of the contributions to eq. (6) explicitly reads

$$\int_0^\Omega d\omega\, \underbrace{(n_+p\,\omega)^{-\epsilon}}_{\text{collinear piece}}\, \underbrace{\frac{1}{\omega^{1+\epsilon}}\frac{1}{(\Omega-\omega)^\epsilon}}_{\text{soft piece}}\,, \tag{7}$$

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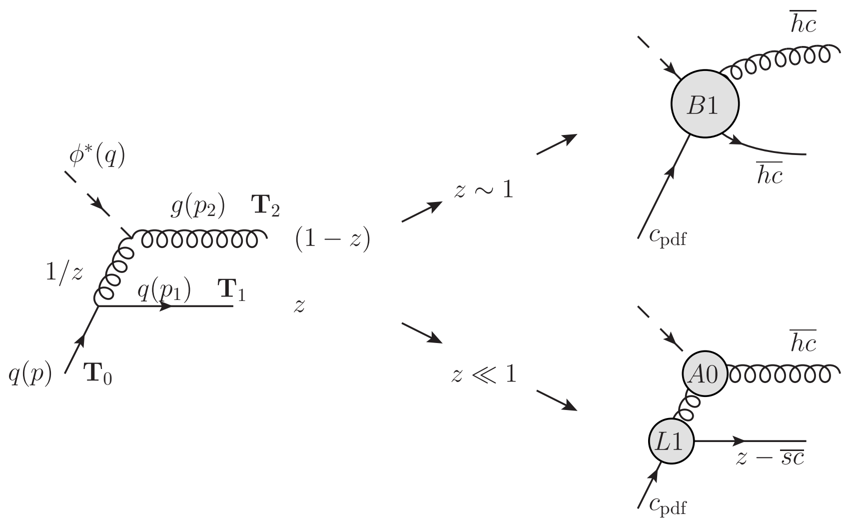

Figure 3: Left: DIS off a Higgs boson. Right: factorization for generic $z \sim 1$, and for small $z \ll 1$.

which is well defined when keeping the exact ϵ dependence in the integrand, but diverges in $d = 4$. This divergence poses a problem for the standard resummation procedure, which relies on defining renormalized factors by subtracting their poles in dimensional regularization and deriving a renormalization group equation for the renormalized function. Large logarithms are summed by evolving one function to the characteristic scale of the other, and then the convolution between the two factors is done. This procedure requires that the final convolution integral of the renormalized factors is well defined, which is not the case in eq. (7).

To explore the problem more in detail, let's consider (partonic) deep inelastic scattering (DIS) as an example. The process is described in terms of structure functions, whose factorization near threshold at LP in terms of parton distributions and a jet function is well known, see [3, 4, 6] and [59] for a SCET derivation. Here we focus instead on the contribution due to off-diagonal qg channel contribution to DIS off a Higgs boson, see figure 3. The study of this channel is particularly useful, because it starts at NLP. Therefore, compared to the Drell-Yan case discussed above, we have to consider only the dynamical contribution, i.e. the NLP matrix element, while the kinematic contribution (the expansion of the phase space in the LP matrix element) is absent.

The structure function for the partonic channel $q(p) + \phi^*(q) \to q(p_1) + g(p_2)$ is written as

$$W_{\phi,q}\Big|_{q\phi^*\to qg} = \int_0^1 dz \left(\frac{\mu^2}{s_{qg} z\bar{z}}\right)^{\epsilon} \mathcal{P}_{qg}(s_{qg}, z)\Big|_{s_{qg}=Q^2\frac{1-x}{x}}\,, \tag{8}$$

where $z \equiv n_- p_1/(n_- p_1 + n_- p_2)$, with $\bar{z} = 1 - z$, and the momentum distribution function reads

$$\mathcal{P}_{qg}(s_{qg}, z) \equiv \frac{e^{\gamma_E \epsilon}\, Q^2}{16\pi^2\Gamma(1-\epsilon)} \frac{|\mathcal{M}_{q\phi^*\to qg}|^2}{|\mathcal{M}_0|^2}\,, \tag{9}$$

where $|\mathcal{M}_0|^2$ denotes the tree-level matrix element squared, averaged (summed) over the spin and colour of the initial (final) state for the leading diagonal channel. At lowest order (diagram on the left of fig. 3) one has

$$\mathcal{P}_{qg}(s_{qg}, z)\Big|_{\rm tree} = \frac{\alpha_s C_F}{2\pi}\frac{\bar{z}^2}{z} + \mathcal{O}(\epsilon, \lambda^2)\,. \tag{10}$$

Integrating and neglecting $O(\epsilon)$ corrections that are *not* multiplied by logarithms (i.e. counting $\epsilon \ll 1$ but $\epsilon \ln(1-x) \sim 1$, $(1-x)^{-\epsilon} \sim 1$ and $\epsilon \ln(\mu/Q) \sim 1$), gives

$$W_{\phi,q}\Big|^{\rm NLP}_{O(\alpha_s),\,\text{leading pole}} = -\frac{1}{\epsilon}\frac{\alpha_s C_F}{2\pi}\left(\frac{\mu^2}{Q^2(1-x)}\right)^{\epsilon}. \tag{11}$$

$W_{\phi,q}\big|_{q\phi^*\to qg}$ represents the contribution to the partonic DIS structure function when only two partons are present in the final state. As such it is an infrared (IR) divergent quantity. In lowest order in α_s, the IR divergence is a single $1/\epsilon$ pole, which arises from the $z \to 0$ region of the integral (8) owing to the $1/z$ behaviour of the tree-level momentum distribution function. The $z \to 0$ limit corresponds to the kinematic configuration where the initial quark transfers all of its momentum to the final-state gluon, and the final-state quark becomes soft. Much like as in the DY case discussed above, it is therefore essential that the integration over z in (8) is done in d dimensions. To investigate further let's calculate the 1-loop correction to the process in the left diagram of fig. 3. Given that the leading order result (10) becomes singular only at the end point $z = 0$, we can safely expand around this limit. Keeping only terms contributing to the leading poles after integration over the phase space, we have

$$\begin{aligned}\mathcal{P}_{qg}(s_{qg},z)|_{1-\text{loop}} = \mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}}\,\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\bigg\{&\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon} \\ &+\mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \left(\frac{\mu^2}{zs_{qg}}\right)^{\epsilon}\right]\bigg\}.\end{aligned} \tag{12}$$

Within SCET this result exhibits a profound problem. For generic $z \sim 1$ the tree amplitude in the left diagram of fig. 3 corresponds to a J^{B1} SCET operator (upper-right diagram of fig. 3) with a quark field in the collinear direction, and a quark and a gluon field in the anti-collinear direction with light-cone momentum fractions z and $\bar{z}$, respectively. The tree-level matching coefficient of this operator is proportional to $1/z$, which gives the $1/z$ behaviour of $\mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}}$ after squaring the amplitude and accounting for a factor of z from the sum of the final-state quark spin. From the general formula for the anomalous dimension of subleading power operators [23, 27], we get the double pole terms with $\mathbf{T}_1\cdot\mathbf{T}_0$ and $\mathbf{T}_2\cdot\mathbf{T}_0$ from the standard cusp anomalous dimension terms. However, one cannot obtain a cusp term for the two fields within the same collinear sector, i.e. the $\mathbf{T}_1\cdot\mathbf{T}_2$ term. In this part, there are three terms involving three different scales. The third, containing the scale zs_{qg}, may be disregarded here, because the dependence on s_{qg} identifies it as a term related to the final-state jet function, rather than the renormalization of the J^{B1} operator at the hard DIS vertex. The first two terms, however, contain the hard scales Q^2 and zQ^2, and they are supposed to be predicted by the corresponding anomalous dimension. However, the anomalous dimension given in [23,27] applies when the convolution of the coefficient function with the anomalous dimension is convergent, which is not the case here. The difference between these two terms is $O(\epsilon)$ and hence does not contribute to the double pole. Instead, the expansion in ϵ produces $1/\epsilon \times \ln z$. However, the important point is that the $1/z$ singularity of the matching coefficient promotes this term to the same leading-pole order $1/\epsilon^3$ as the standard double pole terms after integration over z as in (8). Moreover, the integral over z must itself be regularized due to the singularity at $z = 0$, and the correct result is obtained by *not* expanding (12) before integration. This can easily be seen by comparing (no expansion before integration)

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\,(1-z^{-\epsilon}) = -\frac{1}{2\epsilon^3} \tag{13}$$

to (expansion of (12) before integration)

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(\epsilon\ln z - \frac{\epsilon^2}{2!}\ln^2 z + \frac{\epsilon^2}{3!}\ln^3 z + \cdots\right) = -\frac{1}{\epsilon^3} + \frac{1}{\epsilon^3} - \frac{1}{\epsilon^3} + \cdots . \tag{14}$$

If only the pole part of the integrand were kept, the result would be incomplete. This explains why it was necessary to keep the exact d-dimensional coefficient of the double pole terms in the one-loop momentum distribution function.

How to interpret this result? The lower-right diagram in fig. 3 provides the missing piece of this puzzle. The ultimate reason why the naive SCET approach fails for $z \to 0$ relies on the fact that, for $z \to 0$, the short-distance coefficient C^{B1} of the corresponding operator J^{B1} becomes effectively a function of two scales: $C^{B1}(Q^2, zQ^2)$, with $zQ^2 \ll Q^2$. In this limit the process cannot be described by the EFT diagram in the upper-right diagram in fig. 3; the incoming PDF-collinear quark emits a z-anti-softcollinear quark; the resulting propagator, proportional to $1/z$, is not hard, and cannot be integrated out. Rather, the correct EFT descriptions of the $z \to 0$ limit is given by the lower-right diagram in fig. 3, which implies the *re-factorization* of the two-scale short-distance coefficient C^{B1} according to

$$C^{B1}(Q,z)\,J^{B1}(z) \overset{z\to 0}{\longrightarrow} C^{A0}(Q^2)\int d^4x\,\mathbf{T}\Big[J^{A0},\mathcal{L}_{\xi q_{z-\overline{sc}}}(x)\Big] \;=\; C^{A0}(Q^2)\,D^{B1}(zQ^2,\mu^2)\,J^{B1}_{z-\overline{sc}}. \tag{15}$$

For $z \to 0$ the short-distance coefficient $C^{B1}(Q,z)$ re-factorizes into the coefficient $C^{A0}(Q^2)$ of the LP operator J^{A0} times a "jet" function $D^{B1}(zQ^2,\mu^2)$, which at first order is given by the quark propagator proportional to $1/z$.

As a consequence, a correct treatment of off-diagonal DIS at NLP near threshold would need to include both the upper and lower diagrams in fig. 3, each providing the correct description in the corresponding z regime: $z \sim 1$ and $z \to 0$. Crucially, both contributions would develop endpoint divergences. However, given the discussion above, it is now easier to understand that these are just an artifact of the EFT, due to splitting the matrix element into the two contributions above (the l.h.s and r.h.s of eq. (15)). As such, the endpoint divergences in the two contributions are expected to cancel each other. This allows us to conclude that a well defined resummation by means of standard EFTs methods could be achieved by reshuffling the endpoint divergences among the two contributions, such that both become finite. This is indeed a non-trivial task, which has been recently achieved in the context of SCET II for $H \to \gamma\gamma$, see [37, 40], and in the context of SCET I for thrust, see [50]. This work has been discussed at this conference, too, and we refer to [60] for a short summary. In this talk we illustrate another method, which does not exploit a full EFT treatment, but allows one to achieve the correct resummation of large logarithms and further clarify the non-trivial structure of large logarithms near threshold in presence of soft quarks.

4. Consistency conditions for resummation

So far we have focused on the factorization and resummation of partonic structure functions in DIS, which are IR and ultraviolet (UV) divergent. However, partonic quantities can be defined

PoS(LL2022)057

in terms of a renormalization prescription, which follows from the requirement that an observable must be finite as $\epsilon \to 0$. To be concrete, consider the *hadronic* DIS process $p + \phi^* \to X$. From standard factorization theorems at leading twist in Λ/Q, where Λ denotes the QCD scale, we can write the hadronic tensor as

$$W = \sum_i W_{\phi,i}\, f_i \,, \tag{16}$$

where i sums over all partonic scattering channels and f_i denotes the unrenormalized parton distribution function (PDF) of i in the proton p. Thus f_i contains dimensionally regulated UV divergences. The finite, $\overline{\rm MS}$ subtracted parton distributions and partonic cross sections are related to $W_{\phi,i}$, f_i by

$$\tilde{f}_k = Z_{ki} f_i, \qquad W_{\phi,i} = \tilde{C}_{\phi,k} Z_{ki} \,, \qquad \text{such that} \qquad W_{\phi,i} f_i = \tilde{C}_{\phi,k} \tilde{f}_k \,. \tag{17}$$

For our purposes we need to expand eq. (16) near threshold. Focusing on the NLP terms one has

$$\sum_i (W_{\phi,i} f_i)^{\rm NLP} = W^{\rm NLP}_{\phi,q} f^{\rm LP}_q + W^{\rm NLP}_{\phi,\bar{q}} f^{\rm LP}_{\bar{q}} + W^{\rm NLP}_{\phi,g} f^{\rm LP}_g + W^{\rm LP}_{\phi,g} f^{\rm NLP}_g \,. \tag{18}$$

We regard the PDFs in this equation as the unrenormalized PDFs at the factorization scale μ, and to make the dependence on the collinear and soft-collinear scale explicit, we relate it to a non-perturbative reference PDFs via

$$\begin{aligned} f^{\rm LP}_g(\mu) &= U^{\rm LP}_{gg}(\mu) f_g(\Lambda) \,, \qquad f^{\rm LP}_q(\mu) = U^{\rm LP}_{qq}(\mu) f_q(\Lambda) \qquad (\text{similarly for } \bar{q}) \,, \\ f^{\rm NLP}_g(\mu) &= U^{\rm NLP}_{gg}(\mu) f_g(\Lambda) + U^{\rm NLP}_{gq}(\mu) \left(f_q(\Lambda) + f_{\bar{q}}(\Lambda)\right) . \end{aligned} \tag{19}$$

The hadronic cross section should be finite for any choice of non-perturbative initial conditions $f_g(\Lambda)$, $f_q(\Lambda)$ and $f_{\bar{q}}(\Lambda)$. For the off-diagonal quark-gluon channel we focus on the terms proportional to $f_q(\Lambda)$, given by

$$\sum_i (W_{\phi,i} f_i)^{\rm NLP}\Big|_{\propto f_q(\Lambda)} = \left(W^{\rm NLP}_{\phi,q} U^{\rm LP}_{qq} + W^{\rm LP}_{\phi,g} U^{\rm NLP}_{gq}\right) f_q(\Lambda) \,. \tag{20}$$

The requirement that W must be finite implies so-called consistency relations, which allow one to deduce the expansion in ϵ of unrenormalized partonic quantities $W_{\phi,i}$ based on partial information. Indeed, the first LL resummation of the quark-gluon splitting function was obtained in [61] from the requirement that the DIS cross section is finite, together with additional assumptions on the all-order colour structure as well as an exponentiation ansatz for the full partonic cross section.

Here we consider a stronger form of consistency relations from pole cancellations, that can be obtained when the regions of virtuality relevant to the observable are known. The different scaling of every region with the dimensionless parameters of the problem implies a larger number of consistency relations. In case of DIS near threshold, the factorization formula involves hard and collinear physics related to the scales Q and Λ, which is non-perturbative and factorized into the PDFs. Near threshold the small invariant mass of the final state introduces a new scale into the problem, which is also the source of the large logarithms that we wish to sum. In this section we consider the DIS partonic cross section in Mellin space, according to the standard definition

$g(N) \equiv \int_0^1 dx\, x^{N-1} g(x)$. The $x \to 1$ limit corresponds then to $N \to \infty$ in moment space. The four relevant virtualities are:

$$\begin{aligned} &\text{hard}, \;\; p^2 = Q^2 && \text{anti-hardcollinear}, \;\; p^2 = Q^2\lambda^2 = Q^2/N \\ &\text{collinear}, \;\; p^2 = \Lambda^2 && \text{softcollinear}, \;\; p^2 = \Lambda^2\lambda^2 = \Lambda^2/N \end{aligned} \tag{21}$$

The anti-hardcollinear virtuality arises from the requirement of a small-mass final state X. In the adopted large-momentum frame, its large momentum is in the opposite direction of the incoming proton, hence "anti-hardcollinear". We also need a softcollinear virtuality $\Lambda/N \ll \Lambda$, which accounts for the anomalously small momentum of the target remnant as $x \to 1$ [59].

The calculation of the DIS process is imagined to be strictly factorized into contributions from the different virtualities. A multi-loop diagram is considered as a sum of terms, in which every loop momentum has one of the above virtualities, in the spirit of the strategy of expanding by regions [62]. Each loop is then associated with a factor $(\mu^2/p^2)^\epsilon$ times a function of ϵ, which will usually be singular. According to this reasoning we express the NLP contribution to DIS as

$$\begin{aligned} \sum_i (W_{\phi,i} f_i)^{\rm NLP} &= f_q(\Lambda) \times \frac{1}{N} \sum_{n=1} \left(\frac{\alpha_s}{4\pi}\right)^n \frac{1}{\epsilon^{2n-1}} \sum_{k=0}^{n} \sum_{j=0}^{n} c_{kj}^{(n)}(\epsilon) \left(\frac{\mu^{2n} N^j}{Q^{2k} \Lambda^{2(n-k)}}\right)^\epsilon \\ &\quad + f_{\bar q}(\Lambda),\; f_g(\Lambda) \;\; \text{terms}, \end{aligned} \tag{22}$$

i.e., the perturbative expansion of the NLP contribution is expressed in terms of $(n+1)^2$ coefficients $c_{kj}^{(n)}(\epsilon)$ at order n. The consistency relations follow from the requirement that the sum of all terms is non-singular as $\epsilon \to 0$. In particular, this gives immediately

$$\sum_{k=0}^{n} \sum_{j=0}^{n} j^r k^s c_{kj}^{(n)} = 0 \qquad \text{for } s + r < 2n-1, \; r,s \geq 0\,. \tag{23}$$

After some elaboration it is possible to show [42] that these equations allow one to determine the $(n+1)^2$ coefficients in terms of three unknown for each order n. However, two of the three "initial conditions" at every n can be fixed trivially. In the absence of collinear and softcollinear loops $(k = n)$, there must be at least one anti-hardcollinear loop, since the final state cannot be made up of hard modes for $x \to 1$. This implies

$$c_{n0}^{(n)} = 0\,, \tag{24}$$

for all n. Similarly, without any hard or anti-hardcollinear loops ($k = 0$), the necessary off-diagonal $q \to qg$ splitting always produces a softcollinear quark. Thus there must be at least one softcollinear loop, such that

$$c_{00}^{(n)} = 0 \qquad \text{for all } n\,. \tag{25}$$

We are left with a single unknown coefficient at each order n. Hence, at each order in perturbation theory we can reconstruct the whole result by knowing the contribution of a single region.

This is where the result in eq. (12) comes to play. Let us recall that eq. (12) gives the 1-loop virtual correction to the tree-level diagram in figure 3. In this equation, setting $\bar z = 1$, the contribution proportional to the scales $(\mu^2/Q^2)^\epsilon$ and $[\mu^2/(zQ^2)]^\epsilon$ corresponds to the hard region, i.e. the coefficient $c_{21}^{(2)}$. As we will discuss shortly, SCET reasoning allows one to expect that such contribution exponentiate: under this assumption, eq. (12) gives

$$\mathcal{P}_{qg,\text{hard}}(s_{qg},z) = \frac{\alpha_s C_F}{2\pi}\frac{1}{z}\exp\left\{\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left[(\mathbf{T}_2\cdot\mathbf{T}_0+\mathbf{T}_1\cdot\mathbf{T}_2)\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} + (\mathbf{T}_1\cdot\mathbf{T}_0-\mathbf{T}_1\cdot\mathbf{T}_2)\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}\right]+\mathcal{O}\left(\frac{1}{\epsilon}\right)\right\}, \tag{26}$$

which provides us with the whole tower of coefficients $c_{n1}^{(2)}$, i.e., the remaining "initial condition" per loop that we needed. This information is sufficient to determine the complete hadronic cross section. After some elaboration, we are able to determine the unknown terms in eq. (20), namely, $W_{\phi,q}^{\text{NLP}}$ and U_{gq}^{NLP}:

$$W_{\phi,q}^{\text{NLP,LL}} = -\frac{1}{2N}\frac{C_F}{C_F-C_A}\frac{\epsilon N^\epsilon}{N^\epsilon-1}\left\{\exp\left[\frac{\alpha_s C_F}{\pi}\frac{N^\epsilon-1}{\epsilon^2}\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}\right]-\exp\left[\frac{\alpha_s C_A}{\pi}\frac{N^\epsilon-1}{\epsilon^2}\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}\right]\right\}, \tag{27}$$

$$U_{gq}^{\text{NLP,LL}} = -\frac{1}{2N}\frac{C_F}{C_F-C_A}\frac{\epsilon N^\epsilon}{N^\epsilon-1}\left\{\exp\left[\frac{\alpha_s C_F}{\pi}\frac{1-N^\epsilon}{\epsilon^2}\left(\frac{\mu^2}{\Lambda^2}\right)^{\epsilon}\right]-\exp\left[\frac{\alpha_s C_A}{\pi}\frac{1-N^\epsilon}{\epsilon^2}\left(\frac{\mu^2}{\Lambda^2}\right)^{\epsilon}\right]\right\}. \tag{28}$$

From here it is also possible to obtain the corresponding renormalized coefficients, according to eq. (17), and we refer to [42] for a throughout derivation.

It remains to justify the exponentiation hypothesis in eq. (26). Indeed, even without a proper treatment of endpoint divergences, the exponentiation of the 1-loop hard region can be explained within the refactorization condition of eq. (15). Focusing on the r.h.s. of eq. (15), and working in $d = 4-2\epsilon$, the evolution of the coefficient $C^{B1}(Q,z)$ can be obtained as a two-step procedure, in which the LP short-distance coefficient C^{A0} and the jet D^{B1} are evolved to a common scale μ. The anomalous dimension of C^{A0} is well known, and the one of D^{B1} can be determined by means of a region calculation, obtaining

$$\left[C^{A0}\left(Q^2,\mu^2\right)\right]_{\text{bare}} = C^{A0}\left(Q^2,Q^2\right)\exp\left[-\frac{\alpha_s C_A}{2\pi}\frac{1}{\epsilon^2}\left(\frac{Q^2}{\mu^2}\right)^{-\epsilon}\right],$$

$$\left[D^{B1}\left(zQ^2,\mu^2\right)\right]_{\text{bare}} = D^{B1}\left(zQ^2,zQ^2\right)\exp\left[-\frac{\alpha_s}{2\pi}(C_F-C_A)\frac{1}{\epsilon^2}\left(\frac{zQ^2}{\mu^2}\right)^{-\epsilon}\right]. \tag{29}$$

Replacing the appropriate values $\mathbf{T}_1\cdot\mathbf{T}_0 = C_A/2-C_F$, $\mathbf{T}_2\cdot\mathbf{T}_0 = \mathbf{T}_1\cdot\mathbf{T}_2 = -C_A/2$ in eq. (26), we see that the evolution of the r.h.s of eq. (15) according to eq. (29) reproduces eq. (26), thus providing a SCET-based justification for the "soft quark Sudakov" exponentiation conjecture.

Let us conclude this section with an interesting observation: it is remarkable that, in the leading-pole approximation, the full result for $W_{\phi,q}^{\text{NLP,LL}}$ in eq. (27) follows from the exponentiation conjecture for the hard-only amplitude, eq. (26), by a simple substitution. Let us define

$$A \equiv \frac{\alpha_s(C_F-C_A)}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}, \qquad S \equiv \frac{\alpha_s C_A}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}. \tag{30}$$

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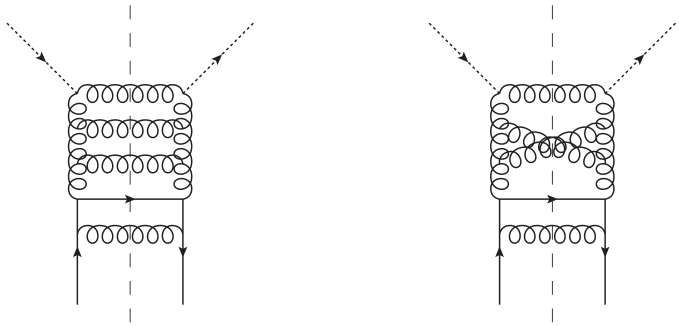

Figure 4: Left: a ladder graph contributing to the DIS qg channel at NLP LL; right: a crossed-ladder graph.

Then, integrating the amplitude in eq. (26) against the phase space in eq. (8) gives the hard-only contribution to the structure function, which reads

$$W_{\phi,q}\Big|^{\rm hard}_{q\phi^*\to qg} = \frac{1}{N}\frac{\alpha_s C_F}{2\pi\epsilon}\left(\frac{\mu^2 N}{Q^2}\right)^{\epsilon} \exp\left[-S\right] \times \frac{\exp(-A)-1}{A}. \tag{31}$$

In the same notation, the full structure function of eq. (27) reads

$$W^{\rm NLP,LL}_{\phi,q} = -\frac{1}{N}\frac{\alpha_s C_F}{2\pi\epsilon}\left(\frac{\mu^2 N}{Q^2}\right)^{\epsilon} \exp\left[S\left(N^{\epsilon}-1\right)\right] \times \frac{\exp(A\left(N^{\epsilon}-1\right))-1}{A\left(N^{\epsilon}-1\right)}, \tag{32}$$

i.e., the full structure function can be obtained from the result in the hard region by replacing $A \to A\,(1-N^{\epsilon})$, $S \to S\,(1-N^{\epsilon})$. The appearance of the factor $(N^{\epsilon}-1)$ is characteristic of the leading-pole solution.

5. Resummation within a diagrammatic approach

In section 2 we have discussed also another approach, based on diagrammatic methods in QCD. Within this framework one defines the collinear and soft matrix elements in fig. 2 in terms of fields and Wilson lines in QCD. It is then possible to study the exponentiation of logarithmic contributions by means of a diagrammatic analysis, determining which diagrams contribute to a given power- and logarithmic-accuracy, and investigating their combinatorial structure (see e.g. [16]). This procedure is typically performed in dimensional regularization, such that one should not have to deal with endpoint divergences at any stage of the computation. The quark-gluon channel in DIS provides an interesting example, where we can compare the SCET approach discussed above with the corresponding calculation based on diagrammatic methods.

The starting point for the diagrammatic analysis [48] also begins from the consistency relations discussed in the previous section. As we concluded there, the whole DIS cross section at NLP can be reconstructed once a single "initial condition" per loop in a given momentum region is known. Within the SCET approach it is convenient to consider the hard region, because the latter can be determined to all orders within the refactorization procedure, as discussed in the previous section. Within a diagrammatic analysis, instead, it proves convenient to consider the soft-anticollinear region. Inspecting the tree-level diagram on the left in figure 3, it is easy to realize that the quark-gluon-quark interaction vertex, where the initial collinear quark is converted into a collinear

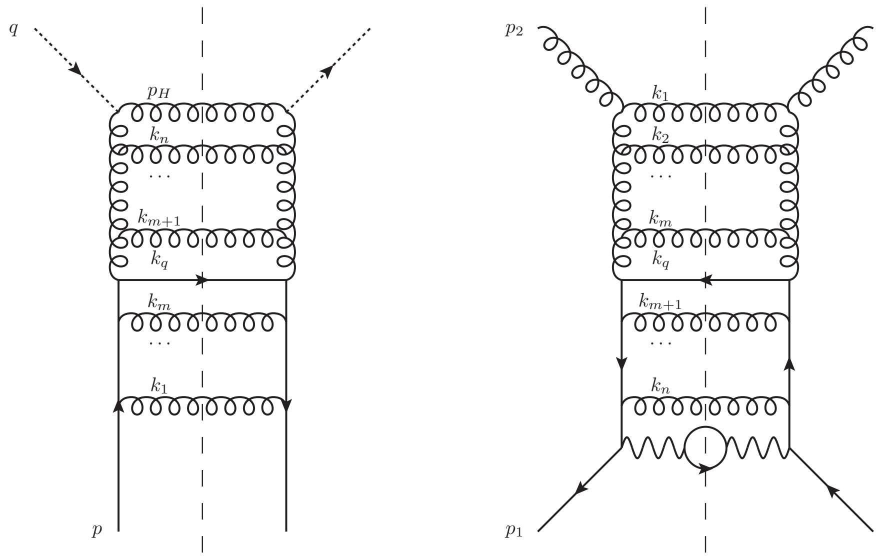

Figure 5: Left: ladder diagram contributing at LL accuracy to the qg channel of Higgs-induced DIS, and right: ladder diagram contributing to the $g\bar{q}$ channel in Drell-Yan.

gluon by means of a soft-anticollinear quark emission, provides a power suppression by a factor of $\lambda \sim \sqrt{1-x}$. Two such vertices are present at the level of the matrix element squared, thus providing the required suppression to NLP, or $\lambda^2 \sim (1-x)$. Any other emission in the soft-anticollinear region must therefore be given in terms of soft gluon emissions at LP, for which the eikonal approximation can be used. The diagram contributing can be further reduced by noticing that a) leading logarithms only arise from the kinematic region in which the transverse momenta of the emitted partons are strongly ordered; and b) one may reduce the set of relevant Feynman diagrams for the squared matrix element to those having a pure ladder form, as shown in the left diagram of fig. 4. Crossed ladders, such as the graph on the right in fig. 4, do not contribute at LL. In non-abelian theories such as QCD, this property is not guaranteed in general gauges, but can be made manifest by choosing to define the polarization states of the emitted gluons in a particular way. Upon choosing a reference vector c_μ, one may define physical gluon polarization vectors $\epsilon_\mu(k)$ via the simultaneous requirements

$$k \cdot \epsilon(k) = c \cdot \epsilon(k) = 0\,. \tag{33}$$

If in addition c is a null vector ($c^2 = 0$), the sum over physical gluon polarization states has the form

$$\sum_{\text{pols.}} \epsilon^\dagger_\mu(k)\epsilon_\nu(k) = -\eta_{\mu\nu} + \frac{k_\mu c_\nu + k_\nu c_\mu}{c \cdot k}\,. \tag{34}$$

As explained in detail in refs. [63, 64], the kinematic dominance of uncrossed gluon ladders occurs for the explicit choice

$$c = q' \equiv q + xp\,, \tag{35}$$

where q and p are defined as in fig. 4, and $x = Q^2/(2p\cdot q)$ is the Bjorken x. This reasoning allows us to conclude that the LL contribution to the qg channel in Higgs-induced DIS at order $n+1$ is given by the left ladder diagram in fig. 5, which corresponds to the amplitude

$$\overline{|\mathcal{M}_{qh\to qg_1\ldots g_n}|^2}$$

$$= \frac{|\phi_h|^2 C_F^{m+1} C_A^{n-m} g_s^{2(n+1)}}{8\mu^{(d-4)(n+1)}} \left(\prod_{i=1}^{n} \frac{2q\cdot p\, p\cdot k_i}{q'\cdot k_i} \right) \mathrm{Tr}[\not{p}\gamma^\beta \not{k}_q \gamma^\alpha] \left(-\eta_{\alpha\beta} + \frac{q'_\alpha p_{H,\beta} + q'_\beta p_{H,\alpha}}{q'\cdot p_H} \right)$$

$$\times \frac{1}{(p\cdot k_1)^2 [p\cdot(k_1+k_2)]^2 \ldots [p\cdot(k_1+\ldots+k_m+k_q)]^2 \ldots [p\cdot(k_1+\ldots+k_n+k_q)]^2}, \tag{36}$$

where we highlight the eikonal structure of the n-soft gluon emissions, and ϕ_h represents the effective Higgs-gluon coupling. The calculation of the full structure function requires us to integrate the matrix element squared above against the $(n+2)$-phase space. Given that the matrix element squared is already contributing at NLP, the phase space can be approximated to LP, which entails a significant simplification: the phase space factorizes into single-gluon phase spaces in Laplace space. After some elaboration we get the LL contribution at order $n+1$ (with $\mu^2 = Q^2$):

$$W_{\phi,q}^{(n+1)} = -\left(\sum_{m=0}^{n} C_F^{m+1} C_A^{n-m} \right) \frac{2}{\epsilon} \frac{N^\epsilon}{N} \left(\frac{4N^\epsilon}{\epsilon^2} \right)^n \frac{1}{(n+1)!}, \tag{37}$$

which can be resummed into a closed form:

$$W_{\phi,q}^{\rm soft}\Big|_{\rm LL} = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{4\pi} \right)^n W_{\phi,q}^{(n)} = -\frac{1}{2N} \frac{C_F}{C_F - C_A} \frac{\epsilon N^\epsilon}{N^\epsilon - 1} \left\{ \exp\left[\frac{\alpha_s C_F}{\pi} \frac{N^\epsilon}{\epsilon^2} \right] - \exp\left[\frac{\alpha_s C_A}{\pi} \frac{N^\epsilon}{\epsilon^2} \right] \right\}. \tag{38}$$

The full amplitude can be easily reconstructed by exploiting the consistency relations. As expected, we recover eq. (27). At this point it is interesting to notice that, writing eq. (38) as

$$W_{\phi,q}^{\rm soft}\Big|_{\rm LL} = -\frac{1}{N} \frac{\alpha_s C_F}{2\pi\epsilon} \left(\frac{\mu^2 N}{Q^2} \right)^\epsilon \exp\left[S N^\epsilon \right] \times \frac{\exp(A N^\epsilon) - 1}{A N^\epsilon}, \tag{39}$$

where the functions A and S have been defined in eq. (30), the calculation of the soft region gives the full result by means of the substitution $AN^\epsilon \to A(N^\epsilon - 1)$, $SN^\epsilon \to S(N^\epsilon - 1)$, which is indeed consistent with the observation made around eqs. (31) and (32).

Let us conclude this section by mentioning that the methods of section 4 and the diagrammatic methods discussed here can be applied to the calculation of other off-diagonal processes, such as the quark-gluon channel contribution to DY, $g(p_1)\bar{q}(p_2) \to \gamma^*(q) \to e^+(q_1)e^-(q_2)$. Focusing on the diagrammatic method, in this case it is possible to show [48] that one has to consider the tower of ladders in the right diagram of fig. 4. This gives once again the soft contribution to the qg channel in DY, and then the full result can be reconstructed by exploiting consistency relations, obtaining the resummed (bare) partonic cross section

$$W_{{\rm DY},g\bar{q}}^{\rm NLP,LL} = -\frac{T_R}{2(C_F - C_A)} \frac{1}{N} \frac{\epsilon(N^{\epsilon-1})}{N^\epsilon - 1} \exp\left[\frac{4a_s C_F (N^\epsilon - 1)}{\epsilon^2} \right]$$

$$\times \left\{ \exp\left[\frac{4a_s C_F N^\epsilon (N^\epsilon - 1)}{\epsilon^2} \right] - \exp\left[\frac{4a_s C_A N^\epsilon (N^\epsilon - 1)}{\epsilon^2} \right] \right\}, \tag{40}$$

and after some work, for which we refer to [48], one finds the renormalized partonic cross section

$$\widetilde{C}_{{\rm DY},g\bar{q}}\Big|_{\rm LL} = \frac{T_R}{C_A - C_F} \frac{1}{2N \ln N} \left[e^{8C_F a_s \ln^2 N} \mathcal{B}_0[4a_s(C_A - C_F)\ln^2 N] - e^{(2C_F + 6C_A) a_s \ln^2 N} \right], \tag{41}$$

where $\mathcal{B}_0(x) = \sum_{n=0}^{\infty} \frac{B_n}{(n!)^2} x^n$, and B_n are Bernoulli numbers. This result reproduces an earlier conjecture in ref. [65], and can be obtained also within the methods discussed in section 4, [66].

6. Conclusions

The factorization of scattering processes near threshold beyond leading power is nontrivial – it involves configurations of collinear and soft momenta, that can be described in terms of matrix elements obtained within a diagrammatic [20,54] or a SCET-based [18,30,36] approach. Within the latter, traditional resummation methods fail [28,36], due to the appearance of endpoint divergences in the convolution between short-distance coefficients, collinear and soft functions.

This problem can be studied conveniently in off-diagonal channels of $2 \to 1$ or $1 \to 2$ processes, such as the quark-gluon channel in DIS and DY, which starts at NLP, and where endpoint divergences appear already in the factorization of the partonic cross sections at LL accuracy.

The requirement of pole cancellation in the hadronic cross section can be used to obtain consistency conditions. These can then be used together with the assumption of exponentiation of the 1-loop hard-region contribution to the partonic cross section, to achieve the resummation of NLP LL logarithms [42]. Within SCET, the exponentiation hypothesis can be justified in the context of a re-factorization involving the short-distance coefficient representing the hard-region contribution, which has been exploited in [37, 40, 50] to construct factorization theorems free of endpoint divergences.

Consistency conditions can be used as well in combination with diagrammatic methods [48]. In this case it is possible to show that the soft region contribution to the partonic cross section can be determined and resummed to all orders in terms of ladder diagrams. The full cross section can then be reconstructed by exploiting the consistency conditions.

The resummation of LLs at NLP in off-diagonal channels, together with the resummation of LLs in diagonal channels previously obtained in [24,28,31,33] completes the resummation of LLs at NLP in $2 \to 1$ and $1 \to 2$ processes. The methods discussed in this talk will be useful to extend the resummation of NLP large logarithms near threshold at NLL accuracy and beyond.

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PoS(LL2022)057

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PoS(LL2022)057

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