Particle escape from supernova remnant shocks:
gamma-ray and cosmic-ray signatures

Silvia Celli$^{a,b,*}$ and Giovanni Morlino$^c$

$^a$Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, D69117 Heidelberg, Germany
$^b$Sapienza Università di Roma, Piazzale Aldo Moro 5, 00185, Roma, Italy
$^c$Osservatorio Astrofisico di Arcetri, Largo Enrico Fermi 5, Firenze, Italia

E-mail: silvia.celli@mpi-hd.mpg.de

In the context of the supernova remnant (SNR) paradigm for the origin of Galactic cosmic rays (CRs), the escape process of accelerated particles represents a fundamental piece of information to interpret both the observed CR spectrum and the gamma-ray spectral signatures emerging from these sources. Under the assumption that in the spatial region immediately outside of the remnant the diffusion coefficient is suppressed with respect to the average Galactic one, we found that a significant fraction of particles can still be located inside the SNR long time after their nominal release from the acceleration region. This fact results into a gamma-ray spectrum arising from hadronic collisions that resembles a broken power law, similar to those observed in several middle-aged SNRs. Above the break, the spectral steepening is determined by the diffusion coefficient outside of the SNR and by the time dependence of maximum energy. Consequently, the comparison between SNR data and model predictions will possibly allow to determine these two quantities. Additionally, by further assuming that protons and electrons are accelerated at SNR shocks with the same slope, CR spectral measurements on Earth can then be reproduced if electrons are injected with a spectrum steeper than protons for energies above 10 GeV. A possible scenario that can in principle justify the observed steeper electron spectrum relies on the combination of energy losses, due to synchrotron radiation in an amplified magnetic field, and time dependent acceleration efficiency.
1. Introduction

The process that allows CRs to escape from their sources and be released into the Galaxy is still largely unknown, mostly because its comprehension relies on details of the acceleration process and magnetic field evolution. Given the large uncertainties of current theoretical models [1,2], here we adopt a phenomenological approach, consisting into a simplified description of the particle transport in spherical symmetry, capable of catching the particle decoupling from the SNR shock within a parametric description of the escape time [3]. In particular, a time-dependent solution for the density distribution of both protons and electrons is obtained, as described in Sec. 2. The implications of an escaping flux for the spectrum of CRs injected into the Galaxy and multi-wavelength signatures from SNRs are discussed in Sec. 3. Finally, conclusions are derived in Sec. 4.

2. The SNR dynamical evolution and the particle maximum energy

The particle acceleration is believed to be highly efficient during the initial stages of the remnant evolution, when the shock is almost in free-expansion as the mass of the supernova (SN) ejecta $M_{\text{ej}}$ dominates over the swept-up mass. Deceleration occurs later on, from the onset of the Sedov-Taylor (ST) stage, when the expansion starts to become adiabatic, at a time that for explosions occurring in a uniform environment of mass density $\rho_0$ reads as:

$$t_{\text{Sed}} \approx 1.6 \times 10^3 \, \text{yr} \left( \frac{E_{\text{SN}}}{10^{51} \, \text{erg}} \right)^{-1/2} \left( \frac{M_{\text{ej}}}{10 \, M_{\odot}} \right)^{5/6} \left( \frac{\rho_0}{1 \, m_p / \text{cm}^3} \right)^{-1/3},$$

$m_p$ being the proton mass, and $E_{\text{SN}}$ the kinetic energy released at the SN explosion. We refer to the analytical parametrization of [4] for the description of the temporal evolution of the shock radius $R_{\text{sh}}$ and speed $u_{\text{sh}}$ during the ejecta-dominated (ED) and ST stages, as well as along the transition. A maximum value of the particle momentum $p_{\text{max}}$, though not naturally accounted for in the Diffusive Shock Acceleration (DSA) theory, is expected to exist in order to limit the spectral energy density of accelerated particles. However, a self-consistent description of the maximum energy achievable in the acceleration mechanism in a non-stationary framework requires the correct modelling of the evolution of the magnetic turbulence, which is supposed to be self-generated by the same accelerated particles, and possibly damped through frictional effects and wave cascade. Such a complete description does not exist yet; we here use a quite general recipe, where the maximum momentum at first increases with time, as long as the shock is actively accelerating particles, and then it decreases during the ST phase according to a power law in time [see e.g. 5]:

$$p_{\text{max},0}(t) = \begin{cases} p_M \left( t/t_{\text{Sed}} \right) & \text{if } t < t_{\text{Sed}} \\ p_M \left( t/t_{\text{Sed}} \right)^{-\delta} & \text{if } t \geq t_{\text{Sed}} \end{cases},$$

where $p_M$ represents the absolute maximum momentum, achieved at $t = t_{\text{Sed}}$. Note that $\delta$ is a free parameter of the model, bounded to be positive, whose value strongly depends on the temporal evolution of the magnetic turbulence [3]. By inverting Eq. (2), we can also define the escape time for particles of given momentum $p$:

$$t_{\text{esc}}(p) = t_{\text{Sed}} \left( p/p_M \right)^{-1/\delta},$$
corresponding to the time when these particles cannot be confined anymore by the turbulence and start escaping from the shock. The onset of the escape process in the acceleration scenario introduces a unique feature in the evolution of the particle distribution, that will behave differently before and after $t_{\text{esc}}(p)$. In fact, at times smaller than $t_{\text{esc}}(p)$, particles closely follow the shock evolution as they are strictly tightened to the turbulence. On the other hand, at later times, when the turbulence starts to fade out, particles behave disconnected by the shock, and freely diffuse in the space. Particles evolving in these two regimes will be named respectively \textit{confined particles} and \textit{non-confined (or escaping) particles}.

2.1 Particle propagation in SNRs

The transport equation regulates the evolution of the particle density in the plasma velocity and magnetic fields of the shock region and around: the accelerated particles are subject to advection, diffusion and adiabatic losses. Under the assumption that the internal structure of the moving plasma is such that its velocity profile is given by

$$u(t, r) = \left(1 - \frac{1}{\sigma}\right) \frac{u_{\text{sh}}(t)}{R_{\text{sh}}(t)} r,$$

$\sigma$ being the compression ratio at the shock ($\sigma = 4$ for strong shocks), an analytical solution can be found for both the confined and the non-confined density function of protons during the ST evolutionary stage. We refer to [3] for details on the methods and the solutions of the particle density in the two different propagation regimes, the confined and the escaping distributions, which coincide at the escape time. The main assumptions of the computations are: i) a constant efficiency in converting the shock bulk kinetic energy into relativistic protons, $\xi_{\text{CR},p}$ (see e.g [7]); ii) a featureless power law in momentum for the acceleration spectrum, as predicted by DSA, with spectral slope $\alpha$; iii) a stationary and homogeneous diffusion coefficient, given by

$$D(p) \equiv \chi D_{\text{Gal}}(p) = \chi 10^{28} \left(\frac{p c}{10 \text{ GeV}}\right)^{1/3} \text{ cm}^2 \text{ s}^{-1},$$

where the parameter $\chi$ quantifies the difference with respect to the average Galactic diffusion coefficient $D_{\text{Gal}}(p)$, which is expected to exist in the accelerator region.

With respect to protons, the propagation of electrons is further affected by energy losses, as revealed by many observations of radiation (from radio to X rays) in several SNRs [8], thus requiring the switch towards a numerical treatment for the solution of their transport equation. These consist of synchrotron and inverse Compton emissions, impacting the particle spectrum. To evaluate them, an estimate of the magnetic field strength at the shock in needed, as well as of its temporal evolution in the remnant interior while it is expanding. The value of the magnetic field at the shock is the result of both amplification, that we here account for parametrically as due to proton-self amplification and hence connected to the maximum energy of protons, and compression at the shock of the circumstellar magnetic field, amounting to a factor $VTT$ for a randomly oriented field. Additionally, the evolution of the downstream field is further affected by adiabatic losses, analogously to particles.

The instantaneous electron spectrum at the shock is assumed proportional to the proton spectrum, with a normalization factor $K_{ep}$ that accounts for possibly different injection efficiencies of electrons and protons [9]. Nonetheless, its cutoff is located at the maximum energy which is
determined by the condition \( t_{acc} = \min(t_{SNR}, \tau_{loss}) \), namely acceleration time either limited by SNR age or by loss time. In the loss dominated case, a super-exponential cutoff is present [10, 11]. In particular, when energy losses are proportional to \( E^2 \), like in the case of synchrotron and inverse Compton processes, the loss-dominated cutoff is \( \propto \exp\left(-\left(p/p_{max,c}\right)^2\right) \). The electron maximum energy, as limited by energy losses, can be estimated starting from the energy loss rate due to synchrotron plus IC scattering, which is

\[
\frac{dE}{dt}_{\text{syn+IC}} = -\frac{\sigma_T c}{6\pi} \left(\frac{E}{m_e c^2}\right)^2 \left(B^2 + B_{eq}^2\right),
\]

where \( \sigma_T \) is the Thomson cross section and \( m_e \) the mass of the electron, while \( B_{eq}^2 = 8\pi U_{rad} \) is the equivalent magnetic field associated to the interstellar radiation field of energy density \( U_{rad} \).

We refer to [9] for details of the numerical computation: the resulting maximum energy achieved at the shock is shown in Fig. 1, as compared to the proton maximum energy given by Eq. (2) in the PeVatron scenario, namely \( p_M = 1 \text{ PeV/c} \) for protons at \( t = t_{Sed} \). As shown in Fig. 1, the electron maximum energy remains quite limited during the ST stage, resulting smaller than 50 TeV. The downstream electron distribution function is then obtained accounting for the full energy loss history from the time of acceleration \( t' \) to the current time \( t \) [13].

3. Radiation and CR signatures of particle escape

Hadronic collisions between the target gas located inside the remnant and the accelerated protons, both confined and escaping, would give rise to gamma-ray fluxes. These are reported in
Concerning the electron spectrum, measurements at Earth suggest that electrons are released with a spectrum steeper than protons by $\Delta s_{ep} \sim 0.3$ for energies above $\sim 10$ GeV and by $\Delta s_{ep} \sim 1.2$ above $\sim 1$ TeV. Within the context of the model here presented, we have explored two possible scenarios that can in principle justify steeper electron spectra: i) energy losses due to synchrotron radiation in an amplified magnetic field, and ii) time dependent acceleration efficiency. We account for magnetic field amplification produced by either CR induced instabilities or by magneto-hydrodynamic (MHD) instabilities my means of a parametric description. As shown in Fig. 2(b), both mechanisms are required to explain the electron spectrum. In particular synchrotron losses can only produce a significant electron steepening above $\sim 1$ TeV, while a time dependent acceleration can explain the spectrum at lower energies if the electron injection into DSA is inversely proportional to the shock speed.

Fig. 2(a) for a typical middle-aged SNR, assumed to have behaved as a PeVatron at the Sedov time: as shown there, the contribution from escaping protons still propagating inside the SNR is sizeable, in case the diffusion coefficient is suppressed by a factor $\chi > 1$, particularly at the highest energies. A clear signature of the presence of escaping particles would be given by a spectral hardening in the $> 50$ TeV domain, that will be accessible with next-generation instruments as LHAASO and CTA. Additionally, for escaping protons propagating in the right vicinity of the shock, a high-energy halo emission might arise, according to the diffusion properties of the region [3].

With respect to the spectrum of CRs injected along the entire history of the SNR up to the start of the radiative stage, we find for protons consistent results with [14, 15], namely that: (i) if the acceleration spectrum is steeper than $p^{-4}$, the spectrum injected in the Galaxy will show the same steepness, thus coinciding with the acceleration spectrum; (ii) if the acceleration spectrum is flatter than $p^{-4}$, the spectrum injected in the Galaxy will be a $p^{-3}$ power law, regardless of the acceleration spectrum. Clearly, any time dependence of $\xi_{CR,p}$ might modify the final spectrum released in the Galaxy. Moreover, the end of the acceleration is expected to produce some additional signatures in the injected spectrum, as discussed in [3].
4. Conclusions

We have presented a phenomenological model for the description of particle escape from an SNR shock aimed at evaluating the effects produced by the escape process on the spectrum of particles contained in the remnant and those located immediately outside of the shock region, as well as at exploring radiative signatures of these occurrences. We find that the possible PeVatron behavior of SNRs can be investigated with detailed spectral analysis in the very-high-energy regime by next generation instruments. Additionally, by extending the measurements of the CR-electron spectrum from ground, the same instruments might be able to clarify the processes ongoing at the accelerator site, thus providing constraints to the present model.

References