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Impact of hadronic interactions and conservation laws on cumulants of conserved charges in a dynamical model

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Understanding the phase diagram of quantum chromodynamics (QCD) by measuring fluctuations of conserved charges in heavy-ion collisions is one of the main goals of various different experimental programs. Within this work, we calculate the role of hadronic interactions and momentum cuts on cumulants of conserved charges up to fourth order in a system in equilibrium within a hadronic transport approach (SMASH). We analyse the results of the full SMASH hadron gas and extract proton and baryon number cumulants. We find modifications of the scaled variance at large baryon chemical potential and good agreement between experimental results and this work.

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1. Introduction

Understanding the phase diagram of QCD sparks large interest in both experimental and theoretical physics. On the experimental side, there is the ongoing beam energy scan program (BES) which uses heavy-ion collisions (HIC) at varying collisional energies to scan the QCD phase diagram at different temperatures and chemical potentials [1]. One of the many observables which are thought to be sensitive to a possible phase transition between hadronic and partonic matter are cumulants of conserved charges. However the cumulants in experimental measurements show a dependency on the size of the rapidity window Δy, which is a sign that the net charge on a global scale is fixed event by event in HIC. This limits the comparability to theoretical models where the starting point is the grand-canonical ensemble. Here the total number of particles can fluctuate whereas in the canonical ensemble the number of particles is fixed. Further limitations in measurements are volume fluctuations of the initial fireball or the usage of net proton number as a proxy of the net baryon number fluctuation.

In this work we want to address the problem of understanding the effect of global charge conservation in the hadronic stage of the HIC. For this purpose we use a transport model which incorporates hadronic interactions and investigate the effects of these processes on cumulants of conserved charges or other types of particles.

2. Model and Methodology

The calculations are performed in the hadronic transport model SMASH [2, 3]. SMASH has been successfully used to describe heavy-ion collisions in the low energy regime, as an afterburner to ultra-relativistic heavy-ion collisions and to calculate properties of the hadronic medium e.g. transport coefficients. In this work we employ a box with periodic boundary conditions simulating infinite matter. We incorporate the geometric collision criterion which uses the geometric interpretation of the cross-section and it states that two particles collide if their relative distance $d_\perp < \sqrt{\sigma_{\mathrm{tot}}/\pi}$ with σ_{tot} being the total cross section. We want to investigate the impact of hadronic interactions such as $2 \leftrightarrow 1$ and $2 \leftrightarrow 2$ processes. Where possible, total cross sections are fitted to experimental data and hadrons up to masses of $\sim 2.3\,\mathrm{GeV}$ are included in the model. It is worth mentioning that all charges (B, Q, S) are perfectly conserved at each interaction. At initialization the particles are distributed uniformly in the box with momenta according to the equilibrium Boltzmann-distribution $f_{\mathrm{eq}} \sim e^{-E/T}$ and when dynamically propagated the code runs from interaction to interaction until a given final time is reached.

The cumulants of the net baryon number are calculated from a box simulation from SMASH. On an event-by-event basis the box is initialized with a fixed number of particles and the system is evolved until thermal and chemical equilibrium is reached. Further it has been tested that the density is distributed isotropically in the box after the system is dynamically evolved over time. After equilibrating each event, the net or total particle number $N_p^{\mathrm{net/tot}} = N_p \mp N_{\bar{p}}$ are counted in defined subvolumes of size V and the cumulants of these distributions are presented as a function of the the fraction $x = (V/V_0)$. We want to focus on the ratios of cumulants which are well known and defined in [4] (see Eq.11-12). We additionally investigate the effect of incorporating cuts in transverse momentum $p_T = \sqrt{p_x^2 + p_y^2}$ on the cumulants of conserved charges.

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3. Results

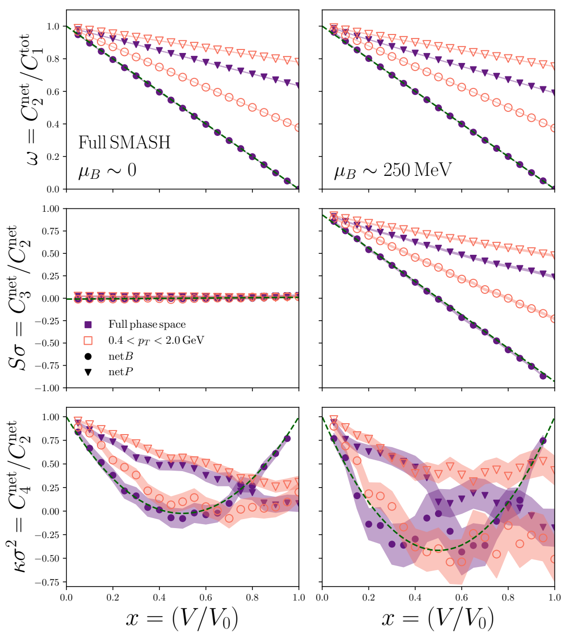

Figure 1: Ratios of cumulants up to fourth order of the full SMASH hadron gas. Left column shows the result for $\mu_B \sim 0$ and the right column for $\mu_B \sim 250$ MeV. In full symbols the fluctuations within full phase space are shown whereas open symbol show the fluctuations including a p_T-cut for baryon (circles) and proton number fluctuations (triangles). Analytic expectations are presented in green dashed lines.

In this section we present our results for the baryon and proton number fluctuations within the full SMASH hadron gas. At initialization, the multiplicities of one fully equilibrated event are taken as an input for each individual event. We consider two different values for the baryon chemical potential, $\mu_B \sim 0$ and $\mu_B \sim 250$ MeV. After the final time step, all unstable particles are forced to decay such that only stable decay products are left in the analysis. Fig. 1 shows the ratios of cumulants of the baryon and proton number fluctuation as a function of the size of the subvolume for different values of chemical potential and in full phase space or including momentum cuts.

We find perfect agreement for the baryon number cumulants in full phase space compared to analytic calculations performed in [5]. The proton number fluctuations do not follow the analytic curves simply because the global net proton number is not a conserved charge. However there are still effects of global baryon conservation present e.g. in the scaled variance (upper left plot), where the slope is reduced but not equal to zero. This can be seen also in the skewness and kurtosis. The scaled variance is not influenced by the chemical potential whereas the skewness is, since it is sensitive to asymmetries in the underlying distribution. The kurtosis $\kappa\sigma^2$ has a strong dependence on μ_B. When comparing to analytic expectations it is important to incorporate the fluctuations of the total charge number, which increase the fourth cumulant. At $\mu_B \sim 0$ baryon annihilation processes occur much more often than at $\mu_B \sim 250$ MeV which enhances $\kappa\sigma^2$ around $x = 1/2$.

Fig. 2 shows the scaled variance which is also presented in the upper left and upper right plot of Fig. 1 but with the focus on small values of x. The first thing to notice is that in both full phase space and with the inclusion of a p_T-cut the cumulants go to the Poisson limit, meaning the fluctuations go to 1 for $x \to 0$. This happens because when the size of the subvolume gets smaller than the correlation length of the underlying system, the relevant physics cannot be grasped by the cumulants. Interestingly in the case of a non-zero baryon chemical potential the fluctuations are enhanced with respect to $\mu_B \sim 0$ at small values of x, originating from an increase in the variance C_2. This enhancement is also present in the proton number fluctuations in both full and restricted phase space.

We now compare our results to experimental measurements from the ALICE collaboration [6] where the scaled variance of protons was measured. One difficulty to overcome is that in this work the cumulants are calculated in coordinate space whereas ALICE measures in bins of momentum space. However it was argued (e.g. in [7]) that there is an equivalence between momentum and coordinate space at these high energies using $x \approx N_{ch}(\Delta y)/(N_{ch}(\infty)) \approx \mathrm{erf}\left(\Delta y/(2\sqrt{2}\sigma_y)\right)$ with $\sigma_y = 3.86$ measured by ALICE [8]. Fig. 3 shows the comparison between the SMASH and experimental measurement. The result was obtained by simply rescaling the variable x with the inverse of the expression for Δy above. For this calculation the p_T cut was adjusted to $0.5 < p_T < 1.6$ GeV to match with [6]. One can see that the correlations of the proton scaled variance including a cut in p_T match with the experimental measurements. This results shows that the fluctuations originating from global charge conservation are still present in the proton number fluctuation which represent a subset of all baryons. This comparison is not supposed to be a perfect explanation for the measurement as there are many different other factors from the dynamics of the heavy-ion collision contributing to this measurement. But it shows that the size of correlation extracted in this work roughly matches with experimental results.

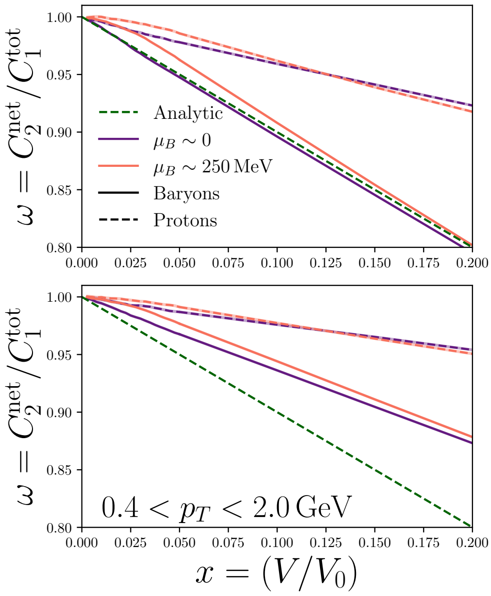

Figure 2: Scaled variance of the SMASH hadron gas at zero (purple) and finite (red) value of the chemical potential in full phase space (top) and including a p_T cut (bottom). The results are presented for the baryon (straight line) and proton number (dashed line) fluctuations. The analytic expectation is shown in green.

4. Summary and outlook

In this work we calculated baryon and proton number cumulants including the effects of global charge conservation as a function of subvolume sizes in a box with periodic boundary conditions using a dynamical hadronic model with realistic interactions. We showed that the behavior of the baryon number cumulants follow the expected behavior which can be calculated analytically and that the proton number fluctuations are still influenced by the global baryon number conservation even though the net proton number is not strictly conserved. We further showed that at $\mu_B \sim 250$ MeV the scaled variance of both baryon and proton number fluctuation are modified compared to the analytic expectation. Finally we compared our result to an experimental measurement and showed that our model produces comparable correlations within the proton fluctuations.

This work can further be extended to study fluctuations of electric charge or strangeness as they are naturally conserved in our calculations. We can further study the diffusion of cumulants in an expanding system.

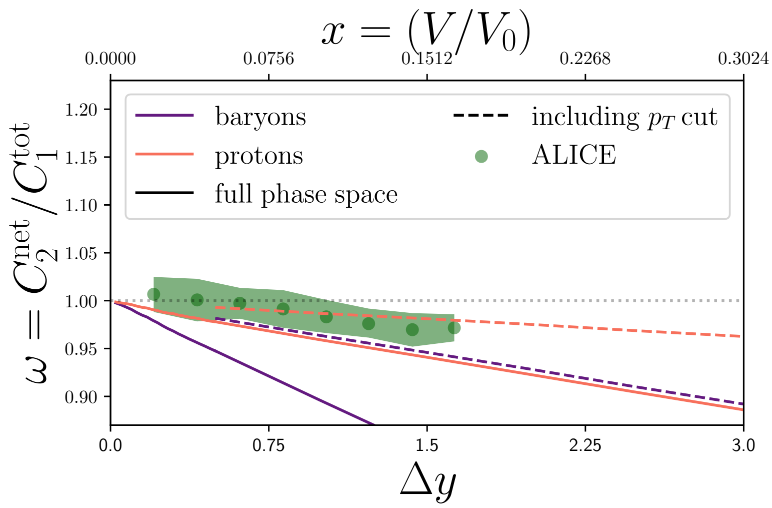

Figure 3: The fluctuations of the net baryon (proton) number are shown in purple (red) lines. Results in the full phase space are in full lines while the pT-cut is shown in dashed lines.

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