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Vector and axial-vector coefficient functions for DVCS at NNLO

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We calculate the two-loop coefficient functions for the (axial-) vector flavor-nonsinglet contributions in processes with one real and one virtual photon. We present the analytic expressions for the coefficient functions in momentum fraction space and estimate their effect on the physical observable. The calculated NNLO corrections appear to be rather large and have to be taken into account.

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1. Introduction

A new generation of experimental facilities provide a wealth of data on hard exclusive reactions in the coming decade. It is expected that these data will have a very high precision and provide a much deeper insight in the hadron structure. Therefore, a pressing question is whether hard exclusive hadronic reactions are under sufficient theoretical control for fully quantitative predictions. Deeply-virtual Compton scattering (DVCS) [1–3] is believed to be the "gold-plated" process that will lead to an understanding of the three-dimensional structure of the proton. The theoretical framework for the QCD description of DVCS is based on collinear factorization in terms of generalized parton distributions (GPDs) [4–6] and is well understood at the leading-twist level. The next-to-next-to leading order (NNLO) analysis of parton distributions has become the standard in this field [7], so that the NNLO precision for DVCS is necessary as well. The NNLO accuracy implies that one needs to know three-loop evolution equations for GPDs and two-loop coefficient functions (CFs) in the operator product expansion (OPE) of the DVCS amplitude.

This program was recently completed for the flavor-nonsinglet sector. The three-loop evolution equations were obtained in [8, 9] and two loop CFs were derived recently in [10–12]. Below we discuss briefly the method used in [10, 11] and present two-loop expressions for the nonsinglet CFs.

2. Preliminaries

The amplitude of the DVCS process is given by a matrix element of the time-ordered product of two electromagnetic currents

$$\mathcal{A}_{\mu\nu}(q,q',p) = i\int d^4x\, e^{-iqx}\langle p'|T\{j^{\rm em}_\mu(x) j^{\rm em}_\nu(0)\}|p\rangle\,. \tag{1}$$

Here q, q' are the momenta of the virtual (incoming) and real (outgoing) photons and p, p' are the nucleon momenta in initial and final states. The longitudinal plane is defined by two light-like vectors [13], $n = q'$, $\quad \bar{n} = -q + (1-\tau)q'$, where $\tau = t/(Q^2+t)$, $Q^2 = -q^2$. In the leading-twist approximation the DVCS amplitude is written as a sum of vector and axial-vector contributions

$$\mathcal{A}_{\mu\nu} = -g^{\perp}_{\mu\nu}A_+ + \epsilon^{\perp}_{\mu\nu}A_- + \ldots\,, \tag{2}$$

where the (axial-)vector amplitudes $A_\pm$ can be written in the form

$$A_\pm(\xi, Q^2) = \frac{1}{2}\sum_q e_q^2 \int_{-1}^{1}\frac{dx}{\xi}\mathrm{C}_\pm\Big(x/\xi, Q, \mu\Big)F_q^\pm(x,\xi,t,\mu). \tag{3}$$

Here $F_q^\pm(x,\xi,t,\mu)$ are the corresponding (axial)vector GPD and ξ is the skewedness parameter. The coefficient functions $\mathrm{C}_\pm(x/\xi)$ are analytic functions: they are real in the region $|x/\xi| < 1$ and beyond this region obtained via the $\xi \to \xi - i\epsilon$ prescription.

At the leading and next-to-leading orders the coefficient functions,

$$C(x/\xi) = C^0(x/\xi) + a_s C^{(1)}(x/\xi) + \ldots, \qquad\qquad a_s = \alpha_s/4\pi, \tag{4}$$

take the form [14, 15]

$$C_{\pm}^{(0)}(x/\xi) = \frac{1}{z} \mp \frac{1}{1-z},$$
$$C_{\pm}^{(1)}(x/\xi) = \frac{C_F}{z}\left(\ln^2 z - (2\pm 1)\,\frac{z}{\bar z}\ln z - 9\right) \mp (z \to 1-z)\,, \tag{5}$$

where $z = \frac{1}{2}(1 - x/\xi)$. It was shown in ref. [16] that the coefficient functions in DVCS and DIS processes are (in a conformal field theory (CFT)) related each to other. Since the DIS CFs are known at NNLO [17, 18] one can restore the DVCS CFs without a calculation of Feynman diagrams, as it was done in ref. [15].

3. Conformal OPE

The OPE for the product of two currents has, schematically, the form

$$T\{j_\mu^{\rm em}(x) j_\nu^{\rm em}(0)\} = \sum_{N,k} C_{N,k}\partial_+^k O_N(0). \tag{6}$$

Here $O_N(0)$ are local operators of increasing dimension and C_{Nk} are the corresponding CFs. Contributions of the operators with total derivatives vanish identically in the forward limit, like in DIS, and can be omitted. Thus only the sum over N remains and the necessary CFs, $C_N \equiv C_{N,k=0}$, are known to three-loop accuracy [17, 18]. In off-forward reactions, operators with total derivatives have to be taken into account and one needs to know their coefficients, $C_{N,k}$, as well. In conformal field theories all C_{Nk} with $k > 0$ are completely determined by the CF of non-derivative operator, C_{N0}, [19].

QCD is not a conformal theory in $d = 4$, it becomes a CFT in non-integer $d = 4-2\epsilon$ dimensions at Wilson-Fisher fixed point α_s^*, such that the $\beta(\alpha_s^*) = 0$. One can consider the DVCS process in QCD in non-integer dimensions. The formula (1) remains true with the only modification: the perturbative expansion of CFs involves ϵ-dependent coefficients:

$$C(a_s,\epsilon) = C_0 + a_s\, C^{(1)}(\epsilon) + a_s^2\, C^{(2)}(\epsilon) + \ldots\ldots\,, \qquad C^{(k)}(\epsilon) = C^{(k)} + \epsilon\, C^{(k,1)} + \ldots\,, \tag{7}$$

where the tree-level CFs, $C^{(0)}$, do not depend on ϵ. The “form" of the CFs at the critical point, $C(a_s^*,\epsilon)$, is restricted by the conformal symmetry. he The difference between the physical $d = 4$ and the critical CFs comes, at two loop, from one loop diagrams which have to be calculated with $O(\epsilon)$ terms. Namely

$$C(a_s, 0) = C(a_s, \epsilon_*) + a_s^2\, \beta_0\, C^{(1,1)}. \tag{8}$$

The expansion (6) can be written in a more practical form as follows (for brevity we consider the vector case only)

$$\begin{aligned} \mathrm{T}\left\{j^\mu(x_1) j^\nu(x_2)\right\} = \sum_{N,\text{even}} \frac{\mu^{\gamma_N}}{(-x_{12}^2)^{t_N}} \int_0^1 du \Big\{ & -\frac{1}{2} A_N(u)\eta^{\mu\nu}(x) + B_N(u) g^{\mu\nu} \\ & + C_N(u) x_{12}^\nu \partial_1^\mu - C_N(\bar u) x_{12}^\mu \partial_2^\nu + D_N(u) x_{12}^2 \partial_1^\mu \partial_2^\nu \Big\}\, O_N^{x_{12}\ldots x_{12}}(x_{21}^u)\,. \end{aligned} \tag{9}$$

Here $x_{12} = x_1 - x_2$, $\bar{u} = 1 - u$, $x_{21}^u = \bar{u}x_2 + ux_1$, $\partial_k^\mu = \frac{\partial}{\partial x_k^\mu}$, $\eta^{\mu\nu}(x) = g^{\mu\nu} - \frac{2x_{12}^\mu x_{12}^\nu}{x_{12}^2}$ and

$$O_N^{x\ldots x}(y) = x_{\mu_1}\ldots x_{\mu_N} O_N^{\mu_1\ldots\mu_N}(y)\,, \tag{10}$$

where $O_N^{\mu_1\ldots\mu_N}(y)$ are the leading-twist conformal operators with anomalous dimension $\gamma_N(\alpha_s)$ and $t_N = 2 - \epsilon_* - \frac{1}{2}\gamma_N(a_s)$. Conformal invariance and current conservation $\partial^\mu j_\mu = 0$ lead to constraints on the functional form of the invariant functions $A_N(u), \ldots, D_N(u)$. One obtains

$$A_N(u) = a_N\, u^{j_N-1}\bar{u}^{j_N-1}\,, \qquad\qquad B_N(u) = b_N\, u^{j_N-1}\bar{u}^{j_N-1}\,, \tag{11}$$

with $j_N = N + 1 - \epsilon_* + \frac{1}{2}\gamma_N(a_s)$ is the so-called conformal spin. The expression for the functions $C_N(u)$ and $D_N(u)$ can be found in ref. [10].

We assume the conventional parametrization for the nucleon matrix element of the conformal operator

$$\langle p'|n^{\mu_1}\ldots n^{\mu_N} O_{\mu_1\ldots\mu_N}|p\rangle = \sum_k \left(-\frac{1}{2}\right)^k f_N^{(k)} P_+^{N-k}\Delta_+^k = P_+^N f_N(\xi)\,, \quad f_N(\xi) \equiv \sum_k f_N^{(k)}\xi^k. \tag{12}$$

Note that $f_N^{(0)} = f_N^{\rm DIS}$. Sandwiching (9) between the nucleon states (in DIS and DVCS kinematics) one derives after Fourier transform a representation for the DVCS amplitude A_+ involving the DIS coefficient functions $C_1(N)$

$$A_+(\xi, Q^2) = \sum_{N,\text{even}} f_N(\xi)\left(\frac{1}{2\xi}\right)^N C_1\left(N, \frac{Q^2}{\mu^2}, a_s, \epsilon_*\right)\frac{\Gamma(\frac{d}{2}-1)\Gamma(2j_N)}{\Gamma(j_N)\Gamma(j_N+\frac{1}{2}-1)}. \tag{13}$$

At the same time the amplitude A_+ is given by the convolution (3) of the GPD with the coefficient function we are interested in. The GPD can be expanded over a set of functions P_N whose form is defined by the operators (10). Since the CF depends on the ratio x/ξ it is convenient to put $\xi = 1$

$$F^+(\xi = 1, Q^2) = \sum_{N,\text{even}} r_N P_N(x)\,. \tag{14}$$

The functions P_N and the coefficients r_N depend on a renormalization scheme. In a special scheme, the so-called conformal scheme [16] [1] the functions P_N and the coefficients r_N are known:

$$P_N(x) = (1-x^2)^{\lambda_N - 1/2} C_{N-1}^{(\lambda_N)}(x), \qquad\qquad \lambda_N = \frac{3}{2} + \beta(a_s) + \frac{1}{2}\gamma_N(a_s). \tag{15}$$

The explicit expressions for the coefficients r_N can be found in [10]. The moments of the CF in the conformal scheme (CS) with respect to the functions P_N take the form :

$$\int_{-1}^1 dx\, \mathbf{C}(x, Q^2, a_s) P_{N-1}^{(\lambda_N)}(x) = C_1\left(N, \frac{Q^2}{\mu^2}, a_s, \epsilon_*\right)\frac{2\Gamma(\frac{d}{2}-1)\Gamma(\lambda_N+\frac{1}{2})\Gamma(N-1+2\lambda_N)}{\sigma_N\,\Gamma(2\lambda_N)\Gamma(j_N+\frac{1}{2}-1)}\,, \tag{16}$$

[1] The conformal scheme is fixed by a requirement for the generator of special conformal transformations, S_+, to have a canonical form (for more details see [8]). The operator U which transforms the $\overline{\rm MS}$ scheme to the CS scheme is known now with two-loop accuracy [20].

where σ_N are the eigenvalues of the operator U, see ref. [10] for more details. This equation unambiguously determines the coefficient function $\mathbf{C}(x)$. The effective method to restore this function is the following: let us look for $\mathbf{C}(x)$ in the form

$$\mathbf{C}(x) = \int_{-1}^{1} dx'\, C^{(0)}(x')K(x',x)\,, \tag{17}$$

where $C^{(0)}$ is the LO coefficient function, Eq. (5), and $K(x',x)$ is the kernel of the SL$(2,\mathbb{R})$ invariant operator. It implies that the functions $P_N(x)$ are the eigenfunctions of the operator K, $\int dx' K(x,x')P_N(x') = K(N)P_N(x)$. The eigenvalues $K(N)$ are fixed by Eq. (16). Any SL$(2,R)$ invariant operator is uniquely determined by its spectrum and can be effectively restored. Evaluating the integral (17) one obtains the CF in the CS scheme. Finally, one transforms the coefficient function to the $\overline{\text{MS}}$ scheme:

$$C(x) = \int_{-1}^{1} dx' \mathbf{C}(x')\mathrm{U}(x',x)\,. \tag{18}$$

The final answer for the two-loop (axial-)vector nonsinglet CFs takes the form [10, 11]:

$$C_\pm^{(2)}(x/\xi) = \beta_0 C_F\, C_\pm^{(\beta)}(x/\xi) + C_F^2 C_\pm^{(P)}(x/\xi) + \frac{C_F}{N_c} C_\pm^{(NP)}(x/\xi)\,, \tag{19}$$

where

$$\begin{aligned}
C_\pm^{(\beta)}(x/\xi) &= \Big\{\frac{2}{z}\Big(\mathrm{H}_{100} - \frac{1}{2}\mathrm{H}_{110} - \mathrm{H}_{000}\Big) + \Big(\frac{10}{3z} \pm \frac{1}{\bar z}\Big)\mathrm{H}_{00} - \Big(\frac{1\pm 1}{\bar z} + \frac{14}{3z}\Big)\mathrm{H}_{10} - \frac{\zeta_2}{z}\mathrm{H}_1 \\
&\quad - \Big(\frac{19}{9z} + \frac{10\pm 13}{6\bar z}\Big)\mathrm{H}_0 - \frac{1}{z}\Big(\frac{457}{24} + \frac{11\mp 3}{3}\zeta_2 + \zeta_3\Big)\Big\} \mp (z \to \bar z), \\
C_\pm^{(P)}(x/\xi) &= \Big\{\frac{2}{z}\Big(6\mathrm{H}_{0000} - \mathrm{H}_{1000} - 2\mathrm{H}_{200} - \mathrm{H}_{1100} - \mathrm{H}_{120} - \mathrm{H}_{210} + \mathrm{H}_{1110}\Big) \mp \frac{2}{\bar z}\mathrm{H}_{000} + \frac{2}{\bar z}\mathrm{H}_{20} \\
&\quad + \frac{4}{z}\mathrm{H}_{110} - \Big(\frac{8}{z} - \frac{(2\pm 2)}{\bar z}\Big)\mathrm{H}_{100} - \Big(\frac{12\pm 1}{\bar z} + \frac{38}{3z}\Big)\mathrm{H}_{00} + \Big(\frac{3\pm 3}{\bar z} + \frac{28\mp 6}{3z}\Big)\mathrm{H}_{10} \\
&\quad + \frac{2}{z}\zeta_2\Big(\mathrm{H}_{11} - \mathrm{H}_2 - \mathrm{H}_{10} - 4\mathrm{H}_{00}\Big) + \frac{2}{\bar z}\Big(\frac{218\pm 5}{12} \pm (3\pm 2)\zeta_2 \mp 2\zeta_3\Big)\mathrm{H}_0 \\
&\quad + \frac{2}{z}\Big(3\zeta_2 + 16\zeta_3 - \frac{32}{9}\Big)\mathrm{H}_0 + \frac{1}{z}\Big(\frac{701}{24} + \frac{25\mp 9}{3}\zeta_2 + (41\mp 2)\zeta_3 + 3\zeta_2^2\Big)\Big\} \mp (z \leftrightarrow \bar z), \\
C_\pm^{(NP)}(x/\xi) &= \Big\{12(\bar z \pm z)\Big(\mathrm{H}_{20} + \mathrm{H}_{110} - \zeta_3\Big) + 12(1\pm 1)\Big(\mathrm{H}_{10} - \mathrm{H}_0\Big) - \Big(\frac{4}{z} \mp \frac{2}{\bar z}\Big)\Big(\mathrm{H}_{30} + \mathrm{H}_{210} + \mathrm{H}_{31}\Big) \\
&\quad - \frac{6}{z}\mathrm{H}_{200} + \Big(\frac{8}{z} \mp \frac{2}{\bar z}\Big)\mathrm{H}_4 + \Big(\frac{6}{z} \mp \frac{4}{\bar z}\Big)\mathrm{H}_{22} \mp \frac{(8\pm 2)}{\bar z}\Big(\mathrm{H}_3 - \mathrm{H}_{20}\Big) - \frac{6(1\mp 1)}{z}\mathrm{H}_{10} \\
&\quad + \Big[\frac{4}{z}\Big(\frac{2}{3} - \zeta_2\Big) \mp \frac{2}{\bar z}(\zeta_2 - 3 \pm 2)\Big]\mathrm{H}_{00} - \Big[\frac{4\zeta_2 - 4}{z} - \frac{21\mp 17 \pm 6\zeta_2}{3\bar z}\Big]\mathrm{H}_2 \\
&\quad + \Big[\frac{2}{z}\Big(7\zeta_3 - \frac{16}{9}\Big) \pm \frac{1}{\bar z}\Big(\frac{35\pm 2}{3} + (8\pm 2)\zeta_2 - 6\zeta_3\Big)\Big]\mathrm{H}_0 \\
&\quad - \frac{1}{z}\Big(3\zeta_2^2 + (13\mp 7)\zeta_2 - 6(5\mp 1)\zeta_3 + \frac{73}{12}\Big)\Big\} \mp (z \leftrightarrow \bar z)\,,
\end{aligned} \tag{20}$$

where $z = (x-\xi)/2\xi$ and $\mathrm{H}_{\vec n} \equiv \mathrm{H}_{\vec n}(z)$ are harmonic polylogarithms [21].

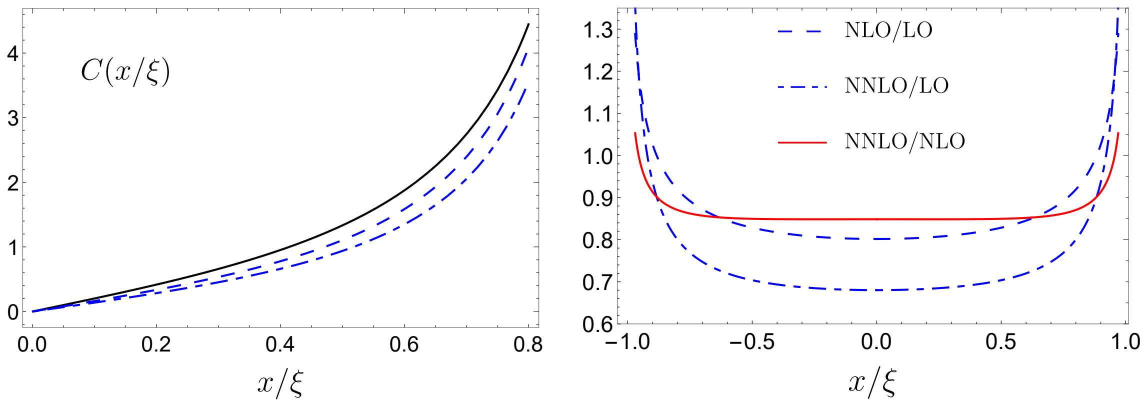

Figure 1: The DVCS vector CF $C(x/\xi)$ in Eqs. (5), (19) at $\mu = Q = 2$ GeV in the ERBL region $x < \xi$. The LO (tree-level), NLO (one-loop) and NNLO (two-loop) CFs are shown by the black solid, blue dashed and blue dash-dotted curves on the left panel, respectively. The right panel shows the ratios NLO/LO (dashed), NNLO/LO (dash-dotted) and NNLO/NLO (solid).

4. Numerical estimates

The numerical results in this section are obtained assuming the photon virtuality $Q^2 = 4$ GeV^2 and the corresponding value of the strong coupling $a_s(4\,\mathrm{GeV}^2) = \alpha_s(4\,\mathrm{GeV}^2)/(4\pi) = 0.02395$. The results for vector CFs are shown in Figs. 1 and 2, respectively. In the first figure, we also show on the right panel the ratios of NLO and NNLO to the leading order (LO) contribution, NLO/LO and NNLO/LO, and the ratio NNLO/NLO. It is seen that the NNLO (two-loop) and NLO (one-loop) contributions to the CF have the same sign and are negative with respect to the LO (tree-level) result in the bulk of the kinematic region apart from the end points $|x| \to |\xi|$ where the loop corrections are positive and dominated by the contributions of threshold double-logarithms. We observe that the NNLO contribution is significant. In the ERBL region, it is generally about 10% of the LO result (a factor two below NLO). In the DGLAP region it is less important and in fact negligible for the real part at $x/\xi > 2$, and for the imaginary part at $x > 4\xi$.

In order to estimate corrections to the Compton form factors,

$$\mathcal{H}(\xi) = \int_{-1}^{1} \frac{dx}{\xi} C(x/\xi) H(x,\xi)\,. \tag{21}$$

we used the GPD model from Ref. [5, Eq. (3.331)], which is based on the so-called double-distributions ansatz and allows for a simple analytic representation: It turns out that the NNLO correction to the absolute value of the Compton form factor $\mathcal{H}$ is quite large: it is only about a factor two smaller than the NLO correction and decreases the Compton form factor by about 10% in the whole kinematic range. The NNLO correction for the phase proves to be much smaller.

5. Summary

Using an approach based on conformal symmetry we have calculated the two-loop CFs in DVCS in the $\overline{\mathrm{MS}}$ scheme for the flavor-nonsinglet vector contributions. Analytic expressions for

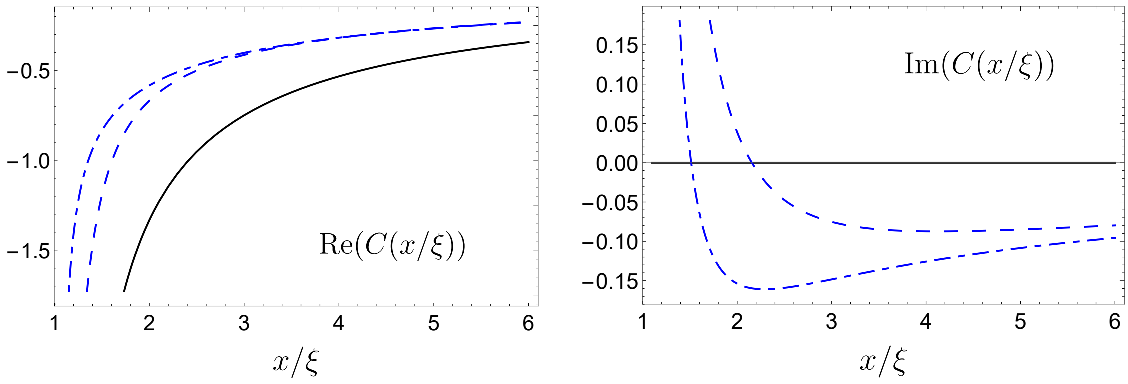

Figure 2: The DVCS vector CF $C(x/\xi)$ in Eqs. (5), (19) at $\mu = Q = 2$ GeV analytically continued into the DGLAP region $x > \xi$: real part on the left and imaginary part on the right panel. The LO (tree-level), NLO (one-loop) and NNLO (two-loop) CFs are shown by the black solid, blue dashed and blue dash-dotted curves. Note, that imaginary part of the LO CF contains a local term $\sim \delta(x - \xi)$ (not shown).

the (axial-)vector CF in momentum fraction space at $\mu = Q$ are presented in Eq. (20). Numerical estimates in Sect. 4 suggest that the two-loop contribution gives rise to a $\sim 10\%$ correction to the Compton form factor, which is significantly above the projected accuracy at the JLAB 12 GeV facility and the Electron Ion Collider.

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