Effective transverse momentum in multijet production at hadron colliders

Chiara Savoini[a,*]

[a]*Physik Institut, Universität Zürich, Winterthurerstrasse 190, CH-8057 Zürich, Switzerland*

E-mail: csavoi@physik.uzh.ch

We consider the class of inclusive hadron collider processes in which one or more hard jets are produced, possibly accompanied by colourless particles (such as Higgs boson(s), vector boson(s) with their leptonic decays, and so forth). We propose a new variable that smoothly captures the N+1 to N jet transition. This variable, that we dub k_T^{ness}, represents an effective transverse momentum controlling the singularities of the cross section when the additional jet is unresolved. The k_T^{ness} variable offers novel opportunities to perform higher-order QCD calculations by using non-local subtraction schemes. We study the singular behavior of the N+1-jet cross section when $k_T^{\text{ness}} \to 0$ and, as a phenomenological application, we use the ensuing results to evaluate next-to-leading order corrections to H+jet and Z+2 jet production at the LHC. We show that k_T^{ness} performs extremely well as a resolution variable and appears to be very stable with respect to hadronization and multiple-parton interactions.

Loops and Legs in Quantum Field Theory - LL2022,
25-30 April, 2022
Ettal, Germany

---

*Speaker

1. Introduction

Jets are collimated bunches of hadrons that represent the fingerprints of the high-energy partons (quarks and gluons) produced in the hard-scattering interaction. Since they are ubiquitous at the Large Hadron Collider (LHC), a classification of the events into jet bins according to the jet multiplicity is mandatory for new physics searches and precise studies of the Standard Model (SM).

An accurate description of jet processes requires an understanding of Quantum CromoDynamics (QCD) across a wide range of energy scales, from the hard-scattering event to low-energy physics phenomena such as hadronization or multiple-parton interactions (MPI). Therefore, it is important to design observables that are sensitive to the different aspects of the underlying hard-scattering event and capable to capture any deviation from the leading order (LO) energy flow. These observables are usually named as *jet resolution variables* [1, 2].

If a process has N hard jets at Born level, a good N-jet resolution variable should smoothly capture the transition from the N to $N + 1$ jet configuration, allowing for a discrimination between the signal and possible backgrounds. Moreover, jet resolution variables have been used to formulate non-local subtraction methods and to match fixed-order computations with parton shower simulations.

A prominent example of 0-jet resolution variables, for processes that do not involve any jet at Born level, is the transverse momentum (q_T) of the tagged colourless system. This observable has been used as a slicing variable to formulate a non-local subtraction method, called q_T-*subtraction* [3], which has been successfully applied, up to NNLO, on a wide range of color singlet processes ($pp \rightarrow V, H, VV, HH, \gamma\gamma, \gamma\gamma\gamma, V\gamma$) [6] and heavy-quark pair production [4, 5]. The main drawback is that q_T cannot regularise final-state collinear singularities, implying that it is not a good N-jet resolution variable for $N > 0$.

Another commonly used observable is (N)-jettines, τ_N [8], which is so far the only player in the game as a slicing variable for multijet processes. It has proved a successful resolution variable, up to NNLO, for colour-singlet processes [9–13] and vector or Higgs boson production in association with a jet [14–16]. In the context of non-local subtraction schemes, the efficiency of the calculation is subject to the size of the missing power corrections, which in general depends on the choice of the resolution variable. It is well known that q_T features linear or quadratic power corrections while τ_N is affected by logarithmically enhanced power corrections (already at NLO), which could spoil the converge of the slicing method. This represents one of the reasons that prompted us to explore other jet resolution variables which could feature milder power corrections (linear or quadratic) compared to τ_N.

The first variable we studied, for processes with a single jet accompanied by a colourless system (F), is called q_T-*imbalance* [17] and represents the natural extension of the transverse momentum in the colour-singlet case. It is defined as the transverse momentum of the F+ jet system after applying an anti-k_T clustering algorithm with radius R. This observable has been resummed up to next-to-leading logarithmic accuracy (NLL) and used as a slicing parameter at NLO for H+jet, showing linear missing power corrections. The weak point is that the dependence on the additional cutting radius R makes the extension of the computation to higher orders more complicated, since some ingredients are known only in the small-R approximation. This implies that we would have to deal with missing power corrections in the slicing parameter and in the radius R. Moreover,

q_T-imbalance is blind to radiation emitted in certain phase space regions and it features non-global logarithms starting at NNLO.

The requirement of globalness induced us to define another jet resolution variable called ΔE_T, which is the difference between the transverse energy and the transverse momentum of the colourless system. This variable has a more involved structure than q_T-imbalance and it offers new interesting features to study. The non-trivial azimuthal dependence of ΔE_T is responsible for the presence of non-vanishing spin-correlations (already at NLO) and different beam functions with respect to the ones used in q_T-subtraction. It shows a different scaling in each singular region and the missing power corrections in the slicing parameter are linear logarithmically enhanced as in the case of τ_N.

After several studies, we came out with a new global dimensionful observable called k_T^{ness} [7], which is able to capture the $N \to N + 1$ jet transition and it can be defined for any number of jets. This variable takes its name form the k_T jet clustering algorithm and it represents an effective transverse momentum, describing the limit in which the additional jet is unresolved. If the unresolved radiation is close to the colliding beams or the event has no jets at Born level, k_T^{ness} reduces to the transverse momentum (q_T) of the hard system. On the other hand, if the unresolved radiation is emitted collinear to one of the final-state jets, k_T^{ness} describes the relative transverse momentum of the hard system with respect to the jet direction.

In this contribution we will discuss the formulation of a subtraction scheme based on k_T^{ness} and we will present some phenomenological applications at NLO, commenting a number of interesting features that this variable is offering.

2. k_T^{ness}-slicing @NLO

We consider the inclusive hard-scattering process

$$h_1(P_1) + h_2(P_2) \to j(p_1) + j(p_2) + ... + j(p_N) + F(p_F) + X \tag{1}$$

where two hadrons h_1 and h_2 with momenta P_1 and P_2 collide and produce N final-state hard jets with momenta p_1, $p_2...p_N$. The hard jets can be accompanied by a generic colourless system F with total momentum p_F.

As mentioned above, the definition of k_T^{ness} is based on the *exclusive k_T clustering algorithm* until N+1 jets remain. If we are performing an $N^k LO$ computation, the role of k_T^{ness} is to discriminate between the fully unresolved region ($k_T^{\text{ness}} = 0$) and the region where at least one additional radiation is resolved ($k_T^{\text{ness}} > 0$). In the latter region, the infrared (IR) singularity structure can be at most $N^{k-1}LO$-type. At NLO, it is not necessary to run the clustering algorithm and the definition of k_T^{ness} directly coincides with the minimum among the usual distances d_{ij} and d_{iB} between the partons $i = 1, ..., N$ in Eq. (1). For the details of the recursive procedure we refer to Ref. [7].

We computed the singular behaviour of the cross section for the process in Eq. (1) at NLO, as $k_T^{\text{ness}} \to 0$. The master formula, for the partonic cross section, is given by

$$d\hat{\sigma}_{\text{NLO}}^{\text{F+N jets+X}} = \mathcal{H}_{\text{NLO}}^{\text{F+N jets}} \otimes d\hat{\sigma}_{\text{LO}}^{\text{F+N jets}} + \left[d\hat{\sigma}_{\text{LO}}^{\text{F+(N+1) jets}} - d\hat{\sigma}_{\text{NLO}}^{\text{CT,F+Njets}} \right]_{k_T^{\text{ness}}/M > r_{\text{cut}}}, \tag{2}$$

where M is a hard scale and r_{cut} is a dimensionless slicing parameter. The main ingredients in Eq. (2) can be listed as:

- $d\hat{\sigma}_{\mathrm{LO}}^{\mathrm{F+(N+1)\,jets}}$, which includes the real contribution associated to configurations where N hard jets are accompanied by additional QCD radiation with finite k_T^{ness};
- $d\hat{\sigma}_{\mathrm{NLO}}^{\mathrm{CT,F+Njets}}$, a non-local *counterterm* that captures the singular behaviour of the real contribution in the $r_{cut} \to 0$ limit;
- $\mathcal{H}_{\mathrm{NLO}}^{\mathrm{F+N\,jets}}$, the so-called *hard-collinear coefficient* which lives at $k_T^{\mathrm{ness}} = 0$ and encodes the genuine one-loop virtual contribution plus finite reminders necessary to restore the unitarity of the cross section.

The computation of the counterterm and of the finite remainders in $\mathcal{H}_{\mathrm{NLO}}^{\mathrm{F+N\,jets}}$ has been performed in $d = 4 - 2\epsilon$ dimensions, by organising the relevant terms in each singular region (IS collinear, soft, FS collinear) and by exploiting the IR factorisation of the real matrix element in presence of unresolved radiation. The soft and collinear singularities cancel out with those from the virtual contribution and the remaining sensitivity to the IR kinematics appears in $d\hat{\sigma}_{\mathrm{NLO}}^{\mathrm{CT,F+Njets}}$ as powers of $\log(k_T^{\mathrm{ness}})$. The explicit expression of the non-local counterterm, in the partonic channel ab, is

$$
d\hat{\sigma}_{\mathrm{NLO}\,ab}^{\mathrm{CT,F+Njets}} = \frac{\alpha_\mathrm{S}}{\pi} \frac{dk_T^{\mathrm{ness}}}{k_T^{\mathrm{ness}}} \left\{ \left[\ln \frac{Q^2}{(k_T^{\mathrm{ness}})^2} \sum_\alpha C_\alpha - \sum_\alpha \gamma_\alpha - \sum_i C_i \ln(D^2) - \sum_{\alpha \neq \beta} \langle \mathbf{T}_\alpha \cdot \mathbf{T}_\beta \rangle \ln\left(\frac{2 p_\alpha \cdot p_\beta}{Q^2} \right) \right] \times \right.
$$
$$
\left. \times \delta_{ac}\delta_{bd}\delta(1-z_1)\delta(1-z_2) + 2\delta(1-z_2)\delta_{bd}P_{ca}^{(1)}(z_1) + 2\delta(1-z_1)\delta_{ac}P_{db}^{(1)}(z_2) \right\} \otimes d\hat{\sigma}_{\mathrm{LO}\,cd}^{\mathrm{F+N\,jets}},
$$
$$(3)$$

where the index i labels the final-state partons with colour charges $\mathbf{T}_i$ ($\mathbf{T}_i^2 = C_i$) and momenta p_i ($\sum_i p_i = q$, $Q^2 = q^2$), while α and β label initial and final-state partons. The parameter D enters the definition of k_T^{ness} and the symbol $\langle \mathbf{T}_\alpha \cdot \mathbf{T}_\beta \rangle$ stands for the born-normalised colour-correlated tree level matrix element for the partonic process contributing to $d\hat{\sigma}_{\mathrm{LO}\,cd}^{\mathrm{F+N\,jets}}$ (a sum over all the possible final-state parton flavours is understood).

The hard-collinear coefficient $\mathcal{H}_{cd;ab}^{\mathrm{F+N\,jets}}$, which has to be convoluted with the partonic cross section $d\hat{\sigma}_{\mathrm{LO}\,cd}^{\mathrm{F+N\,jets}}$, can be schematically written as

$$
\mathcal{H}_{cd;ab}^{\mathrm{F+N\,jets}} = (\mathbf{HS})_{cd} C_{ca} C_{db} \prod_{i=1,\ldots,N} J_i \,, \tag{4}
$$

where C_{ca} and C_{db} are the customary collinear functions appearing in the q_T subtraction formalism, J_i are the jet functions describing collinear radiation to each of the final-state parton i. The explicit expressions of the jet functions are provided in the Appendix of Ref. [7].

The contribution $(\mathbf{HS})_{cd}$ is given by

$$
(\mathbf{HS})_{cd} = \frac{\langle \mathcal{M}_{cd} | \mathbf{S} | \mathcal{M}_{cd} \rangle}{|\mathcal{M}_{cd}^{(0)}|^2} \tag{5}
$$

where $|\mathcal{M}_{cd}\rangle$ is the UV renormalised virtual amplitude after the subtraction of infrared singularities, which admits a perturbative expansion in $\alpha_\mathrm{S}(\mu_R)$. The soft-parton factor $\mathbf{S}$ is an operator in colour space and can be expanded as

$$
\mathbf{S} = 1 + \frac{\alpha_\mathrm{S}(\mu_R)}{\pi}\mathbf{S}^{(1)} + O(\alpha_\mathrm{S}^2) \,, \tag{6}
$$

where $\mathbf{S}^{(1)}$ is related to the integral, over the radiation phase space, of the soft-subtracted current $\mathbf{J}^2_{\mathrm{sub}}$. Again, for more details, we refer to the Appendix of Ref. [7].

Before concluding this section, we remind that the subtraction formula in Eq. (2) is valid for an arbitrary number of jets. We will show in the following how the subtraction works in the case of processes with one and two jets at Born level.

3. Phenomenological applications

We consider proton-proton collisions at the LHC with $\sqrt{s} = 13$ TeV and, for all processes we studied, we set the following parameters:

$$G_F = 1.16639 \times 10^{-5}\,GeV^{-2} \qquad (G_\mu - scheme)$$
$$m_Z = 91.1876\,GeV\,, \qquad \Gamma_Z = 2.4952\,GeV$$
$$m_W = 80.385\,GeV\,, \qquad m_H = 125\,GeV$$
$$\text{anti-}k_T \text{ clustering algorithm with } R = 0.4$$
$$\texttt{NNPDF\_nlo\_as\_0118}\,.$$

We have first applied Eq. (2) to compute the cross section of H + jet production.

We require the leading jet to have $p_T^j > 30$ GeV, we choose the scales $\mu_F = \mu_R = m_H$ and we set $D = 1$ in the definition of k_T^{ness}. In order to compare the behaviour of our slicing variable against 1-jettiness, we define a dimensionless variable $r = \mathcal{T}_1/\sqrt{m_H^2 + (p_T^j)^2}$ $(r = k_T^{\mathrm{ness}}/\sqrt{m_H^2 + (p_T^j)^2})$, where $\sqrt{m_H^2 + (p_T^j)^2}$ corresponds to the hard scale M in Eq. (2).
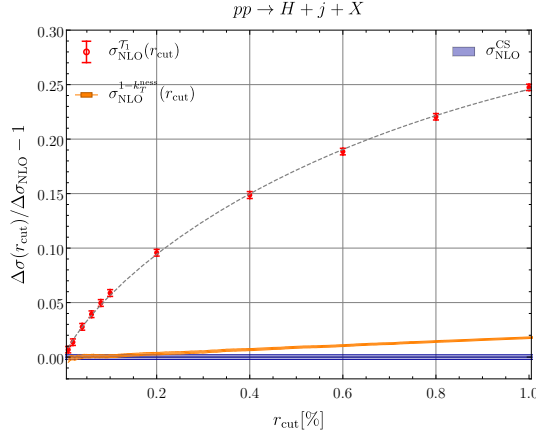

Figure 1: The NLO correction $\Delta\sigma$ for the H + jet cross section computed with 1-jettiness (red points) and 1-k_T^{ness} (orange). The r_{cut} dependence is compared to the (r_{cut} independent) result obtained with dipole subtraction using MCFM (blue).

In Fig. 1 we study the behavior of the NLO correction $\Delta\sigma$ as a function of r_{cut} for the jettiness and k_T^{ness} calculations, normalised to the result obtained with Catani-Seymour (CS) dipole subtraction. The main message is that both jettiness and k_T^{ness} results nicely converge to the expected one but the r_{cut} dependence is very different for the two calculations. In the case of jettiness, the missing

power corrections are logarithmically enhanced while, in the case of k_T^{ness}, the r_{cut} dependence is consistent with a purely linear behavior. This implies that one could choose a sufficiently large value of r_{cut} (e.g. $r_{\text{cut}} = 1\%$) without missing a big portion ($\approx 3\%$) of the exact cross section and avoiding numerical instabilities in the integration of $d\hat{\sigma}_{\text{LO}}^{\text{F+(N+1) jets}}|_{k_T^{\text{ness}}/M > r_{\text{cut}}}$, due to too small r_{cut} values.
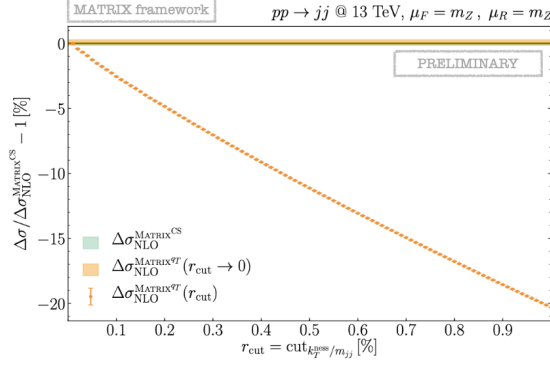

Figure 2: The NLO correction $\Delta\sigma$ for the dijet cross section computed with 2-k_T^{ness} (orange). The r_{cut} dependence is compared to the (r_{cut} independent) result obtained with dipole subtraction using MATRIX (green).

In order to be convinced that the missing power corrections are at most linear at NLO, we studied also processes with 2 jets at Born level. In Fig. 2 we present preliminary results for dijet production. We impose a cut of 30 GeV on the transverse momentum of the jets and we require $\mu_F = \mu_R = m_Z$. We arbitrarily choose $D = 1$ in the definition of k_T^{ness} and the hard scale M is set to the invariant mass of the dijet system.

As in the case of H+jet, the result nicely converges to the CS computation and the r_{cut} dependence is compatible with a purely linear behaviour. Compared to Fig. 1, it seems that the coefficient of the linear power corrections is larger and, at $r_{\text{cut}} = 1\%$, we are off by 20% with respect to the exact result. Nevertheless, a purely linear behavior would in general imply a faster convergence of the computation with respect to logarithmically enhanced missing power corrections.

Finally, we have considered $Z + 2$ jet production, by imposing the following fiducial cuts:

$$66 GeV \leq m_{\ell\ell} \leq 116 GeV$$
$$p_T^j > 30 GeV , \quad |\eta| < 4.5$$
$$p_T^l \geq 20 GeV , \quad |\eta_\ell| \leq 2.5$$
$$\Delta R_{\ell\ell} > 0.2 , \quad \Delta R_{\ell j} > 0.5 .$$

The factorization and renormalization scales are set to m_Z, the hard scale M corresponds to the transverse mass of the dilepton system and the parameter D is set to $D = 0.1$ in this case.

In Fig. 3 (left panels) we show the p_T distribution of the hardest jet at LO and NLO, computed with k_T^{ness} subtraction (using $r_{\text{cut}} = 0.05\%$) and with CS. In the ratio panel, we observe an excellent agreement between the two results at the few permille level, confirming again the goodness of k_T^{ness} as a slicing variable. The three panels on the right display the NLO correction $\Delta\sigma$ as a function of r_{cut} in the (anti-)quark-gluon, gluon-gluon and (anti-)quark-(anti-)quark partonic channels compared

to the corresponding result obtained with CS. As found for the other processes, the results nicely converge to the CS values in all the channels, and also in this case the r_{cut} dependence is linear.
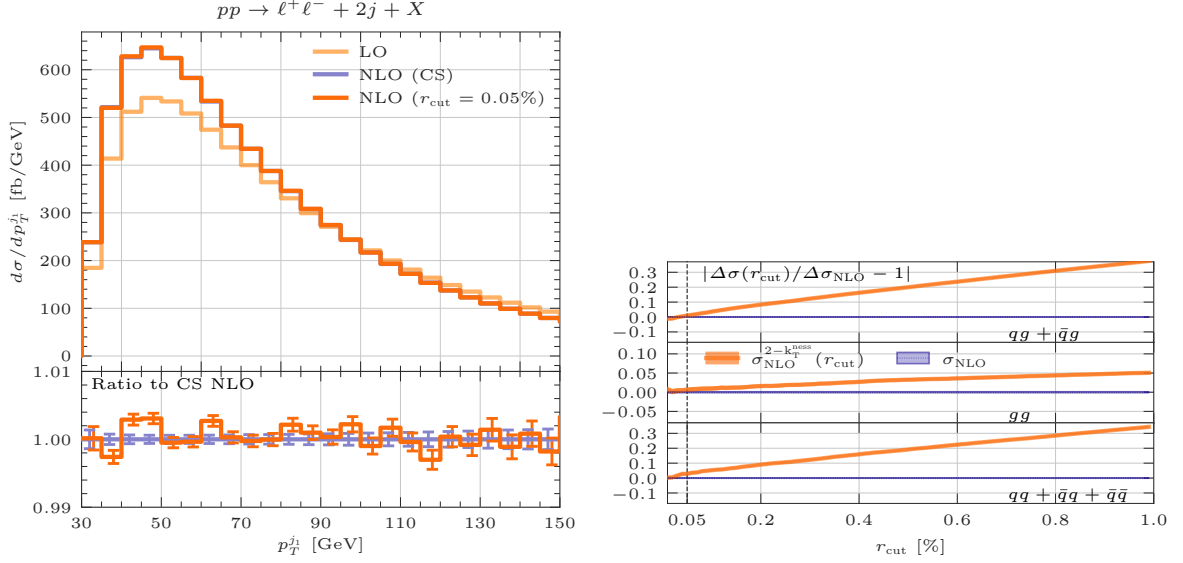

Figure 3: $Z + 2$ jet production at NLO: k_T^{ness}-subtraction against CS. The p_T distribution of the leading jet (left panels) at LO (yellow) and NLO (orange: k_T^{ness}, blue: CS). NLO corrections $\Delta\sigma$ as a function of r_{cut} in the three partonic channels (right panels).

So far, we have shown how k_T^{ness} behaves as a slicing variable. In view of potential applications of k_T^{ness} as a probe of jet production in hadron collisions, we have also studied the stability of our new variable under hadronization and MPI. For the details on the outcome of our study, we will refer to Ref. [7].

4. Conclusions

In this contribution we have discussed the new global observable k_T^{ness} introduced in Ref. [7].

The variable represents and effective transverse momentum controlling the singularities of the $N + 1$-jet cross section when the additional jet is unresolved. We have shown that k_T^{ness} can be used as a *slicing variable* to formulate a non-local subtraction method, analogously to what is done for q_T and jettiness. We have computed all necessary ingredients to build the subtraction at NLO and we have used the results to evaluate the NLO cross section for processes with one and two jets at Born level, finding complete agreement with the predictions obtained with standard tools.

The missing power corrections are under much better control and are purely linear. We verified that this scaling behavior, which is due to the fact that k_T^{ness} is an effective transverse momentum, is still linear for process with three jets and we expect that it holds for an arbitrary number of jets, since no additional perturbative ingredients appear beyond the three jet case. The extension of the k_T^{ness} subtraction method at NNLO would require a significant amount of conceptual and theoretical work, but the perturbative coefficients entering the NLO and NNLO fixed-order computation are fundamental ingredients to study the all-order structure of k_T^{ness} via a *resummation* procedure.

We also expect that, being an effective transverse momentum, k_T^{ness} could be used as a resolution variable when matching fixed-order computations with k_T-ordered *parton showers*.

Acknowledgments

References

[1] A. Banfi, G. P. Salam and G. Zanderighi, *JHEP* **08** (2004), 062 [hep-ph/0407287].

[2] A. Banfi, G. P. Salam and G. Zanderighi, *JHEP* **06** (2010), 038 [hep-ph/1001.4082].

[3] S. Catani and M. Grazzini, Phys. Rev. Lett. **98** (2007), 222002 [hep-ph/0703012].

[4] S. Catani, S. Devoto, M. Grazzini, S. Kallweit and J. Mazzitelli, *JHEP* **07** (2019), 100 [hep-ph/1906.06535].

[5] S. Catani, S. Devoto, M. Grazzini, S. Kallweit and J. Mazzitelli, *JHEP* **03** (2021), 029 [hep-ph/2010.11906].

[6] M. Grazzini, S. Kallweit and M. Wiesemann, *Eur. Phys. J. C* **78** (2018) no.7, 537 [hep-ph/1711.06631].

[7] L. Buonocore, M. Grazzini, J. Haag, L. Rottoli and C. Savoini, [hep-ph/2201.11519].

[8] I. W. Stewart, F. J. Tackmann and W. J. Waalewijn, Phys. Rev. Lett. **105** (2010), 092002 [hep-ph/1004.2489].

[9] J. Gaunt, M. Stahlhofen, F. J. Tackmann and J. R. Walsh, *JHEP* **09** (2015), 058 [hep-ph/1505.04794].

[10] R. Boughezal, J. M. Campbell, R. K. Ellis, C. Focke, W. Giele, X. Liu, F. Petriello and C. Williams, *Eur. Phys. J. C* **77** (2017) no.1, 7 [hep-ph/1605.08011].

[11] J. M. Campbell, R. K. Ellis, Y. Li and C. Williams, *JHEP* **07** (2016), 148 [hep-ph/1603.02663].

[12] J. M. Campbell, T. Neumann and C. Williams, *JHEP* **11** (2017), 150 [hep-ph/1708.02925].

[13] G. Heinrich, S. Jahn, S. P. Jones, M. Kerner and J. Pires, *JHEP* **03** (2018), 142 [hep-ph/1710.06294].

[14] R. Boughezal, X. Liu and F. Petriello, *Phys. Rev. D* **94** (2016) no.11, 113009 [hep-ph/1602.06965].

[15] R. Boughezal, J. M. Campbell, R. K. Ellis, C. Focke, W. T. Giele, X. Liu and F. Petriello, *Phys. Rev. Lett.* **116** (2016) no.15, 152001 [hep-ph/1512.01291].

[16] R. Boughezal, C. Focke, W. Giele, X. Liu and F. Petriello, *Phys. Lett. B* **748** (2015), 5-8 [hep-ph/1505.03893].

[17] L. Buonocore, M. Grazzini, J. Haag and L. Rottoli, *Eur. Phys. J. C* **82** (2022) no.1, 27 [hep-ph/2110.06913].