PROCEEDINGS OF SCIENCE

Automating Antenna Subtraction in Colour Space

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We present the colourful antenna subtraction method, a reformulation of the antenna subtraction scheme for next-to-next-to-leading order (NNLO) calculations in QCD. The aim of this new approach is to achieve a general and process-independent construction of the subtraction infrastructure at NNLO. We rely on the predictability of the infrared singularity structure of one- and two-loop amplitudes in colour space to generate virtual subtraction terms and, subsequently, we define an automatable procedure to systematically infer the expression of the real subtraction terms, guided by the correspondence between unintegrated and integrated antenna functions. To demonstrate the applicability of the described approach, we compute the full colour NNLO correction to gluonic three-jet production $pp(gg) \to ggg$, for gluons-only subprocesses.

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1. Introduction

In this talk we present the colourful antenna subtraction formalism for gluonic processes, which is described in detail in [1].

The NNLO antenna subtraction method [2, 3] is based up to now on the identification of single and double real radiation patterns in colour-ordered subprocess contributions and has been applied successfully in computing NNLO corrections to a variety of hadron-collider processes [4–13]. However, the efficiency of the present formulation scales poorly with the number of external partons and its application to processes involving four or more external partons is extremely challenging. One reason for this is the proliferation of infrared limits in real emission corrections for high-multiplicity processes. Moreover, in the context of antenna subtraction, the treatment of contributions beyond the leading colour approximation is highly non-trivial, due to the appearance at the matrix element level of incoherent interferences between different colour orderings, which cannot be straightforwardly addressed with the traditional technique. The structure of subleading-colour contributions becomes more involved at high multiplicities and thus represents a major obstacle. Because of these issues, the extension of antenna subtraction to new applications has required a significant amount of process-dependent work in the past years.

It is the objective of the colourful antenna subtraction method to overcome these limitations and to achieve a more general and process-independent formulation of the antenna subtraction. The primary goals are the definition of a systematic procedure for the generation of the entire subtraction infrastructure at NNLO and the more efficient treatment of colour correlations within matrix elements to directly retain the full N_c-dependence.

The main idea behind the new approach consists in relying on the predictability of the singularity structure of one- and two-loop amplitudes in colour space to automatically generate virtual subtraction terms, which cancel the explicit poles of virtual corrections. Subsequently, the one-to-one correspondence between antenna functions and their integrated counterparts is exploited to systematically infer real subtraction terms which can be used to remove the divergent behaviour of real emission corrections in the infrared limits. The colourful antenna subtraction method at NLO and NNLO is briefly described in the following.

2. Colourful antenna subtraction at NLO

The NLO QCD correction to an n-jet partonic cross section with parton species a and b in the initial state is given by [3]:

$$\mathrm{d}\hat{\sigma}_{ab,NLO} = \int_n \left(\mathrm{d}\hat{\sigma}^V_{ab,NLO} + \mathrm{d}\hat{\sigma}^{MF}_{ab,NLO}\right) + \int_{n+1} \mathrm{d}\hat{\sigma}^R_{ab,NLO}, \tag{1}$$

where the symbol $\int_n$ indicates an integration over the n final state particles. $\mathrm{d}\hat{\sigma}^V_{ab,NLO}$ and $\mathrm{d}\hat{\sigma}^R_{ab,NLO}$ respectively represent the virtual and real corrections, while $\mathrm{d}\hat{\sigma}^{MF}_{ab,NLO}$ is the NLO mass factorization counterterm. Due to the emergence of infrared divergences in both the virtual and real corrections, a subtraction procedure is needed to numerically evaluate (1). In the context of antenna subtraction, this is achieved constructing a real subtraction term $\mathrm{d}\hat{\sigma}^S_{ab,NLO}$ [3], which locally removes the singular behaviour of $\mathrm{d}\hat{\sigma}^R_{ab,NLO}$ in the IR limits and can be analytically integrated over the phase space of the unresolved radiation. This latter feature is required to obtain from $\mathrm{d}\hat{\sigma}^S_{ab,NLO}$ the virtual subtraction term $\mathrm{d}\hat{\sigma}^T_{ab,NLO}$, which cancels the explicit poles of the virtual correction and contains the mass factorization contribution. The NLO cross section can then be reformulated as [3]:

$$\mathrm{d}\hat{\sigma}_{ab,NLO} = \int_n \left[\mathrm{d}\hat{\sigma}^V_{ab,NLO} - \mathrm{d}\hat{\sigma}^T_{ab,NLO}\right] + \int_{n+1} \left[\mathrm{d}\hat{\sigma}^R_{ab,NLO} - \mathrm{d}\hat{\sigma}^S_{ab,NLO}\right], \tag{2}$$

with

$$\mathrm{d}\hat{\sigma}^T_{ab,NLO} = -\int_1 \mathrm{d}\hat{\sigma}^S_{ab,NLO} - \mathrm{d}\hat{\sigma}^{MF}_{ab,NLO}. \tag{3}$$

In the following we give an overview of how the virtual and the real subtraction terms are constructed in the colourful antenna subtraction approach, mainly to introduce important concepts, which will be crucial for its application at NNLO.

The singularity structure of renormalized $(n+2)$-parton one-loop amplitudes in QCD can be described in colour space with [14]:

$$|\mathcal{A}^1_{n+2}\rangle = \boldsymbol{I}^{(1)}\left(\epsilon,\mu_r^2\right)|\mathcal{A}^0_{n+2}\rangle + |\mathcal{A}^{1,\mathrm{fin}}_{n+2}(\mu_r^2)\rangle, \tag{4}$$

where μ_r is the renormalization scale, $|\mathcal{A}^{1,\mathrm{fin}}_{n+2}(\mu_r^2)\rangle$ is a finite remainder and $\boldsymbol{I}^{(1)}\left(\epsilon,\mu_r^2\right)$ is Catani's IR insertion operator [14], which can be rewritten as

$$\boldsymbol{I}^{(1)}\left(\epsilon,\mu_r^2\right) = \sum_{(i,j)} (\boldsymbol{T}_i\cdot\boldsymbol{T}_j)\,\mathcal{I}^{(1)}_{ij}\left(\epsilon,\mu_r^2\right), \tag{5}$$

where in the last line the sum runs over pairs of partons. For the gluons-only case that is considered in this talk, we only need the expression of $\mathcal{I}^{(1)}_{i_gj_g}\left(\epsilon,\mu_r^2\right)$ at $N_f=0$:

$$\mathcal{I}^{(1)}_{i_gj_g}\left(\epsilon,\mu_r^2\right) = \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\left[\frac{1}{\epsilon^2}+\frac{b_0}{\epsilon}\right]\left(\frac{-s_{ij}}{\mu_r^2}\right)^{-\epsilon}, \qquad b_0 = \frac{11}{6}. \tag{6}$$

Using (5) it is possible to write down the IR singularity structure of the virtual correction in the following general way:

$$\begin{aligned}\mathcal{P}oles\left(\hat{\sigma}^V_{gg,NLO}\right) \;=\; & \mathcal{N}^V_{NLO}\int \mathrm{d}\Phi_n(p_3,\ldots,p_{n+2};p_1,p_2)\,J^{(n)}_n(\{p\}_n) \\ & \times\mathcal{P}oles\left[\sum_{(i_g,j_g)}\langle\mathcal{A}^0_{n+2}|\boldsymbol{T}_{i_g}\cdot\boldsymbol{T}_{j_g}|\mathcal{A}^0_{n+2}\rangle\,2\,\mathrm{Re}\left(\mathcal{I}^{(1)}_{i_gj_g}\left(\epsilon,\mu_r^2\right)\right)\right],\end{aligned} \tag{7}$$

where the factor $\mathcal{N}^V_{NLO}$ is an appropriate overall normalization. In the colourful antenna subtraction approach, we exploit the previous result to directly construct the NLO virtual subtraction term. To do so, we define a NLO singularity dipole operator in colour space for an $(n+2)$-parton process:

$$\begin{aligned}\boldsymbol{\mathcal{J}}^{(1)}(\epsilon) \;=\; & \sum_{(i,j)\geq 3}(\boldsymbol{T}_i\cdot\boldsymbol{T}_j)\,\mathcal{J}^{(1)}_2(i_g,j_g) + \sum_{i\neq 1,2}(\boldsymbol{T}_1\cdot\boldsymbol{T}_i)\,\mathcal{J}^{(1)}_2(1_g,i_g) \\ & + \sum_{i\neq 1,2}(\boldsymbol{T}_2\cdot\boldsymbol{T}_i)\,\mathcal{J}^{(1)}_2(2_g,i_g) + (\boldsymbol{T}_1\cdot\boldsymbol{T}_2)\,\mathcal{J}^{(1)}_2(1_g,2_g).\end{aligned} \tag{8}$$

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The first sum in the previous formula runs over all pairs of gluons in the final state, the second and the third sums include all pairs with an initial-state gluon (respectively 1_g and 2_g) and a final-state one and the last term addresses the configuration where both gluons are in the initial state. The scalar functions $\mathcal{J}_2^{(1)}(i,j)$ are colour stripped one-loop integrated dipoles [3, 15], given by a combination of integrated three-parton tree-level antenna functions and NLO mass factorization kernels. The explicit expressions of the gluon-gluon integrated dipoles for final-final (FF), initial-final (IF) and initial-initial (II) configurations are the following:

$$\begin{aligned} \mathcal{J}_2^{(1)}(i_g,j_g) &= \frac{1}{3}\mathcal{F}_3^0(s_{ij}), \\ \mathcal{J}_2^{(1)}(1_g,j_g) &= \frac{1}{2}\mathcal{F}_{3,g}^0(s_{1j}) - \frac{1}{2}\Gamma_{gg}^{(1)}(x_1), \\ \mathcal{J}_2^{(1)}(1_g,2_g) &= \mathcal{F}_{3,gg}^0(s_{12}) - \frac{1}{2}\Gamma_{gg}^{(1)}(x_1)\,\delta_2 - \frac{1}{2}\Gamma_{gg}^{(1)}(x_2)\,\delta_1, \end{aligned} \tag{9}$$

where $\delta_i = \delta(1-x_i)$. The functions $\mathcal{F}_3^0$, $\mathcal{F}_{3,g}^0$ and $\mathcal{F}_{3,gg}^0$ are gluon-gluon three-parton integrated antenna functions [2, 16]. The integrated dipoles in (9) incorporate the mass factorization counterterm, as indicated by the presence of the gluon-gluon splitting kernels $\Gamma_{gg}^{(1)}$. The poles carried by the mass factorization kernels cancel with poles in the integrated initial-final and initial-initial antenna functions associated with initial-state collinear divergences. The remaining ϵ-poles exactly match the ones of the virtual matrix element, once the operator in (8) is evaluated on the corresponding Born-level amplitude in colour space. In particular, at one loop the following relation holds:

$$\mathcal{P}oles\left[\mathcal{J}_2^{(1)}(i_g,j_g)\right] = \mathcal{P}oles\left[\mathrm{Re}\left(\mathcal{I}_{i_g j_g}^{(1)}\left(\epsilon,\mu_r^2\right)\right)\right]. \tag{10}$$

It is then possible to express the NLO virtual subtraction term as

$$\begin{aligned} \mathrm{d}\hat{\sigma}_{gg,NLO}^T &= \mathcal{N}_{NLO}^V \int \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\mathrm{d}\Phi_n(p_3,\ldots,p_{n+2};x_1p_1,x_2p_2)J_n^{(n)}(\{p\}_n) \\ &\quad \times 2\,\langle\mathcal{A}_{n+2}^0|\boldsymbol{\mathcal{J}}^{(1)}(\epsilon)|\mathcal{A}_{n+2}^0\rangle\,. \end{aligned} \tag{11}$$

We remark that (11) is a completely general result in the case of gluon scattering: it is valid for any number of external legs and retains the full N_c dependence.

The real subtraction term at NLO is systematically obtained from (11) relying on the one-to-one correspondence between integrated and unintegrated antenna functions:

$$\mathcal{X}_3^0(s_{ij}) \leftrightarrow X_3^0(i,k,j), \tag{12}$$

where $\mathcal{X}_3^0(s_{ij})$ is the integrated antenna function obtained integrating the tree-level three-parton antenna function $X_3^0(i,k,j)$ over the phase space of the unresolved parton k. Due to this relation, once the virtual subtraction term is obtained, the structure of the real subtraction term can be completely determined by inserting an unresolved gluon between each pair of hard radiators appearing in the integrated dipoles. The procedure to obtain $\mathrm{d}\hat{\sigma}_{NLO,gg}^S$ from $\mathrm{d}\hat{\sigma}_{NLO,gg}^T$ can be formulated as follows:

1. Removal of the splitting kernels from the integrated dipoles;

2. Transition from integrated three-parton antenna functions to unintegrated ones:

$$\begin{array}{llcl} \text{FF:} & \mathcal{F}_3^0(s_{ij}) & \to & 3\, f_3^0(i,k,j), \\ \text{IF:} & \mathcal{F}_{3,g}^0(s_{1i}) & \to & 2\, f_{3,g}^0(1,k,i), \\ \text{II:} & \mathcal{F}_{3,gg}^0(s_{12}) & \to & F_{3,gg}^0(1,k,2), \end{array} \tag{13}$$

3. Momenta relabeling within colour interferences and jet functions according to the accompanying antenna function;

4. Sum over permutations of the $n+3$ momenta to cover all possible IR limits;

5. Dressing of the obtained expression with the appropriate phase space and overall coefficient factor.

We introduce the following notation for the procedure that we have just described:

$$\mathrm{d}\hat{\sigma}_{gg,NLO}^{S} = -\mathcal{I}ns\left[\mathrm{d}\hat{\sigma}_{gg,NLO}^{T}\right], \tag{14}$$

where the minus sign in (14) follows from (3).

We remark that the knowledge of the unintegrated antenna functions required for the construction of the real subtraction term is a crucial premise for the application of this method. Indeed, the described unintegration procedure allows for a systematic assembly of such ingredients and not for an actual direct generation of the structures required to remove the IR divergences of real emission corrections.

3. Colourful antenna subtraction at NNLO

The NNLO QCD correction to an n-jet cross section is given by:

$$\begin{aligned} \mathrm{d}\hat{\sigma}_{ab,NNLO} &= \int_n \left(\mathrm{d}\hat{\sigma}_{ab,NNLO}^{VV} + \mathrm{d}\hat{\sigma}_{ab,NNLO}^{MF,2}\right) \\ &+ \int_{n+1} \left(\mathrm{d}\hat{\sigma}_{ab,NNLO}^{RV} + \mathrm{d}\hat{\sigma}_{ab,NNLO}^{MF,1}\right) + \int_{n+2} \mathrm{d}\hat{\sigma}_{ab,NNLO}^{RR}, \end{aligned} \tag{15}$$

where $\mathrm{d}\hat{\sigma}_{ab,NNLO}^{VV}$ represents the double virtual correction, $\mathrm{d}\hat{\sigma}_{ab,NNLO}^{RV}$ the real virtual correction and $\mathrm{d}\hat{\sigma}_{ab,NNLO}^{RR}$ the double real correction. The mass factorization counterterm is split into two terms associated with n- and $(n+1)$-particle final states

The singular behaviour of both the double real and real virtual corrections in the IR limits must be subtracted and the explicit poles in the double virtual and real virtual matrix elements need to be properly removed. To achieve this, the NNLO cross section is rewritten in the context of antenna subtraction as [3]:

$$\begin{aligned} \mathrm{d}\hat{\sigma}_{ab,NNLO} &= \int_n \left[\mathrm{d}\hat{\sigma}_{ab,NNLO}^{VV} - \mathrm{d}\hat{\sigma}_{ab,NNLO}^{U}\right] + \int_{n+1} \left[\mathrm{d}\hat{\sigma}_{ab,NNLO}^{RV} - \mathrm{d}\hat{\sigma}_{ab,NNLO}^{T}\right] \\ &+ \int_{n+2} \left[\mathrm{d}\hat{\sigma}_{ab,NNLO}^{RR} - \mathrm{d}\hat{\sigma}_{ab,NNLO}^{S}\right], \end{aligned} \tag{16}$$

where the subtracted quantities are the double virtual, the real virtual and the double real subtraction term. These contributions have the following form [3]:

$$\begin{aligned} \mathrm{d}\hat{\sigma}^{S}_{ab,NNLO} &= \mathrm{d}\hat{\sigma}^{S,1}_{ab,NNLO} + \mathrm{d}\hat{\sigma}^{S,2}_{ab,NNLO}\,, \\ \mathrm{d}\hat{\sigma}^{T}_{ab,NNLO} &= \mathrm{d}\hat{\sigma}^{VS}_{ab,NNLO} - \int_1 \mathrm{d}\hat{\sigma}^{S,1}_{ab,NNLO} - \mathrm{d}\hat{\sigma}^{MF,1}_{ab,NNLO}\,, \\ \mathrm{d}\hat{\sigma}^{U}_{ab,NNLO} &= -\int_1 \mathrm{d}\hat{\sigma}^{VS}_{ab,NNLO} - \int_2 \mathrm{d}\hat{\sigma}^{S,2}_{ab,NNLO} - \mathrm{d}\hat{\sigma}^{MF,2}_{ab,NNLO}\,. \end{aligned} \tag{17}$$

Analogously to the NLO case, we first address the IR poles of the double virtual contribution. The singularity structure of renormalized two-loop amplitudes in QCD is known [14] and can be described in colour space by:

$$|\mathcal{A}^2_{n+2}\rangle = \boldsymbol{I}^{(1)}\left(\epsilon,\mu_r^2\right)|\mathcal{A}^1_{n+2}\rangle + \boldsymbol{I}^{(2)}\left(\epsilon,\mu_r^2\right)|\mathcal{A}^0_{n+2}\rangle + |\mathcal{A}^{2,\mathrm{fin}}_{n+2}(\mu_r^2)\rangle\,, \tag{18}$$

where, as before, $|\mathcal{A}^{2,\mathrm{fin}}_{n+2}(\mu_r^2)\rangle$ is a finite remainder. The two-loop Catani IR insertion operator [14] can be written as:

$$\begin{aligned} \boldsymbol{I}^{(2)}\left(\epsilon,\mu_r^2\right) &= -\frac{\beta_0}{\epsilon}\sum_{(i,j)}\mathcal{I}^{(1)}_{ij}\left(\epsilon,\mu_r^2\right)\boldsymbol{T}_i\cdot\boldsymbol{T}_j + \sum_{(i,j)}\mathcal{I}^{(2)}_{ij}\left(\epsilon,\mu_r^2\right)\boldsymbol{T}_i\cdot\boldsymbol{T}_j \\ &\quad -\frac{1}{2}\sum_{(i,j)}\sum_{(k,l)}\mathcal{I}^{(1)}_{ij}\left(\epsilon,\mu_r^2\right)\mathcal{I}^{(1)}_{kl}\left(\epsilon,\mu_r^2\right)(\boldsymbol{T}_i\cdot\boldsymbol{T}_j)(\boldsymbol{T}_k\cdot\boldsymbol{T}_l), \end{aligned} \tag{19}$$

where

$$\mathcal{I}^{(2)}_{ij}\left(\epsilon,\mu_r^2\right) = e^{-\epsilon\gamma_E}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon}+K\right)\mathcal{I}^{(1)}_{ij}\left(2\epsilon,\mu_r^2\right) - \mathcal{H}^{(2)}_{ij}\left(\epsilon\right)\,, \tag{20}$$

with

$$\beta_0 = \frac{11}{6}N_c, \quad K = \left(\frac{67}{18}-\frac{\pi^2}{6}\right)N_c\,. \tag{21}$$

The poles of the double virtual cross section for gluons-only processes are therefore given by:

$$\begin{aligned} &\mathcal{P}oles\left(\hat{\sigma}^{VV}_{gg,NNLO}\right) = \mathcal{N}^{VV}_{NNLO}\int \mathrm{d}\Phi_{n+2}(p_3,\ldots,p_{n+2};p_1,p_2)\,J^{(n+2)}_{n+2}(\{p\}_n) \\ &\times\ \mathcal{P}oles\Big\{\sum_{(i_g,j_g)} 2\mathrm{Re}\left[\mathcal{I}^{(1)}_{i_gj_g}\left(\epsilon,\mu_r^2\right)\right]\left[\langle\mathcal{A}^1_{n+2}|\boldsymbol{T}_{i_g}\cdot\boldsymbol{T}_{j_g}|\mathcal{A}^0_{n+2}\rangle + \langle\mathcal{A}^0_{n+2}|\boldsymbol{T}_{i_g}\cdot\boldsymbol{T}_{j_g}|\mathcal{A}^1_{n+2}\rangle\right] \\ &\quad -\frac{1}{2}\sum_{(i_g,j_g)}\sum_{(k_g,l_g)} 2\mathrm{Re}\left[\mathcal{I}^{(1)}_{i_gj_g}\left(\epsilon,\mu_r^2\right)\right]2\mathrm{Re}\left[\mathcal{I}^{(1)}_{l_gk_g}\left(\epsilon,\mu_r^2\right)\right]\langle\mathcal{A}^0_{n+2}|(\boldsymbol{T}_{i_g}\cdot\boldsymbol{T}_{j_g})(\boldsymbol{T}_{k_g}\cdot\boldsymbol{T}_{l_g})|\mathcal{A}^0_{n+2}\rangle \\ &\quad -\frac{\beta_0}{\epsilon}\sum_{(i,j)} 2\mathrm{Re}\left[\mathcal{I}^{(1)}_{i_gj_g}\left(\epsilon,\mu_r^2\right)\right]\langle\mathcal{A}^0_{n+2}|\boldsymbol{T}_{i_g}\cdot\boldsymbol{T}_{j_g}|\mathcal{A}^0_{n+2}\rangle \\ &\quad +\sum_{(i,j)} 2\mathrm{Re}\left[\mathcal{I}^{(2)}_{i_gj_g}\left(\epsilon,\mu_r^2\right)\right]\langle\mathcal{A}^0_{n+2}|\boldsymbol{T}_{i_g}\cdot\boldsymbol{T}_{j_g}|\mathcal{A}^0_{n+2}\rangle\Big\}. \end{aligned} \tag{22}$$

In analogy with (8), we define a two-loop insertion operator in colour space:

$$\begin{aligned} \boldsymbol{\mathcal{J}}^{(2)}(\epsilon) &= N_c\sum_{(i,j)\geq 3}(\boldsymbol{T}_i\cdot\boldsymbol{T}_j)\,\mathcal{J}^{(2)}_2(i_g,j_g) + N_c\sum_{i\neq 1,2}(\boldsymbol{T}_1\cdot\boldsymbol{T}_i)\,\mathcal{J}^{(2)}_2(\hat{1}_g,i_g) \\ &\quad +N_c\sum_{i\neq 1,2}(\boldsymbol{T}_2\cdot\boldsymbol{T}_i)\,\mathcal{J}^{(2)}_2(\hat{2}_g,i_g) + N_c\,(\boldsymbol{T}_1\cdot\boldsymbol{T}_2)\,\mathcal{J}^{(2)}_2(\hat{1}_g,\hat{2}_g)\,. \end{aligned} \tag{23}$$

The two-loop colour stripped integrated dipoles $\mathcal{J}_2^{(2)}$ have a more involved structure with respect to their one-loop counterparts. The expressions of the gluon-gluon $\mathcal{J}_2^{(2)}$ are given by [3, 15]:

$$\begin{aligned}
\mathcal{J}_2^{(2)}(i_g, j_g) &= \frac{1}{4}\mathcal{F}_4^0 + \frac{1}{3}\mathcal{F}_3^1 + \frac{1}{3}\frac{b_0}{\epsilon}\left(\frac{|s_{ij}|}{\mu_r^2}\right)^{-\epsilon}\mathcal{F}_3^0 - \frac{1}{9}\left[\mathcal{F}_3^0 \otimes \mathcal{F}_3^0\right], \\
\mathcal{J}_2^{(2)}(\hat{1}_g, j_g) &= \frac{1}{2}\mathcal{F}_{4,g}^0 + \frac{1}{2}\mathcal{F}_{3,g}^1 + \frac{1}{2}\frac{b_0}{\epsilon}\left(\frac{|s_{1j}|}{\mu_r^2}\right)^{-\epsilon}\mathcal{F}_{3,g}^0 - \frac{1}{4}\left[\mathcal{F}_{3,g}^0 \otimes \mathcal{F}_{3,g}^0\right] - \frac{1}{2}\overline{\Gamma}_{gg}^{(2)}(x_1)\,\delta_2, \\
\mathcal{J}_2^{(2)}(\hat{1}_g, \hat{2}_g) &= \mathcal{F}_{4,gg}^{0,adj} + \frac{1}{2}\mathcal{F}_{4,gg}^{0,n.adj} + \mathcal{F}_{3,gg}^1 + \frac{b_0}{\epsilon}\left(\frac{|s_{12}|}{\mu_r^2}\right)^{-\epsilon}\mathcal{F}_{3,gg}^0 - \left[\mathcal{F}_{3,gg}^0 \otimes \mathcal{F}_{3,gg}^0\right] \\
&\quad - \frac{1}{2}\overline{\Gamma}_{gg}^{(2)}(x_1)\,\delta_2 - \frac{1}{2}\overline{\Gamma}_{gg}^{(2)}(x_2)\,\delta_1, \qquad (24)
\end{aligned}$$

where we omitted the dependence on the scale s_{ij} in the integrated antennae. At NNLO, as expected, we see the appearance of integrated four-parton antennae ($\mathcal{F}_4^0$, $\mathcal{F}_{4,g}^0$, $\mathcal{F}_{4,gg}^{0,adj}$, $\mathcal{F}_{4,gg}^{0,n.adj}$), integrated three-parton one-loop antennae ($\mathcal{F}_3^1$, $\mathcal{F}_{3,g}^1$, $\mathcal{F}_{3,gg}^1$) and a convolution of two three-parton integrated antennae, as well as two-loop mass factorization kernels for initial-final and initial-initial configurations. In analogy with (10), we can relate the pole structure of (24) to the insertion operators in (19):

$$\mathcal{P}oles\left[N_c\,\mathcal{J}_2^{(2)}(i_g, j_g) - \frac{\beta_0}{\epsilon}\mathcal{J}_2^{(1)}(i_g, j_g)\right] = \mathcal{P}oles\left[\mathrm{Re}\left(\mathcal{I}_{gg}^{(2)}\left(\epsilon, \mu_r^2\right) - \frac{\beta_0}{\epsilon}\mathcal{I}_{gg}^{(1)}\left(\epsilon, \mu_r^2\right)\right)\right]. \qquad (25)$$

Thus, using (8) and (23) we can construct a general expression for the double virtual subtraction term in colour space:

$$\begin{aligned}
\mathrm{d}\hat{\sigma}_{gg,NNLO}^{U} &= \mathcal{N}_{NNLO}^{VV}\int\frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\mathrm{d}\Phi_n(p_3,\ldots,p_{n+2}; x_1p_1, x_2p_2)J_n^{(n)}(\{p\}_n) \\
&\times 2\Big\{\langle\mathcal{A}_{n+2}^0|\boldsymbol{\mathcal{J}}^{(1)}(\epsilon)|\mathcal{A}_{n+2}^1\rangle + \langle\mathcal{A}_{n+2}^1|\boldsymbol{\mathcal{J}}^{(1)}(\epsilon)|\mathcal{A}_{n+2}^0\rangle - \frac{\beta_0}{\epsilon}\langle\mathcal{A}_{n+2}^0|\boldsymbol{\mathcal{J}}^{(1)}(\epsilon)|\mathcal{A}_{n+2}^0\rangle \\
&- \langle\mathcal{A}_{n+2}^0|\boldsymbol{\mathcal{J}}^{(1)}(\epsilon)\otimes\boldsymbol{\mathcal{J}}^{(1)}(\epsilon)|\mathcal{A}_{n+2}^0\rangle + \langle\mathcal{A}_{n+2}^0|\boldsymbol{\mathcal{J}}^{(2)}(\epsilon)|\mathcal{A}_{n+2}^0\rangle\Big\}. \qquad (26)
\end{aligned}$$

The real virtual and the double real subtraction terms are systematically inferred from (26). A detailed description of how this is achieved can be found in [1] and it is too lengthy to be repeated here. A sketch of the procedure can be described in terms of the following steps:

- insertion of an unresolved parton to generate part of the real virtual subtraction term;
- addition of suitable contributions at the real virtual level to ensure the cancellation of spurious ϵ-poles and to avoid the oversubtraction of IR divergences in single unresolved limits;
- insertion of a second single unresolved parton at the real virtual level in combination with the simultaneous insertion of two unresolved partons at the double virtual level to obtain the double real subtraction term.

The simultaneous insertion of a pair of unresolved gluons mentioned in the last step is required since the integrated version of a four-particle antenna is obtained after analytic integration over the double unresolved antenna phase space and so it can not be equated to the iterated insertion

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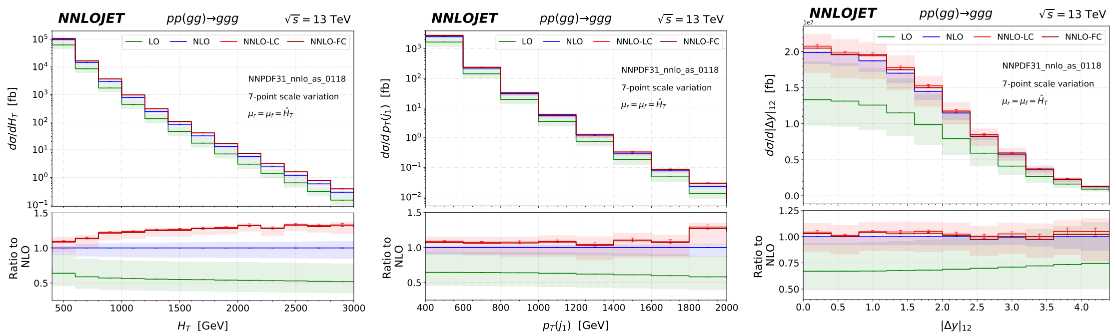

Figure 1: Differential distributions in $H_T = \sum_{j\in jets} p_{T,j}$ (left), transverse momentum of the leading jet (centre) and rapidity difference between the two leading jets (right). NNLO-LC and NNLO-FC respectively indicate the NNLO correction in the leading colour approximation and in full colour.

of a single gluon. This is the only genuinely new operation required at NNLO at is denoted by the operator $\mathcal{I}ns_2\,[\cdot]$. Nevertheless from a practical standpoint this operation is very similar to the application of $\mathcal{I}ns\,[\cdot]$ with two unresolved partons.

4. Results and conclusions

We implemented the colourful antenna subtraction method to construct the subtraction infrastructure required for the calculation of the NNLO correction to the gluons-only process $gg \to ggg$. This computation is part of the NNLO correction to 3-jet production, recently presented in [17], and demonstrates the applicability of the colourful antenna approach to the construction of NNLO subtraction terms for a highly non-trivial high-multiplicity process. A selection of results, obtained in the NNLOJET framework, is presented in Figure 1 to illustrate the quality of numerical convergence that can be obtained with the generated subtraction terms.

The natural next step for the development of the described approach is the extension to subprocesses involving quarks. Consistent work has already been performed in this direction and the goal remains the definition of a complete, process-independent and systematic procedure for the generation of the subtraction terms in the context of antenna subtraction.

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