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UV and IR rational terms in two-loop amplitudes: first insights

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In this proceeding, we review the construction of UV rational terms at two loops, and present the first insights of the cancellation mechanism of two-loop IR rational terms with an example in massive QED. The rational terms are crucial ingredients that are needed for the four-dimensional calculation of scattering amplitudes, which is suitable for automation in a numerical framework. Within automated one-loop tools, the numerator of D-dimensionally regularised loop amplitudes are usually constructed through numerical algorithms in $D = 4$ dimensions, while the contributions of the missing $(D-4)$-dimensional parts can be reconstructed a posteriori by means of rational counterterms. At two loops with four-dimensional numerators, we show that the missing $(D-4)$ part can also be reconstructed through the process-independent two-loop rational terms in any renormalisable theories. This opens the door to the development of an efficient two-loop amplitudes automation in the future.

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1. Introduction

Scattering amplitudes are the fundamental ingredients of theoretical predictions for experiments at high-energy colliders. Their accurate determination is of prime importance for high-precision tests of the Standard Model (SM) of particle physics at the Large Hadron Collider (LHC) and at future collider experiments. In the recent past, the field of particle physics has witnessed a revolution in the automation of numerical one-loop amplitudes calculations [1–6], which has opened the door to accurate predictions for a vast spectrum of non-trivial scattering processes at the LHC. Together with recent advances in the computation of multi-loop amplitudes with numerical, semi-analytical and four-dimensional techniques [7–16], the possibility of a two-loop automation to a degree similar to that for the one-loop case is getting closer [17–27]. The development of such tools is highly desirable, since most processes at the LHC require at least two-loop corrections to keep up with experimental accuracy, and its application is not limited to the LHC but also for future colliders.

Loop amplitudes exhibit divergences of ultraviolet (UV) and infrared (IR) nature that are usually regularised in dimensional regularisation, i.e. via analytic continuation in the number D of spacetime dimensions. In this approach, the number of space-time dimensions is handled as a continuous complex-valued parameter D, and the divergences of loop integrals manifest themselves as $1/(D-4)$ poles. While all divergences cancel in physical observables, their interplay with objects of order $(D-4)$ can lead to non-vanishing rational terms. Within numerical frameworks at one loop, such rational terms are most conveniently computed through dedicated methods [28–32], in such a way that the most involved parts of the amplitudes can be constructed through numerical algorithms in four dimensions. The most widely used method of this kind has been presented in [32] and is based on the idea of splitting the numerators of one-loop integrands into four-dimensional parts and $(D-4)$-dimensional remnants. In this way, the former can be built by means of automated numerical algorithms, while the $(D-4)$-dimensional parts of the numerators contribute only in combination with UV poles and can be reconstructed a posteriori by means of process-independent UV rational counterterms, which are explicitly available both for the full SM [33–35] and for beyond SM theories [36, 37]. On the other hand, IR divergences do not give rise to any rational terms at one loop [38], except for trivial process-independent contributions associated with the on-shell renormalisation of external fields. This method is a key ingredient of the efficient and flexible one-loop tools on the market, and its extension to two loops is an important step towards the automation of two-loop calculations.

With this motivation in mind, we develop a theoretical framework for two-loop UV rational terms [39–43], and we explore the cancellation mechanisms of IR rational terms in two-loop IR subtracted amplitudes in a forthcoming paper. This new framework provides the theoretical foundation for calculations of two-loop amplitudes based on numerical algorithms in four dimensions. In the first part of this proceeding, we review the general properties of UV rational terms at one loop and the extensions to two loops. In the second part, as first insights, we demonstrate the cancellation of IR rational terms with an examples in the two-loop massive QED amplitudes.

2. UV rational terms at one and two loops

In this section, we show the reconstruction of the D-dimensional renormalised amplitudes through the UV rational terms with four-dimensional loop numerators. In the following, we refer this procedure as the so-called rational-term-reconstruction.

2.1 One-loop rational-term-reconstruction

For the regularisation of UV divergences, following the 't Hooft–Veltman scheme [44] we keep external states in four dimensions, while loop momenta, metric tensors and Dirac matrices inside the loops are extended to $D = 4 - 2\varepsilon$ dimensions. For the decomposition of these objects into four-dimensional parts and $(D-4)$-dimensional remnants we use the notation

$$\bar{q}^{\mu} = q^{\mu} + \tilde{q}^{\tilde{\mu}}\,, \qquad \bar{\gamma}^{\mu} = \gamma^{\mu} + \tilde{\gamma}^{\tilde{\mu}}\,, \qquad \bar{g}^{\bar{\mu}\bar{\nu}} = g^{\mu\nu} + \tilde{g}^{\tilde{\mu}\tilde{\nu}}\,, \tag{1}$$

where the bar and the tilde are used to mark, respectively, the D-dimensional and $(D-4)$-dimensional parts, while objects without a bar or tilde are four-dimensional.

The amplitude of a one-loop diagram Γ in D dimensions has the form

$$\bar{\mathcal{A}}_{1,\Gamma} = \int \mathrm{d}\bar{q}_1 \, \frac{\bar{\mathcal{N}}(\bar{q}_1)}{D_0(\bar{q}_1)\cdots D_{N-1}(\bar{q}_1)}\,, \tag{2}$$

with the integration measure

$$\mathrm{d}\bar{q} = \mu_0^{2\varepsilon} \frac{\mathrm{d}^D\bar{q}}{(2\pi)^D}\,, \tag{3}$$

where μ_0 is the scale of dimensional regularisation. The denominators in (2) read

$$D_j(\bar{q}_1) = (\bar{q}_1 + p_j)^2 - m_j^2\,, \tag{4}$$

and p_j are combinations of four-dimensional external momenta. The corresponding renormalised amplitude in the **R**-operation notation reads

$$\mathbf{R}\,\bar{\mathcal{A}}_{1,\Gamma} = \bar{\mathcal{A}}_{1,\Gamma} + \delta Z_{1,\Gamma}\,, \tag{5}$$

where $\delta Z_{1,\Gamma}$ denotes the UV counterterm. Now we split the numerator of the amplitude (2) in D-dimensions into

$$\bar{\mathcal{N}}(\bar{q}_1) = \mathcal{N}(q_1) + \tilde{\mathcal{N}}(\bar{q}_1)\,, \tag{6}$$

where $\mathcal{N}(q_1)$ is the four-dimensional part, obtained by projecting the metric tensor, Dirac matrices and the loop momentum to four dimensions. The remnant $\tilde{\mathcal{N}}(\bar{q}_1)$ is of $O(\varepsilon, \tilde{q}_1)$ and will be referred to as the $(D-4)$-dimensional part of the numerator. To keep track of the dimensionality of loop numerators we use the parameter D_n. The amplitude $\bar{\mathcal{A}}_{1,\Gamma}$, defined in (2), is referred to as amplitude in $D_\mathrm{n} = D$ dimensions, while its counterpart in $D_\mathrm{n} = 4$ dimensions corresponds to

$$\mathcal{A}_{1,\Gamma} = \int \mathrm{d}\bar{q}_1 \, \frac{\mathcal{N}(q_1)}{D_0(\bar{q}_1)\cdots D_{N-1}(\bar{q}_1)}\,. \tag{7}$$

Note that here the numerator is projected to four dimensions, while retaining the full D-dependence of the loop momentum in the denominator. Renormalised one-loop amplitudes in $D_\mathrm{n} = D$ and $D_\mathrm{n} = 4$ dimensions are related to each other by

$$\mathbf{R}\,\bar{\mathcal{A}}_{1,\Gamma} = \mathcal{A}_{1,\Gamma} + \delta Z_{1,\Gamma} + \delta\mathcal{R}_{1,\Gamma}\,, \tag{8}$$

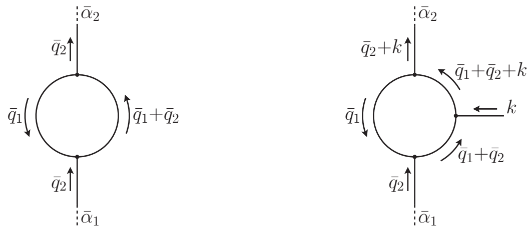


Figure 1: Examples of UV divergent one-loop subtopologies. The loop momentum $\bar{q}_1$ circulates inside the subdiagram, while the two external lines that are going to be embedded in a two-loop diagram depend on the D-dimensional loop momentum $\bar{q}_2$ and carry the Lorentz/Dirac indices $\bar{\alpha}_1, \bar{\alpha}_2$.

where $\delta Z_{1,\Gamma}$ are usual UV counterterms, and $\delta\mathcal{R}_{1,\Gamma}$ are rational counterterms [32]. Since $\delta\mathcal{R}_{1,\Gamma}$ counterterms originate only from the interplay of $\tilde{\mathcal{N}}(\bar{q}_1)$ with poles of UV kind [38], they can be derived once and for all by computing

$$\delta\mathcal{R}_{1,\Gamma} \quad = \quad \bar{\mathcal{A}}_{1,\Gamma} - \mathcal{A}_{1,\Gamma}\,, \tag{9}$$

at the level of UV divergent 1PI vertex functions.

2.2 Extension to one-loop subdiagram

For the rational-term-reconstruction of one-loop subdiagrams inside the two-loop diagrams, an identity of type (8) is also needed. As depicted in Fig. 1, this kind of one-loop (sub)diagrams involve an internal loop momentum $\bar{q}_1$ and an external loop momentum $\bar{q}_2$. Thus the relation (8) needs to be extended to the case of D-dimensional external kinematics.

The extension of the relation (8) can be achieved by observing the following identity between the fully UV- and rational-part subtracted finite remainders

$$\bar{\mathcal{A}}^{\bar{\alpha}}_{1,\gamma}(\bar{q}_2) - \left[-\delta Z^{\alpha}_{1,\gamma}(\bar{q}_2) + \delta\mathcal{R}^{\alpha}_{1,\gamma}(\bar{q}_2)\right] \quad = \quad \mathcal{A}^{\alpha}_{1,\gamma}(q_2) - \left[\delta Z^{\alpha}_{1,\gamma}(q_2) + \delta\tilde{Z}^{\alpha}_{1,\gamma}(\tilde{q}_2) + \mathcal{O}(\varepsilon, \tilde{q}_2)\right]\,, \tag{10}$$

where the lhs is finite remainders in $D_{\rm n} = D$ dimensions, while the rhs corresponds to its counterpart in $D_{\rm n} = 4$ dimensions. This identity contains a new kind of UV counterterm $\delta\tilde{Z}^{\alpha}_{1,\gamma}(\tilde{q}_2)$ that is required for the cancellation of UV divergences of one-loop subdiagram in $D_{\rm n} = 4$ dimensions. This new kind of UV divergences of one-loop subdiagram arises from the D-dimensional external loop momenta entering the loop denominators of $D_{\rm n} = 4$ loop integrals. In renormalisable theories, the $\delta\tilde{Z}^{\alpha}_{1,\gamma}(\tilde{q}_2)$ counterterm is needed only for quadratically divergent selfenergy subdiagrams that contribute in the following form [39]

$$\delta\tilde{Z}^{\alpha}_{1,\gamma}(\tilde{q}_2) \quad = \quad v^{\alpha}\frac{\tilde{q}_2^2}{\varepsilon}\,, \tag{11}$$

where v^{α} is independent of q_2. Hence the formula for the reconstruction of renormalised one-loop subdiagram with D-dimensional external kinematics reads

$$\mathbf{R}\,\bar{\mathcal{A}}^{\bar{\alpha}}_{1,\gamma}(\bar{q}_2) \quad = \quad \mathcal{A}^{\alpha}_{1,\gamma}(q_2) + \delta Z^{\alpha}_{1,\gamma}(q_2) + \delta\tilde{Z}^{\alpha}_{1,\gamma}(\tilde{q}_2) + \delta\mathcal{R}^{\alpha}_{1,\gamma}(q_2) + \mathcal{O}(\varepsilon, \tilde{q}_2)\,. \tag{12}$$

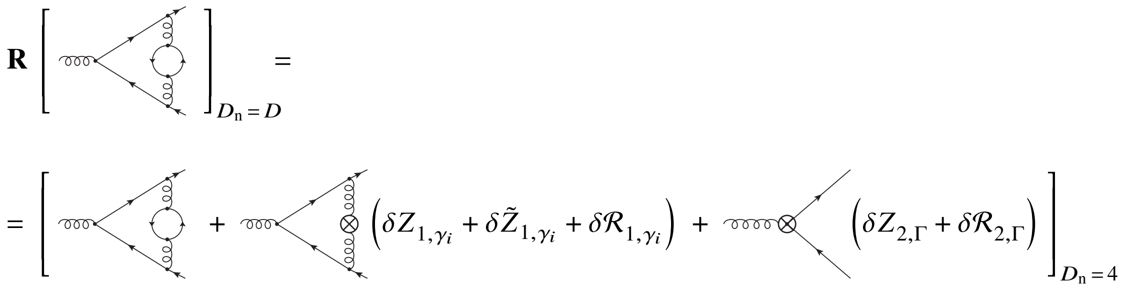


Figure 2: Graphical representation of the master formula (14) for the case of a globally divergent two-loop QCD diagram with a single subdivergence.

2.3 Two-loop rational-term-reconstruction

At two loops, the amplitudes involve subdivergences and additional local two-loop divergences. These two kinds of divergences can be subtracted by means of the **R**-operation. For a single two-loop diagram or a full two-loop vertex function Γ, the renomalised amplitude has the form

$$\mathbf{R}\,\bar{\mathcal{A}}_{2,\Gamma} \;=\; \bar{\mathcal{A}}_{2,\Gamma} + \sum_{\gamma} \delta Z_{1,\gamma_i} \cdot \bar{\mathcal{A}}_{1,\Gamma/\gamma_i} + \delta Z_{2,\Gamma}\,, \tag{13}$$

where $\bar{\mathcal{A}}_{2,\Gamma}$ is the unrenormalised two-loop amplitude in D dimensions. The second term on the rhs subtracts all relevant subdivergences. The corresponding UV divergences are subtracted by the counterterms $\delta Z_{1,\gamma_i}$ and their insertion into the complementary one-loop diagrams Γ/γ_i read $\delta Z_{1,\gamma_i} \cdot \bar{\mathcal{A}}_{1,\Gamma/\gamma_i}$. The counterterm $\delta Z_{2,\Gamma}$ in (13) subtracts the local two-loop divergence that is left after subtraction of the subdivergences.

As demonstrated in [39], the renormalised two-loop amplitude (13) in $D_{\rm n} = D$ dimensions can be reconstructed by amplitudes in $D_{\rm n} = 4$ dimensions plus appropriate rational counterterms. The corresponding master formula reads

$$\mathbf{R}\,\bar{\mathcal{A}}_{2,\Gamma} \;=\; \mathcal{A}_{2,\Gamma} + \sum_{\gamma} \left(\delta Z_{1,\gamma} + \delta\tilde{Z}_{1,\gamma} + \delta\mathcal{R}_{1,\gamma}\right) \cdot \mathcal{A}_{1,\Gamma/\gamma} \;+\; \delta Z_{2,\Gamma} + \delta\mathcal{R}_{2,\Gamma}\,, \tag{14}$$

and is illustrated in Fig. 2. The first term on the rhs is the unrenormalised two-loop amplitude in $D_{\rm n} = 4$ dimensions. The second term contains all required one-loop counterterms—see (12)—for the cancellation of the UV poles of the subdiagrams γ and for the reconstruction of the associated rational parts. As for the remaining two-loop counterterms, $\delta Z_{2,\Gamma}$ is the same UV counterterm as in (13), while the rational counterterm $\delta\mathcal{R}_{2,\Gamma}$ reconstructs all remaining contributions of order ε^{-1} and ε^{0} that originate form the interplay of the $\tilde{\mathcal{N}}$-part of the numerator with local UV divergences.

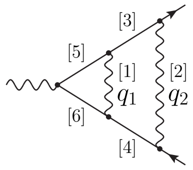
As proved in [39], the $\delta\mathcal{R}_{2,\Gamma}$ terms are process-independent local counterterms, and their renormalisation scheme dependence are discussed in [40] in details. The analytical results of the complete set of two-loop rational counterterms of $\mathcal{O}(\alpha_{\rm s}^2)$ in a generic renormalisation scheme for the full Standard Model are presented in [40, 41].

3. First insights of IR rational terms in massive QED at two loops

At one loop, it is shown in [38] that the IR rational terms vanish for the individual Feynman diagram. At two loops, the treatment of IR rational terms is under investigation. In this section, we show the first insights into the cancellation mechanism of IR rational terms in two-loop massive QED amplitudes with an examples.

3.1 Example: cancellation of IR rational terms

Let us consider the example with soft divergences in the following UV renormalised diagram Γ in $D_\mathrm{n} = D$ dimensions



$$= \int \mathrm{d}\bar{q}_1 \mathrm{d}\bar{q}_2 \, \frac{\bar{\mathcal{N}}_{234}(\bar{q}_2)\; \bar{\mathcal{N}}_{156}(\bar{q}_1,\bar{q}_2)}{\mathcal{D}_{234}(\bar{q}_2)\; \mathcal{D}_{156}(\bar{q}_1,\bar{q}_2)} \;:=\; \bar{\mathcal{A}}_{2,\Gamma}\,, \tag{15}$$

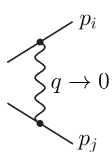
where the indices of numerators and denominators match the ones in the diagram, i.e. $\bar{\mathcal{N}}_{ijk} = \bar{\mathcal{N}}_i \bar{\mathcal{N}}_j \bar{\mathcal{N}}_k$ and $\mathcal{D}_{ijk} = D_i D_j D_k$. The IR divergences are generated by soft photons exchanges in the limit $q_2 \to 0$, and also in the limit $q_1, q_2 \to 0$. These IR divergent configurations can be represented by the following nesting list

$$\mathcal{N}[\Gamma] \;=\; \{\{S_1 S_2\}, \{S_1 S_2, S_2\}, \{S_2\}\}\,, \tag{16}$$

where $\{S_1 S_2\}$ denotes the leading double-soft configuration with $q_1, q_2 \to 0$, which also enters the nested-soft configuration $\{S_1 S_2, S_2\}$. The single soft configuration with only $q_2 \to 0$ is denoted as $\{S_2\}$. Now we can isolate the IR divergences through the integrand-level approximation operator $t_{\sigma\in\mathcal{N}[\Gamma]}$ [45, 46] such that

$$\begin{aligned}\bar{\mathcal{A}}_{2,\Gamma} &= \underbrace{t_{S_1S_2}\,\bar{\mathcal{A}}_{2,\Gamma}}_{\text{double-soft div.}} + \underbrace{\left(t_{S_2}\,\bar{\mathcal{A}}_{2,\Gamma} - t_{S_2} t_{S_1S_2} \bar{\mathcal{A}}_{2,\Gamma}\right)}_{\text{remnant single-soft div.}} + \bar{\mathcal{A}}^{\mathrm{fin}}_{2,\Gamma} \\ &= \underbrace{\int_{\mathrm{d}\bar{q}_2} \frac{\bar{\mathcal{N}}_{234}(0)}{\mathcal{D}_{234}(\bar{q}_2)} \int_{\mathrm{d}\bar{q}_1} \frac{\bar{\mathcal{N}}_{156}(\bar{q}_1,0)}{\mathcal{D}_{156}(\bar{q}_1,0)}}_{=\left(\mathbf{I}^{(1)}_{\mathrm{IR}} + \mathbf{I}^{(1)}_{\mathrm{fin}}\right)\bar{\mathcal{A}}_{1,\Gamma}} + \underbrace{\int_{\mathrm{d}\bar{q}_1\mathrm{d}\bar{q}_2} \frac{\bar{\mathcal{N}}_{234}(0)}{\mathcal{D}_{234}(\bar{q}_2)} \left(\frac{\bar{\mathcal{N}}_{156}(0,0)}{\mathcal{D}_{156}(\bar{q}_1,\bar{q}_2)} - \frac{\bar{\mathcal{N}}_{156}(0,0)}{\mathcal{D}_{156}(\bar{q}_1,0)} \right)}_{=\left(\mathbf{I}^{(2)}_{\mathrm{IR}} - \mathbf{I}^{(1)}_{\mathrm{IR}}\mathbf{I}^{(1)}_{\mathrm{fin}} + \mathbf{I}^{(2)}_{\mathrm{fin}}\right)\mathcal{A}_{0,\Gamma}} + \bar{\mathcal{A}}^{\mathrm{fin}}_{2,\Gamma}\,,\end{aligned} \tag{17}$$

where $\mathbf{I}^{(l)}_{\mathrm{IR}}$ denotes the divergent pole part of the l-loop Catani-Seymour IR operator, while $\mathbf{I}^{(l)}_{\mathrm{fin}}$ is the finite remnant of the IR approximation, and $\bar{\mathcal{A}}^{\mathrm{fin}}_{1,\Gamma}$ denotes the remaning finite piece of the amplitude. By using the Dirac algebra, one can show that the Eikonal approximations of soft photon exchanges yield 4-dim scalar product of external momenta



$$\propto p_i \cdot p_j\,, \tag{18}$$

which implies that we can safely project $\bar{\mathcal{N}}_{234}(0)$ and $\bar{\mathcal{N}}_{156}(0,0)$ into $\mathcal{N}_{234}(0)$ and $\mathcal{N}_{156}(0,0)$ in $D_\mathrm{n} = 4$ dimensions. Now we obtained two key identities in both $D_\mathrm{n} = D$ and $D_\mathrm{n} = 4$ dimensions

$$\bar{\mathcal{A}}_{2,\Gamma} = \left(\mathbf{I}^{(1)}_{\mathrm{IR}} + \mathbf{I}^{(1)}_{\mathrm{fin}}\right) \bar{\mathcal{A}}_{1,\Gamma} + \left(\mathbf{I}^{(2)}_{\mathrm{IR}} - \mathbf{I}^{(1)}_{\mathrm{IR}}\mathbf{I}^{(1)}_{\mathrm{fin}} + \mathbf{I}^{(2)}_{\mathrm{fin}}\right) \mathcal{A}_{0,\Gamma} + \bar{\mathcal{A}}^{\mathrm{fin}}_{2,\Gamma}\Big|_{D_\mathrm{n}=D}, \tag{19}$$

$$\mathcal{A}_{2,\Gamma} = \left(\mathbf{I}^{(1)}_{\mathrm{IR}} + \mathbf{I}^{(1)}_{\mathrm{fin}}\right) \mathcal{A}_{1,\Gamma} + \left(\mathbf{I}^{(2)}_{\mathrm{IR}} - \mathbf{I}^{(1)}_{\mathrm{IR}}\mathbf{I}^{(1)}_{\mathrm{fin}} + \mathbf{I}^{(2)}_{\mathrm{fin}}\right) \mathcal{A}_{0,\Gamma} + \mathcal{A}^{\mathrm{fin}}_{2,\Gamma}\Big|_{D_\mathrm{n}=4}. \tag{20}$$

Notice that at this stage, (19) contains the IR rational terms due to the presence of $\mathbf{I}^{(1)}_{\mathrm{IR}}\,\bar{\mathcal{A}}_{1,\Gamma}$ contribution. However, this type of IR rational terms cancel in the IR-subtracted amplitude. This cancellation can be seen by taking the difference between the IR-subtracted amplitudes in $D_\mathrm{n} = D$ and $D_\mathrm{n} = 4$, which yields

$$\begin{aligned}&\left[\bar{\mathcal{A}}_{2,\Gamma} - \mathbf{I}^{(1)}_{\mathrm{IR}}\,\bar{\mathcal{A}}_{1,\Gamma} - \mathbf{I}^{(2)}_{\mathrm{IR}}\,\bar{\mathcal{A}}_{0,\Gamma}\right]_{D_\mathrm{n}=D} - \left[\mathcal{A}_{2,\Gamma} - \mathbf{I}^{(1)}_{\mathrm{IR}}\,\mathcal{A}_{1,\Gamma} - \mathbf{I}^{(2)}_{\mathrm{IR}}\,\bar{\mathcal{A}}_{0,\Gamma}\right]_{D_\mathrm{n}=4}\\ &= \mathbf{I}^{(1)}_{\mathrm{fin}}\underbrace{\left(\bar{\mathcal{A}}_{1,\Gamma} - \mathcal{A}_{1,\Gamma}\right)}_{O(\varepsilon)} + \underbrace{\left(\bar{\mathcal{A}}^{\mathrm{fin}}_{2,\Gamma} - \mathcal{A}^{\mathrm{fin}}_{2,\Gamma}\right)}_{O(\varepsilon)} = O(\varepsilon)\,,\end{aligned} \tag{21}$$

where in the last line, we used one-loop decomposition $\bar{\mathcal{A}}_{1,\Gamma} = \left(\mathbf{I}^{(1)}_{\mathrm{IR}} + \mathbf{I}^{(1)}_{\mathrm{fin}}\right)\mathcal{A}_{0,\Gamma} + \bar{\mathcal{A}}^{\mathrm{fin}}_{1,\Gamma}$. This shows that the IR-subtracted amplitude is free from IR rational terms, and we have

$$\mathbf{R}_{\mathrm{IR}}\,\bar{\mathcal{A}}_{2,\Gamma} = \mathcal{A}_{2,\Gamma} - \mathbf{I}^{(1)}_{\mathrm{IR}}\,\mathcal{A}_{1,\Gamma} - \mathbf{I}^{(2)}_{\mathrm{IR}}\,\mathcal{A}_{0,\Gamma}\,. \tag{22}$$

The comprehensive discussion of two-loop IR rational terms will be presented in a forthcoming paper.

4. Conclusion

In this report, we briefly reviewed the constructions and properties of two-loop UV rational terms, and we further presented the first insights of the cancellation mechanisum of two-loop IR rational terms in massive QED. The completion of two-loop rational terms studies will open the door to the automated and efficient two-loop amplitudes calculations in a numerical framework.

Acknowledgments

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