Identified hadrons in antenna subtraction at NNLO

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Processes with identified hadrons require the introduction of fragmentation functions to describe the hadronisation of a quark or a gluon into the observed hadron particle. Such identified particles in the final state make the treatment of infrared divergences more subtle, because of additional collinear divergences to be handled. We extend the antenna subtraction method to include hadron fragmentation processes up to next-to-next-to-leading order (NNLO) in QCD in e^+e^- collisions. To this end, we introduce new double-real and real-virtual fragmentation antenna functions in the final-final kinematics, with associated phase space mappings. These antenna functions are integrated over the relevant phase spaces, retaining their dependence on the momentum fraction of the fragmenting parton.

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1. Introduction

In QCD, single-inclusive hadron production can be described by convoluting single-inclusive parton production, which is calculable in perturbation theory, with fragmentation functions (FF, [1, 2]) that parametrise the parton-to-hadron transition as a function of the fractional momentum transfer. Identified hadrons play an increasingly important role in precision measurements at the LHC, for example in cross sections in association with a vector boson or a photon, where they are relevant to determining the flavour decomposition of PDFs.

For notational simplicity, we focus on one hadron (plus jets) production at e^+e^- colliders:

$$e^+ + e^- \longrightarrow H(K_H) + X\,(\,+\text{jets}\,) \tag{1}$$

where we identify a hadron H with momentum K_H and possibly some jets, which may or not contain the identified hadron. The fully differential cross section can be written as

$$\mathrm{d}\sigma^H = \sum_p \int \mathrm{d}\eta\, D_p^H(\eta, \mu_a^2)\, \mathrm{d}\hat{\sigma}_p(\eta, \mu_a^2)\,, \tag{2}$$

where the index p runs over all possible partons in the process, D_p^H is the physical (mass-factorised) fragmentation function describing the collinear fragmentation process of the parton p into the hadron H, and μ_a^2 is the fragmentation scale (which may differ from the renormalisation scale). In the framework of collinear factorisation encoded in (2), the momentum k_p of the identified parton is proportional to the momentum K_H of the identified hadron according to the simple relation $K_H = \eta k_p$.

Beyond leading order, it is well known that infrared divergences of soft and collinear origin appear in the short-distance cross section. They are guaranteed to cancel between real and virtual contributions in sufficiently inclusive observables, but a subtraction method is required in order to deal with such divergences in the intermediate steps of the calculation. In the antenna subtraction formalism [3, 4], the singularities associated to single or double unresolved particles in real emission matrix elements are locally subtracted by means of counterterms built out of antenna functions [5–7]. Each antenna function encodes the radiation pattern between a pair of hard radiators, thus reproducing the behaviour of the matrix element in the singular limits, but being simple enough to be analytically integrated over the unresolved degrees of freedom. The integrated subtraction terms are then added back at the virtual level, so as to cancel the explicit poles appearing in the virtual matrix elements.

However, whenever we identify a parton, we spoil the cancellation of collinear divergences. The physical reason is that by identifying for example a quark we are in the position to distinguish a quark from a collinear quark-gluon pair. These collinear divergences are subtracted from the short-distance cross sections by means of mass factorisation counterterms and absorbed in the bare fragmentation functions, which eventually result in mass-factorised fragmentation functions, the ones appearing in (2). In order to allow for a proper subtraction of final-state collinear divergences, we need to keep track of the momentum fraction of the fragmenting parton in the intermediate layers of the calculation.

In this talk, we introduce fragmentation antenna functions which explicitly depend on the momentum fraction of the fragmenting parton. After integrating over all kinematical variables except the momentum fraction, these fragmentation antenna functions have the proper structure to be combined with the mass factorisation counterterms and result in a cancellation of final-state collinear divergence at the integrand level, *before* performing the convolution with the fragmentation function. The integrated antenna functions are inclusive over unresolved radiation, but in the context of a subtraction scheme they can be used as local subtraction terms for more *exclusive* calculations, such as the process in (1).

We limit ourselves to explain where the subtraction terms have to be modified in order to account for the presence of the identified particle and we provide some details about the integration of the fragmentation antenna functions. Checks on the integrated antenna functions by comparison against known single-inclusive NNLO coefficients, as well as explicit subtraction terms at NLO for hadron-in-jet fragmentation in three-jet final states in e^+e^- annihilation, can be found in [8].

2. Subtraction at NLO

The short-distance one-parton exclusive cross section appearing in (2) admits a perturbative expansion in the renormalised strong coupling constant α_s,

$$\mathrm{d}\hat{\sigma}_p(\eta) = \mathrm{d}\hat{\sigma}_p^{\mathrm{LO}}(\eta) + \left(\frac{\alpha_s}{2\pi}\right)\mathrm{d}\hat{\sigma}_p^{\mathrm{NLO}}(\eta) + \left(\frac{\alpha_s}{2\pi}\right)^2\mathrm{d}\hat{\sigma}_p^{\mathrm{NNLO}}(\eta)\,. \tag{3}$$

For instance, the LO cross section is defined as the integration over the n particle phase space of the tree-level Born partonic cross section:

$$\mathrm{d}\hat{\sigma}_p^{\mathrm{LO}}(\eta) = \int_n \mathrm{d}\hat{\sigma}_p^{\mathrm{B}}(\eta)\,, \tag{4}$$

with

$$\mathrm{d}\hat{\sigma}_p^{\mathrm{B}}(\eta) = \mathcal{N}_{\mathrm{B}}\,\mathrm{d}\Phi_n(k_1,\ldots,k_n;Q)\,\frac{1}{S_n}\,M_n^0(k_1,\ldots,k_n)\,J(\{k_1,\ldots,k_n\}_n,\eta k_p)\,, \tag{5}$$

with $\mathcal{N}_{\mathrm{B}}$ the Born-level normalisation factor, S_n a symmetry factor for final-state particles, M_n^0 the squared tree-level n-particle matrix element and $\mathrm{d}\Phi_n$ the usual phase space for a n-parton final state with total four-momentum Q^μ in $d = 4-2\epsilon$ space-time dimensions. Compared to the standard jet cross sections, the element of novelty here is the modified jet function J, which retains a dependence on the momentum fraction η, similarly to what was done in the photon fragmentation case in [9]. The purpose of the modified jet function is to define jet observables and/or any additional observable depending on the momentum k_p of the identified parton.

The NLO corrections to the one-parton exclusive cross section in (3) contain contributions from real emission of one extra parton and virtual corrections. As it is customary in antenna subtraction, we introduce a real subtraction term $\mathrm{d}\hat{\sigma}_p^{\mathrm{S}}$ and a virtual subtraction term $\mathrm{d}\hat{\sigma}_p^{\mathrm{T}}$, to be subtracted from the real cross section $\mathrm{d}\hat{\sigma}_p^{\mathrm{R}}$ and the virtual cross section $\mathrm{d}\hat{\sigma}_p^{\mathrm{V}}$, respectively. The NLO short-distance cross section can then be written as

$$\mathrm{d}\hat{\sigma}_p^{\mathrm{NLO}}(\eta) = \int_{n+1}\left[\mathrm{d}\hat{\sigma}_p^{\mathrm{R}}(\eta) - \mathrm{d}\hat{\sigma}_p^{\mathrm{S}}(\eta)\right] + \int_n\left[\mathrm{d}\hat{\sigma}_p^{\mathrm{V}}(\eta) - \mathrm{d}\hat{\sigma}_p^{\mathrm{T}}(\eta)\right]\,. \tag{6}$$

Each term in square brackets in (6) is free of infrared divergences and suitable for a numerical implementation. Note the subscript in (6), indicating that each term retains a dependence on the parton which is undergoing the fragmentation process.

The real partonic cross section $\mathrm{d}\hat{\sigma}_p^{\mathrm{R}}$ is given by (5) with an additional parton. It is decomposed according to its colour orderings. As for the real subtraction term $\mathrm{d}\hat{\sigma}_p^{\mathrm{S}}$, it will be given by the sum of several terms, summing over all possible single unresolved partons:

$$\mathrm{d}\hat{\sigma}_p^{\mathrm{S}} = \sum_j \mathrm{d}\hat{\sigma}_{p,j}^{\mathrm{S}} \,. \tag{7}$$

The $\mathrm{d}\hat{\sigma}_{p,j}^{\mathrm{S}}$ are obtained by summing over all colour connections in which the parton j can become unresolved. They are further decomposed in two types of contributions as

$$\mathrm{d}\hat{\sigma}_{p,j}^{\mathrm{S}} = \mathrm{d}\hat{\sigma}_{p,j}^{\mathrm{S,non-id.}p} + \mathrm{d}\hat{\sigma}_{p,j}^{\mathrm{S,id.}p}, \tag{8}$$

where the first term contains all configurations where the identified parton p is not colour-connected to the unresolved parton j, such that we can use the standard NLO subtraction term with final-final kinematics, with p appearing unmodified in the respective reduced matrix element.

In order to subtract the infrared limits involving the unresolved parton j colour-connected to the identified p and a second hard parton k, we newly introduce the following subtraction term

$$\begin{aligned} \mathrm{d}\hat{\sigma}_{p,j}^{\mathrm{S,id.}p} \;=\; & \mathcal{N}_{\mathrm{R}} \mathrm{d}\Phi_{n+1}(k_1,\ldots,k_p,\ldots,k_{n+1};Q)\,\frac{1}{S_{n+1}} \\ & \times X_3^0(k_p,k_j,k_k)\, M_n^0(k_1,\ldots,\tilde{K},\tilde{k}_p,\ldots,k_{n+1})\, J(\{\ldots,\tilde{K},\tilde{k}_p,\ldots\}_n, \eta\, z\, \tilde{k}_p)\,. \end{aligned} \tag{9}$$

where $\mathcal{N}_{\mathrm{R}} = \mathcal{N}_{\mathrm{B}}\, \overline{C}(\epsilon)/C(\epsilon)$, with

$$C(\epsilon) = \frac{(4\pi e^{-\gamma_E})^{\epsilon}}{8\pi^2}\,, \quad \overline{C}(\epsilon) = (4\pi e^{-\gamma_E})^{\epsilon}\,, \tag{10}$$

which are customary normalisation factors in the antenna subtraction formalism. The X_3^0 function is just the standard three-particle tree-level antenna function in the final-final kinematics, depending on the final state momenta which sum up to $q = k_j + k_k + k_p$ with $Q^2 \geq q^2 > 0$. The phase space mapping involves the reconstruction of the momentum fraction z, used to define $\tilde{k}_p = k_p/z$, and of a recoil momentum $\tilde{K}$. The momentum fraction z is defined by projecting the momentum of the fragmenting parton and the momentum of its parent parton pair onto a specific reference four-vector that can be chosen freely. In our case, we choose q as reference direction, resulting in

$$\begin{aligned} z &= \frac{s_{pj}+s_{pk}}{s_{pj}+s_{pk}+s_{jk}}\,, \\ \tilde{K} &= k_j + k_k - (1-z)\frac{k_p}{z}\,, \end{aligned} \tag{11}$$

which satisfies all the required properties. In particular, in the collinear limit $k_p \parallel k_j$, z approaches the momentum fraction of k_p along the common collinear direction. Hence the overall momentum fraction entering the jet function is the product of η and z.

In order to reach the factorisation of the phase space, we follow closely [10] by inserting

$$1 = \int \mathrm{d}^d q\, \delta\left(q - k_p - k_j - k_k\right)\,, \tag{12}$$

and

$$1 = \frac{q^2}{2\pi} \int \frac{\mathrm{d}z}{z} \int [\mathrm{d}\tilde{K}]\, (2\pi)^d \delta\left(q - \frac{k_p}{z} - \tilde{K}\right)\,. \tag{13}$$

Since in (9) we are integrating over k_p, we need to introduce the one-particle phase space for $\tilde{k}_p$. They are related by

$$[\mathrm{d}\tilde{k}_p] = [\mathrm{d}k_p]\, z^{2-d} = [\mathrm{d}k_p]\, z^{-2+2\epsilon}\,, \tag{14}$$

which is due to the fact that $[\mathrm{d}p] \propto E^{d-3}\mathrm{d}E$. Hence, by integrating over q, we get

$$\begin{aligned} \mathrm{d}\Phi_{n+1}(k_1,\ldots,k_p,k_j,k_k,\ldots,k_{n+1};Q) &= \mathrm{d}\Phi_n(k_1,\ldots,\tilde{k}_p,\tilde{K},\ldots,k_{n+1};Q) \\ &\times \frac{q^2}{2\pi}\mathrm{d}\Phi_2(k_j,k_k;q-k_p)\, z^{1-2\epsilon}\,\mathrm{d}z\,. \end{aligned} \tag{15}$$

We define the integrated version of the fragmentation antenna function as

$$\mathcal{X}_3^{0,\mathrm{id}.p}(z) = \frac{1}{C(\epsilon)}\int \mathrm{d}\Phi_2 \frac{q^2}{2\pi}\, z^{1-2\epsilon}\, X_3^0(k_p^{\mathrm{id.}},k_j,k_k)\,, \tag{16}$$

where the two-particle phase space has kinematics

$$q + (-k_p) \to k_1 + k_2\,, \tag{17}$$

with $s_{12} = (q-k_p)^2 = q^2(1-z)$ and

$$z = \frac{2\,k_p\cdot q}{q^2}\,. \tag{18}$$

The fragmenting parton k_p may be regarded as an initial state parton with negative four-momentum, and we are then looking at the $1 \to 3$ process with one identified parton as a $2 \to 2$ scattering process with rescaled invariant mass.

The integrated form of the subtraction term is then

$$\begin{aligned} \int_1 \mathrm{d}\hat{\sigma}^{\mathrm{S,id}.p}_{p,j} &= \mathcal{N}_{\mathrm{V}}\int \mathrm{d}z\,\mathrm{d}\Phi_n(k_1,\ldots,\tilde{k}_p,\tilde{K},\ldots,k_{n+1};Q)\,\frac{1}{S_n} \\ &\times \mathcal{X}_3^{0,\mathrm{id}.p}(z)\, M_n^0(k_1,\ldots,\tilde{k}_p,\tilde{K},\ldots,k_{n+1})\, J(\{\ldots,\tilde{k}_p,\tilde{K},\ldots\}_n,\eta\, z\,\tilde{k}_p)\,, \end{aligned} \tag{19}$$

with $\mathcal{N}_{\mathrm{V}} = \mathcal{N}_{\mathrm{R}}C(\epsilon) = \mathcal{N}_{\mathrm{B}}\,\overline{C}(\epsilon)$. The above expression contains the infrared poles required to cancel explicit poles of the virtual matrix element associated with the colour-connections involving the identified parton momentum p as well as collinear poles proportional to the Altarelli-Parisi splitting functions, which need to be properly subtracted by means of an NLO mass factorisation counterterm.

3. Subtraction at NNLO

Predictions at NNLO require the calculation of three different contributions, namely double-real (RR), real-virtual (RV) and double-virtual (VV) contributions relative to the Born process. Each of these pieces is separately infrared divergent, whereas their sum is guaranteed to be finite. In order to handle the implicit divergences and explicit poles that arise in the intermediate steps of the calculation, three subtraction terms are introduced: a RR subtraction term $\mathrm{d}\hat{\sigma}_p^{\mathrm{S}}$, a RV subtraction term $\mathrm{d}\hat{\sigma}_p^{\mathrm{T}}$ and a VV subtraction term $\mathrm{d}\hat{\sigma}_p^{\mathrm{U}}$, such that the analogue of (6) at NNLO reads

$$\mathrm{d}\hat{\sigma}_p^{\mathrm{NNLO}}(\eta) = \int_{n+2}\left[\mathrm{d}\hat{\sigma}_p^{\mathrm{RR}} - \mathrm{d}\hat{\sigma}_p^{\mathrm{S}}\right] + \int_{n+1}\left[\mathrm{d}\hat{\sigma}_p^{\mathrm{RV}} - \mathrm{d}\hat{\sigma}_p^{\mathrm{T}}\right] + \int_n\left[\mathrm{d}\hat{\sigma}_p^{\mathrm{VV}} - \mathrm{d}\hat{\sigma}_p^{\mathrm{U}}\right]\,, \tag{20}$$

where a dependence on η in the integrands is understood.

The real-real subtraction term $\mathrm{d}\hat{\sigma}_p^{\mathrm{S}}$ is built out of several pieces, each of which accounts for a particular type of unresolved configuration. The first piece, $\mathrm{d}\hat{\sigma}_p^{\mathrm{S},a}$ deals with the the single unresolved limits of the double real matrix elements. Its structure is similar to the NLO real subtration term, already introduced in (7). The second piece, $\mathrm{d}\hat{\sigma}_p^{\mathrm{S},b}$ accounts for the double unresolved limits of the RR matrix element. We can distinguish configurations with the identified parton p colour-connected or not to the pair of unresolved partons j and k. In the latter case, we can use the standard NNLO subtraction term. In the former case, we introduce the subtraction term

$$\begin{aligned}
\mathrm{d}\hat{\sigma}_{p,jk}^{\mathrm{S},b,\mathrm{id}.p} &= \mathcal{N}_{\mathrm{RR}}\,\mathrm{d}\Phi_{n+2}(k_1,\ldots,k_{n+2};Q)\,\frac{1}{S_{n+2}} \\
&\times\Big[X_4^0(k_p,k_j,k_k,k_l)M_n^0(k_1,\ldots,\tilde{k}_p,\tilde{K},\ldots,k_{n+2})\,J(\{\ldots,\tilde{k}_p,\tilde{K},\ldots\}_n,\eta\,z\,\tilde{k}_p) \\
&-X_3^0(k_p,k_j,k_k)\,X_3^0(\tilde{k}_p,\tilde{K},k_l)M_n^0(k_1,\ldots,\tilde{\tilde{k}}_p,\tilde{\tilde{K}},\ldots,k_{n+2})\,J(\{\ldots,\tilde{\tilde{k}}_p,\tilde{\tilde{K}},\ldots\}_n,\eta\,z\,\tilde{\tilde{k}}_p) \\
&-X_3^0(k_j,k_k,k_l)\,X_3^0(k_p,\tilde{k}_{jk},\tilde{k}_{kl})M_n^0(k_1,\ldots,\tilde{\tilde{k}}_p,\tilde{\tilde{K}},\ldots,k_{n+2})\,J(\{\ldots,\tilde{\tilde{k}}_p,\tilde{\tilde{K}},\ldots\}_n,\eta\,z\,\tilde{\tilde{k}}_p)\Big]\,,
\end{aligned} \tag{21}$$

with $\mathcal{N}_{\mathrm{RR}} = \mathcal{N}_{\mathrm{B}}\,\overline{C}(\epsilon)^2/C(\epsilon)^2$. The two products of three-parton antenna function are necessary to subtract the single unresolved limits of the four-parton antenna function, such that $\mathrm{d}\hat{\sigma}_{p,jk}^{\mathrm{S},b,\mathrm{id}.p}$ is active only in the double unresolved limits. They each involve two consecutive NLO phase space mappings, whose results are abbreviated as $(\tilde{\tilde{k}}_p,\tilde{\tilde{K}})$ and where the intermediate momenta $(\tilde{k}_{jk},\tilde{k}_{kl})$ indicate a standard final-final NLO mapping. The genuine NNLO mapping to $(\tilde{k}_p = k_p/z,\tilde{K})$ in the first term is a generalisation of (11) with more than one parton becoming unresolved. Explicitly, it reads:

$$\begin{aligned}
z &= \frac{s_{pj}+s_{pk}+s_{pl}}{s_{pj}+s_{pk}+s_{jk}+s_{pl}+s_{jl}+s_{kl}}\,, \\
\tilde{K} &= k_j+k_k+k_l-(1-z)\frac{k_p}{z}\,.
\end{aligned} \tag{22}$$

with p the fragmenting parton, j and k the two unresolved partons, and l the other final state radiator. Such a mapping satisfies the appropriate limits in all double singular configurations. Moreover, it turns into an NLO phase space mapping in its single unresolved limits, as required in order to cancel the single unresolved limits of the X_4^0 antenna function. The integral of the tree-level four-particle antenna function over the three-particle phase space

$$\mathcal{X}_4^{0,\mathrm{id}.p}(z) = \frac{1}{[C(\epsilon)]^2}\int \mathrm{d}\Phi_3\,\frac{q^2}{2\pi}\,z^{1-2\epsilon}\,X_4^0(k_p^{\mathrm{id}.},k_j,k_k,k_l)\,, \tag{23}$$

reappears at the virtual-virtual level. We integrate the X_4^0 over a three-particle phase space with $2\to3$ kinematics

$$q+(-k_p)\longrightarrow k_1+k_2+k_3\,. \tag{24}$$

The integration is performed with well-known techniques, based on multi-loop calculation technology. Namely, we rewrite the three-particle phase space in terms of cut propagators, in order to express it as a cut through a three-loop vacuum polarisation diagram. Then, we reduce the occurring integrals as linear combinations of a smaller set of master integrals, with the help of REDUZE2 [11]. The master integrals have been calculated by exploiting the differential equations method [12]. We first use EPSILON [13] to find the canonical form [14, 15] of the differential equations. Once expressed in the canonical form, the differential equations can be iteratively solved in term of the harmonic polylogarithms (HPLs) [16], with unknown boundary conditions. In order to impose the boundary conditions, and at the same time check the structure of the master integrals, we exploit the fact that the master integrals for real-real fragmentation antenna functions are related to the master integrals for initial-final antenna functions, reported in Appendix A.1 of [17], by means of the replacement

$$Q^2 \to -q^2 , \quad x \to 1/z , \tag{25}$$

which amounts to an analytic continuation. The whole workflow of the calculation has been implemented in FORM [18].

The other real-real subtraction terms, $\mathrm{d}\hat{\sigma}^{\mathrm{S},c}$ and $\mathrm{d}\hat{\sigma}^{\mathrm{S},d}$, do not require new ingredients: they contain products of tree-level three-parton antenna functions or fragmentation antenna functions in the final-final kinematics, or soft antenna functions [19].

At the real-virtual level, we need to remove the explicit infrared poles of the one-loop matrix element and to also subtract its single unresolved limit. The former purpose is accomplished by the integral of $\mathrm{d}\hat{\sigma}_p^{\mathrm{S},a}$, which combined with mass factorisation terms, results in $\mathrm{d}\hat{\sigma}_p^{\mathrm{T},a}$. The latter purpose requires the introduction of a new subtraction term:

$$\begin{aligned}\mathrm{d}\hat{\sigma}_{p,j}^{\mathrm{T},b,\mathrm{id}.p} \quad &= \quad \mathcal{N}_{\mathrm{RV}}\, \mathrm{d}\Phi_{n+1}(k_1,\ldots,k_{n+1};Q)\,\frac{1}{S_{n+1}} J(\{\ldots,\tilde{k}_p,\tilde{K},\ldots\}_n,\eta\, z\, \tilde{k}_p) \\ &\quad \times \Bigg[X_3^0(k_p,k_j,k_k)\, M_n^1(k_1,\ldots,\tilde{k}_p,\tilde{K},\ldots,k_{n+1}) \\ &\qquad + X_3^1(k_p,k_j,k_k)\, M_n^0(k_1,\ldots,\tilde{k}_p,\tilde{K},\ldots,k_{n+1}) \Bigg] . \end{aligned} \tag{26}$$

with $\mathcal{N}_{\mathrm{RV}} = \mathcal{N}_{\mathrm{B}}\, \overline{C}(\epsilon)^2/C(\epsilon)$, and where we have used the NLO momentum mapping (11). In here, M_n^1 is the one-loop reduced matrix element and X_3^1 is the one-loop three-parton antenna function in the final-final kinematics, whose integral over the two-particle phase space is denoted as

$$\mathcal{X}_3^{1,\mathrm{id}.p}(z) = \frac{1}{C(\epsilon)} \int \mathrm{d}\Phi_2 \frac{q^2}{2\pi}\, z^{1-2\epsilon}\, X_3^1(k_p^{\mathrm{id}.},k_j,k_k)\,; \tag{27}$$

it is reintroduced at the virtual-virtual level. The integration of the X_3^1 fragmentation antenna is performed over a two-particle phase space with $2 \to 2$ kinematics, as in (17). Since the X_3^1 antenna functions are expressed in terms of rational functions of invariants multiplying one-loop bubble and box integrals, we first rewrite the one-loop integrals in terms of propagators, and then we write the two-particle phase space integral of the one-loop antenna functions as a three-loop integral with two cut propagators. The master integrals are determined with the differential equation method, with boundary conditions obtained by internal consistency of the set of equations, or by a direct calculation at $z = 1$. In the real-virtual case, the master integrals for fragmentation antenna functions cannot be inferred from the master integrals for initial-final antenna functions, reported in Appendix A.2 of [17], since the analytic continuation acts differently on the different bubble and box integrals and must be performed prior to the phase space integration. The integrated one-loop squared matrix elements are subsequently renormalised, as described in detail in Section 4.2 of [17].

The last piece needed for the subtraction at the real-virtual level is $\mathrm{d}\hat{\sigma}^{\mathrm{T},c}$, which results from the integration of $\mathrm{d}\hat{\sigma}^{\mathrm{S},c}$, plus additional terms to ensure an IR finite contribution, which are added back at the double virtual level.

At the virtual-virtual level, there are no implicit infrared divergences; the explicit poles of the two-loop matrix element are canceled by the integrated form of the appropiate subtraction terms, together with the double virtual mass factorisation term.

4. Conclusions

In this talk, we have described how identified final-state hadrons can be incorporated in the antenna subtraction formalism for NNLO calculations, which required the introduction of fragmentation antenna functions. These functions retain the information on a final-state parton momentum fraction, in contrast to previously considered antenna functions that were inclusive in the final state parton momenta.

In the description of the formalism, we focused on identified hadron production processes in generic hadronic final states in e^+e^- annihilation, where subtraction terms are constructed from antenna functions with both radiator partons in the final state (final-final kinematics). Such fragmentation antenna functions in final-final kinematics will also appear in the construction of subtraction terms for processes with identified hadrons in deep-inelastic scattering or at hadron colliders: combined with the fragmentation antenna functions in initial-final kinematics already derived in parts in the context of photon fragmentation up to NNLO [9], an extension of the formalism to ep and pp collisions is straightforward.

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