PROCEEDINGS OF SCIENCE

On the Propagation of Relativistic Shocks in Conductive Media

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Relativistic shocks have a central role in high energy astrophysical phenomena, with Gamma-Ray Bursts being the most prominent example. Their propagation in perfectly conductive plasmas has been extensively studied. In the present work we extend the analysis by assuming a finite electrical conductivity for the propagation medium and a finite thickness for the shock front. These two assumptions necessitate the inclusion of an additional jump condition derived through the covariant Gauss-Ampère Law and introduce a dimensionless parameter which depends on the electrical conductivity of the plasma in the shock front, the shock thickness, as well as on the shock's propagation four-velocity. We show that this parameter determines the degree to which the shock interacts with the propagation medium's electromagnetic field and governs shock dynamics.

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1. Introduction

Relativistic shocks possess a central role in a multitude of high energy astrophysical phenomena, by acting as a mechanism for particle acceleration while also providing a way to transfer thermal and kinetic energy to the propagation medium. Relativistic shocks are closely associated with relativistic outflows, produced either by an accreting compact object, during violent processes such as Gamma-Ray Bursts (GRBs), or during a Tidal Disruption Event (TDE). These shocks occur both in the interior of such outflows (internal shocks) or when they interact with their environment [1], [2]. While the propagation of relativistic shocks in perfectly conductive plasmas has been studied by numerous authors, for instance [3] and [4], their propagation in media characterized by a finite electrical conductivity has yet to be examined analytically. In this work we aim to comprehend the effects of the propagation medium's finite electrical conductivity on the properties of the post-shock medium, through the numerical solution of the jump conditions.

2. Jump Conditions for a Medium with Finite Conductivity

Following Taub [5], we express the covariant laws governing relativistic plasma flows in integral form. The tensors are projected on a spacelike four-vector S^{μ}, perpendicular to the timelike shock hypersurface Σ in Minkowski space [6]. The resulting covariant expressions for the jump conditions are:

$$[N^{\mu}]S_{\mu} = 0\,, \quad [T^{\mu\nu}]S_{\nu} = 0\,, \quad [{}^{*}F^{\mu\nu}]S_{\mu} = 0\,, \quad [F^{\mu\nu}]S_{\mu} + \frac{4\pi\mathcal{L}_{sh}}{c}\tilde{J}^{\nu} = 0\,, \tag{1}$$

where $[Q] = Q_2 - Q_1$. The subscripts $1, 2$ denote the propagation and the shocked medium respectively. $\tilde{J}^{\nu}$ is the average four-current across the shock front's width in Minkowski space $\mathcal{L}_{sh}$, which in the shock frame corresponds to its physical width.

The $3 + 1$ decomposition of Eqs. 1 provides the algebraic relations which determine the boundary values at the shock front of all quantities describing the shocked medium as functions of the respective quantities of the propagation medium. In this work we assume shock propagation along $\hat{x}$ and a transverse electromagnetic field: $\boldsymbol{E}_{1,2} = E_{1,2}\hat{y}$, $\boldsymbol{B}_{1,2} = B_{1,2}\hat{z}$. The derived algebraic relations are:

$$\gamma_2\rho_2(\beta_{sh} - \beta_2) = \rho_1\beta_{sh}\,, \tag{2}$$

$$\gamma_2^2(\epsilon_2 + P_2)(\beta_{sh} - \beta_2) - P_2\beta_{sh} + \frac{E_2^2 + B_2^2}{8\pi}\beta_{sh} - \frac{E_2B_2}{4\pi} = \epsilon_1\beta_{sh} + \frac{E_1^2 + B_1^2}{8\pi}\beta_{sh} - \frac{E_1B_1}{4\pi}\,, \tag{3}$$

$$P_2 - \gamma_2^2(\epsilon_2 + P_2)(\beta_{sh} - \beta_2)\beta_2 + \frac{E_2^2 + B_2^2}{8\pi} - \frac{E_2B_2}{4\pi}\beta_{sh} = P_1 + \frac{E_1^2 + B_1^2}{8\pi} - \frac{E_1B_1}{4\pi}\beta_{sh}\,, \tag{4}$$

$$E_2 - \beta_{sh}B_2 = E_1 - \beta_{sh}B_1\,, \tag{5}$$

$$E_2\beta_{sh} - B_2 - E_1\beta_{sh} + B_1 = \frac{4\pi\sigma\xi\mathcal{L}_{sh}}{c\Gamma_{sh}}(\gamma_2(E_2 - \beta_2B_2) + E_1)\,. \tag{6}$$

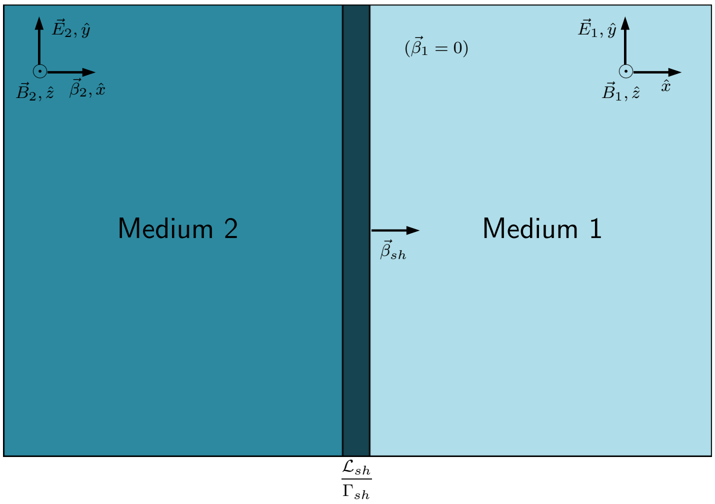

Figure 1: The shock propagates along the $\hat{x}$-axis with velocity β_{sh} in the propagation medium's frame (medium 1). In this frame the shock thickness is $\dfrac{\mathcal{L}_{sh}}{\Gamma_{sh}}$.

The average current in the plasma frame is $\tilde{J}_{co}$ is written as:

$$\tilde{J}_{co} = \sigma \tilde{E}_{co} = \sigma \xi (E_2^{co} + E_1^{co}) \tag{7}$$

with ξ a dimensionless constant of order unity and σ the electrical conductivity of the plasma in the shock front. $\boldsymbol{E}_{co} = \gamma(\boldsymbol{E} + \boldsymbol{\beta} \times \boldsymbol{B})$ is the medium's comoving electric field, while $\Gamma_{sh}\beta_{sh}$ is the shock propagation four-velocity in the propagation medium's rest frame. Due to the chosen geometrical configuration of the electromagnetic field, the current density is perpendicular to the flow velocity. Consequently, it is unaffected by Lorentz boosts between inertial frames of reference.

The system of equations is closed by the inclusion of an Equation of State (EoS). The EoS assumed here is the Taub-Matthews (TM) EoS [7], according to which the fluid's total internal energy is related to its pressure through the relation:

$$\epsilon = \frac{3}{2}P + \sqrt{\frac{9}{4}P^2 + \rho^2 c^4} \tag{8}$$

The numerical solutions are obtained by solving the algebraic relations 2-6 for $\{\rho_2, \epsilon_2, P_2, E_2, B_2\}$ as functions of the corresponding quantities of the propagation medium and γ_2. ρ_2, ϵ_2, P_2 are then substituted into Eq. 8, which is solved numerically for γ_2. The solutions are derived with respect to the value of a dimensionless parameter α defined as:

$$\alpha = \xi \frac{c\gamma_2 \Gamma_{sh}(\beta_{sh} - \beta_2)\mathcal{L}_{sh}}{\eta} \tag{9}$$

with $\eta = \dfrac{c^2}{4\pi\sigma}$ the magnetic diffusivity of the plasma in the shock front and $\gamma_2\Gamma_{sh}(\beta_{sh} - \beta_2) = \tilde{\gamma}_2\tilde{\beta}_2$ the shocked plasma four-velocity in the shock frame.

Defining the dissipative length as $\mathcal{L}_\sigma = \dfrac{c}{4\pi\sigma}$, the previous relation can be rewritten as:

$$\alpha = 10\xi \left(\frac{\tilde{\gamma}_2 \tilde{\beta}_2}{10} \right) \frac{\mathcal{L}_{sh}}{\mathcal{L}_\sigma} \,. \tag{10}$$

$\mathcal{L}_{sh}$ and $\mathcal{L}_\sigma$ are normalized to the gyroradius of the plasma's thermal protons $r_g = \dfrac{\gamma m c u_{Th}}{eB}$, which for a plasma with $T = 10^9\,K$ and $B = 1\,G$ is approximately equal to $410\,m$. The characteristic conductivity value for which $\mathcal{L}_\sigma$ becomes equal to r_g is $56240\,s^{-1}$.

3. Numerical Results

By applying the algorithm detailed in the previous section, we obtained numerical solutions for a shock propagating with a Lorentz factor $\Gamma_{sh} = 100$ in the propagation medium's frame, for three different cases of cold propagation media ($P_1 \ll \rho_1 c^2$) with normalized magnetic energy densities $\mathcal{B} = \dfrac{B_1^2}{8\pi\rho_1 c^2} = 0.1, 1, 10$.

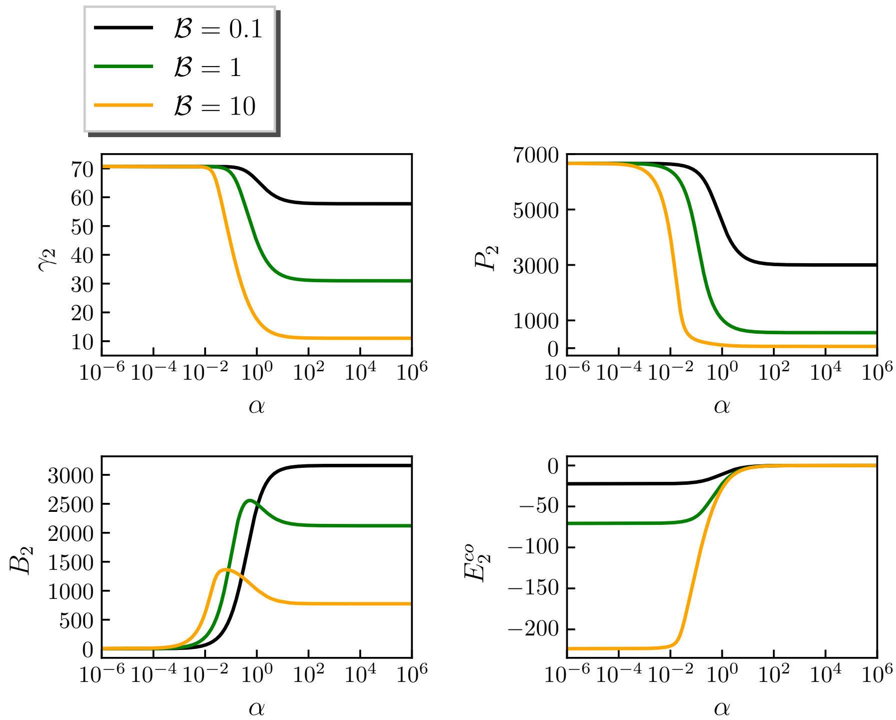

Figure 2: The shocked medium's Lorentz factor γ_2, thermal pressure P_2, magnetic field B_2, and comoving electric field E_2^{co} for a shock propagating with $\Gamma_{sh} = 100$.

We identify two characteristic regimes in our solutions with respect to the electromagnetic field's behavior, corresponding to:

- Vacuum Electrodynamics: $\alpha \ll 1$
- Ideal MHD: $\alpha \gg 1$

For $\alpha \ll 1$, the fluid is essentially decoupled from the electromagnetic field. No current density develops in the shock front, which is why the electric and magnetic field experience no change due to the shock propagating in the conducting propagation medium. The hydrodynamic quantities of the medium obey the purely hydrodynamic relativistic jump conditions, which for strongly relativistic shocks are [8]:

$$\gamma_2 = \frac{\Gamma_{sh}}{\sqrt{2}} \tag{11}$$

$$\gamma_2 \rho_2 = 2\Gamma_{sh}^2 \rho_1 \tag{12}$$

$$P_2 = \frac{2\Gamma_{sh}^2}{3} \rho_1 c^2 \tag{13}$$

On the other hand, for $\alpha \gg 1$, the ideal MHD jump conditions are satisfied.

The post-shock medium's total pressure closely follows the relation $P_{total} \sim \Gamma_{sh}^2$ for strongly relativistic shocks irregardless of the value of α. The same behavior is exhibited by the rest of the post-shock quantities.

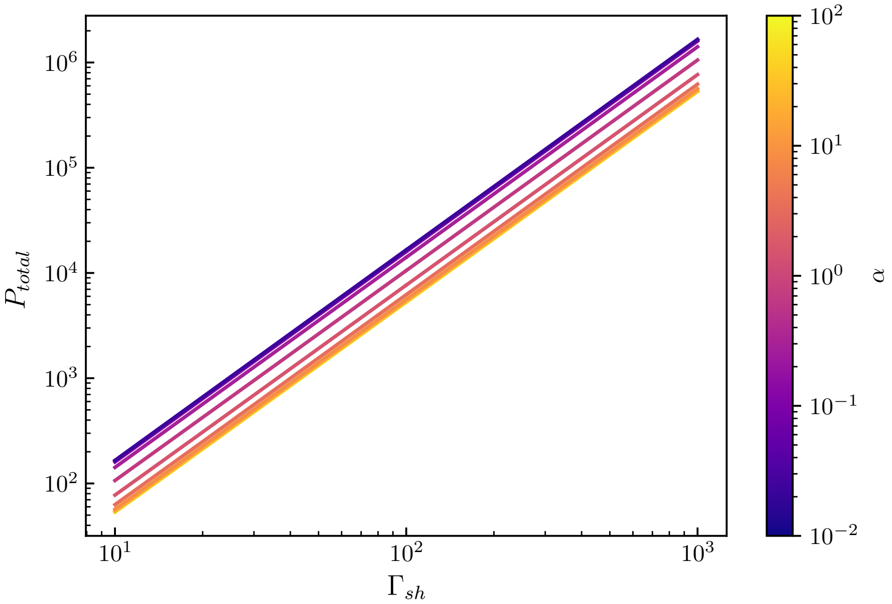

Figure 3: The sum of the thermal and electromagnetic pressure of the post-shock medium with respect to the shock propagation Lorentz factor for varying α.

4. Conclusions

In the present work we considered the propagation of relativistic shocks in magnetized media characterized by a finite electrical conductivity. We derived the covariant equations expressing the jump conditions across a shock front of finite thickness and obtained numerical solutions to the 3+1 decomposed algebraic equations for a strongly relativistic shock propagating in a cold conductive medium with a transverse electromagnetic field. A dimensionless parameter α which depends on the medium's electrical conductivity was defined, the value of which governs shock dynamics. The numerical solutions reveal the existence of two characteristic regimes, one for $\alpha \ll 1$, in which the fluid and the electromagnetic field are decoupled and the only quantities experiencing a jump across the shock front are the hydrodynamic quantities of the propagation medium. In the second regime, for $\alpha \gg 1$, the propagation medium behaves as a perfect conductor and the ideal MHD conditions are obtained.

Aside from providing physical insight into the more realistic picture of shock propagation in finite conductivity media, the present analysis also finds practical applications in the construction of improved Riemann solvers for resistive relativistic magnetohydrodynamics.

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