PROCEEDINGS OF SCIENCE

A neural network reconstruction of the neutron star equation of state via automatic differentiation

Shriya Soma,[a,b,c,*] **Lingxiao Wang,**[a,c] **Shuzhe Shi,**[d] **Horst Stöcker**[a,b,e] **and Kai Zhou**[a]

[a] *Frankfurt Institute for Advanced Studies (FIAS), Ruth-Moufang-Straße 1, D-60438 Frankfurt am Main, Germany*
[b] *Institut für Theoretische Physik, Goethe Universität, Max-von-Laue-Straße 1, D-60438 Frankfurt am Main, Germany*
[c] *Xidian-FIAS International Joint Research Center, Ruth-Moufang-Straße 1, D-60438 Frankfurt am Main, Germany*
[d] *Center for Nuclear Theory, Department of Physics and Astronomy, Stony Brook University, 100 Nicolls Road, Stony Brook, New York, 11794, USA,*
[e] *GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstraße 1, D-64291 Darmstadt, Germany*

E-mail: soma@fias.uni-frankfurt.de, lwang@fias.uni-frankfurt.de, shuzhe.shi@stonybrook.edu, stoecker@fias.uni-frankfurt.de, zhou@fias.uni-frankfurt.de

In this work, we reconstruct the cold and dense matter equation of state (EoS) from the current observational neutron star data. We achieve this by using a physics-based deep learning method that utilizes the Automatic Differentiation technique. A deep neural network, *EoS Network*, is deployed to represent the EoS in a model-independent way. A second neural network, the *TOV-Solver Network*, is trained to solve the Tolman–Oppenheimer–Volkoff (TOV) equations. The *EoS Network* is then combined with the pre-trained *TOV-Solver Network* and a gradient-based approach is implemented to optimize the weights of the *EoS Network*, in an unsupervised manner. Thus, the designed pipeline is trained to optimize the EoS, so as to yield through TOV equations, a mass-radius (M-R) curve that best fits the observations. We present the EoS obtained from this procedure, using the current neutron star observational data. The results are compatible with the reconstructions from earlier works that used conventional methods and also with the limits of tidal deformability obtained from the gravitational wave event, GW170817.

FAIR next generation scientists - 7th Edition Workshop (FAIRness2022)
23-27 May 2022
Paralia (Pieria, Greece)

*Speaker

© Copyright owned by the author(s) under the terms of the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License (CC BY-NC-ND 4.0).

PoS(FAIRness2022)055

1. Introduction

Neutron star (NS) observations provide useful insights into the dense matter equation of state (EoS). The discovery of massive neutron stars [1–3], a binary neutron star merger event, GW170817 [4], and NICER measurements of certain NS radii [5, 6] have all been able to add constraints on the unknown EoS. This work is an attempt to infer the NS EoS from observational data, based on a novel method. We present a physics-based deep learning algorithm that reconstructs the NS EoS using the currently available mass-radius (M-R) observations of NSs. The proposed technique utilizes a deep neural network (*EoS Network*) for a flexible representation of an EoS. The *EoS Network* is then combined with a pre-trained *TOV-Solver Network* (as the name suggests, the *TOV-Solver Network* is trained to output the M-R curve of any input EoS). The combined framework is then optimized in the Automatic Differentiation (AD) framework, to output an EoS which reproduces the M-R observations with the least error. A schematic representation of the algorithm is depicted in Fig. 1.

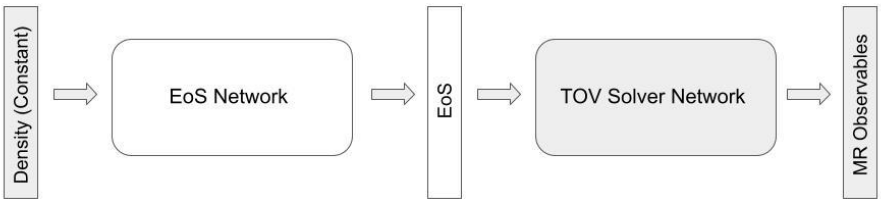

Figure 1: A schematic representation of the proposed algorithm for reconstructing the neutron star EoS via automatic differentiation. The trainable weights in the gradient-based optimization include the parameters from the *EoS Network*. The weights of the pre-trained *TOV-Solver Network* are frozen in this framework.

2. Physics-driven deep learning

Earlier works have proven that physics-driven deep learning methods have the potential to surpass traditional methods in solving inverse problems [7, 8]. In this work, we use the AD framework to invert the TOV equations, i.e. to reconstruct the NS EoS from M-R observations. The pipeline in Fig. 1 consists of two differentiable modules: the *EoS Network*, and the *TOV-Solver Network*. The former uses an unbiased representation for the EoS. It uses a constant density array ρ, and outputs the corresponding pressure, $P_\theta(\rho)$. The latter is a pre-trained emulator for solving the TOV equations [9]. It was trained and tested on a multitude of piece-wise polytropic EoSs and their corresponding M-R curves. When compared to numerical methods like the Euler or Runge-Kutta, the network emulator is superior in computational efficiency ($\sim 10^6$ *sec* quicker). Furthermore, the *TOV-Solver Network* is easily differentiable, an aspect that is critical for applying back-propagation in the AD framework. Linked with the well-trained *TOV-Solver Network*, the *EoS Network* is optimized in an unsupervised way. A detailed structure of the networks, their training procedures and performances can be found in [9].

Assuming N_{obs} number of NS observations, we train the *EoS Network* to fit the M-R output from the devised pipeline to these N_{obs} M-R observations. The optimization procedure deploys a

PoS(FAIRness2022)055

gradient-based algorithm within the AD framework to reduce the loss function, χ^2, which is given as,

$$\chi^2 = \sum_{i=1}^{N_{\text{obs}}} \frac{(M_i - M_{\text{obs},i})^2}{\Delta M^2_{\text{obs},i}} + \frac{(R_i - R_{\text{obs},i})^2}{\Delta R^2_{\text{obs},i}}. \tag{1}$$

Here, the TOV-Solver Network predictions are (M_i, R_i)'s, the observations and their associated uncertainties are $(M_{\text{obs},i}, R_{\text{obs},i})$'s and $(\Delta M_{\text{obs},i}, \Delta R_{\text{obs},i})$'s respectively. The gradients of χ^2 with respect to the weights of the *EoS Network* are,

$$\frac{\partial \chi^2}{\partial \theta} = \sum_{i=1}^{N_{\text{obs}}} \int \left[\frac{\partial \chi^2}{\partial M_i} \frac{\delta M_i}{\delta P_\theta(\rho)} + \frac{\partial \chi^2}{\partial R_i} \frac{\delta R_i}{\delta P_\theta(\rho)} \right] \frac{\partial P_\theta(\rho)}{\partial \theta} \mathrm{d}\rho. \tag{2}$$

In Eq. 2, the terms $\delta P_\theta/\delta\theta$, $\delta(M_i, R_i)/\delta P_\theta(\rho)$ are implicit in the back-propagation algorithm.

Thus, the optimization of the *EoS Network* executed by maximizing the likelihood of observational M-R data, in an unsupervised paradigm. In other words, the aim is to minimize χ^2 (Eq. 1). The observational data is, however, very limited and scattered across the M-R plane with large uncertainties. Moreover, the central density of an observation is unknown. This value is determined in each iteration by the 'closest approach' [10], i.e.

$$\rho_{ci} = \arg\min_{\rho_c} \frac{(M(\rho_c) - M_{\text{obs},i})^2}{\Delta M^2_{\text{obs},i}} + \frac{(R(\rho_c) - R_{\text{obs},i})^2}{\Delta R^2_{\text{obs},i}}. \tag{3}$$

This way, we implement Eq. (3) to reduce the difference between the M-R observations and the M-R curve obtained from the *TOV-Solver Network*. Therefore, the loss in each iteration during the training is,

$$\chi^2 = \sum_{i=1}^{N_{\text{obs}}} \frac{(M(\rho_{ci}) - M_{\text{obs},i})^2}{\Delta M^2_{\text{obs},i}} + \frac{(R(\rho_{ci}) - R_{\text{obs},i})^2}{\Delta R^2_{\text{obs},i}}, \tag{4}$$

where ρ_{ci} is the updated central density. In order to account for measurement uncertainties of the M-R observational data into the reconstructed EoS, we sample several M-R curves from the normal distribution around each observation. The procedure mentioned above is repeated for each M-R curve, thereby resulting in several reconstructed EoSs. We reject the reconstructed EoSs that do not comply with the causal condition or those that fail to support a $1.9 M_\odot$ star. With the remaining EoSs, we define a posterior distribution, which factors in the uncertainties of the M-R data.

3. Results

In order to examine the potential of the proposed method, we conducted several tests on mock M-R data [9]. We further tested the performance of the algorithm on reconstructing a couple of RMF EoSs [9]. Here, we present the results of the reconstructed EoS from existing M-R observations of NSs [5, 6, 11–13]. We fit each observational uncertainty with a 1D normal distribution for both the mass and radius, independently [14]. From the distribution, we sample several points for mass and radius, consequently sampling a multitude of M-R curves. The proposed method is then applied on each M-R curve to reconstruct several EoSs. On filtering the EoSs as descibed in the previous section, we obtain a posterior distribution for the reconstructed EoS. We depict the 68%

PoS(FAIRness2022)055

confidence interval (CI) of the reconstructed EoS in the left panel of Fig. 2, as a red shaded band. The corresponding M-R curve is represented by the red band in the right panel of Fig. 2. We further deduce the tidal deformability of the reconstructed EoS and test the results against the data from GW170817.The tidal deformability of a $1.4 M_\odot$ neutron star, $\Lambda_{1.4}$, from the reconstructed EoS is estimated at $\Lambda_{1.4} = 224^{+107.3}_{-107.3}$ (95% CI). This range lies within the estimated range of $\Lambda_{1.4} = 190^{+390}_{-120}$, obtained from the gravitational wave event, GW170817 [15].

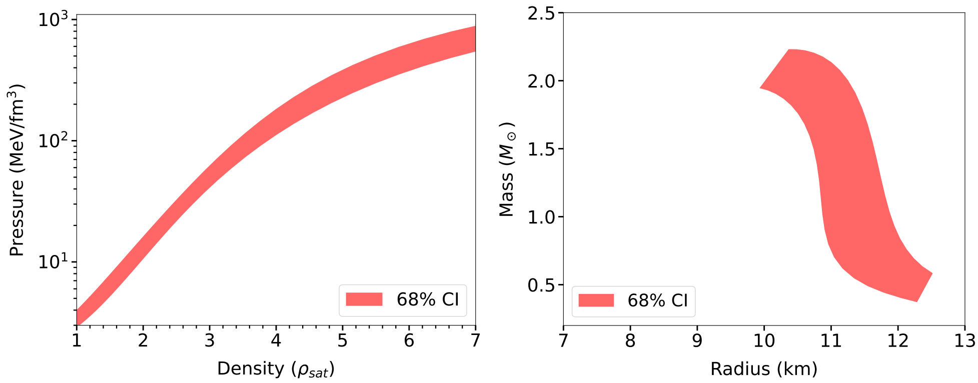

Figure 2: The left panel depicts the 68% CI of the reconstructed EoS from the proposed algorithm. The corresponding M-R curve is plotted in the right panel.

Any bias introduced by the *TOV-Solver Network* can be eradicated by using the TOV equations instead. This method would involve calculation of the linear response of an M-R curve to a change in the EoS. Furthermore, the lack of NS observations below $1 M_\odot$, or a physical explanation for the production of such NSs, gives rise to small deviations in the low-density region of the reconstructed EoS, even in an ideal scenario where uncertainties are ignored [9]. The maximum mass of a non-rotating NS is still under speculation, and could add further inference on the NS EoS. With the upcoming telescopes and gravitational-wave detectors, an improved precision is expected on NS observations. In addition to an anticipated increase in the number of observations, this would imply a finer reconstruction of the NS EoS in future.

This work is supported by HGS-HIRe for FAIR, Deutscher Akademischer Austauschdienst (DAAD), GSI-F&E, BMBF, SAMSON AG, Xidian-FIAS International Joint Research Center (XFIJR), INFN, U.S. Department of Energy, Office of Science, Office of Nuclear Physics, grant No. DE-FG88ER40388 and the Walter Greiner Gesellschaft.

References

[1] J. Antoniadis et al., *A Massive Pulsar in a Compact Relativistic Binary*, Science **340** (2013) 6131 [1304.6875].

[2] E. Fonseca et al., *Refined Mass and Geometric Measurements of the High-mass PSR J0740+6620*, Astrophys. J. Lett. **915** (2021) L12 [2104.00880].

[3] R.W. Romani, D. Kandel, A.V. Filippenko, T.G. Brink and W. Zheng, *PSR J1810+1744: Companion Darkening and a Precise High Neutron Star Mass*, Astrophys. J. Lett. **908** (2021) L46 [2101.09822].

PoS(FAIRness2022)055

[4] LIGO Scientific, Virgo collaboration, *Properties of the binary neutron star merger GW170817*, *Phys. Rev. X* **9** (2019) 011001 [1805.11579].

[5] M.C. Miller et al., *PSR J0030+0451 Mass and Radius from NICER Data and Implications for the Properties of Neutron Star Matter*, *Astrophys. J. Lett.* **887** (2019) L24 [1912.05705].

[6] T.E. Riley et al., *A NICER View of the Massive Pulsar PSR J0740+6620 Informed by Radio Timing and XMM-Newton Spectroscopy*, *Astrophys. J. Lett.* **918** (2021) L27 [2105.06980].

[7] S. Shi, K. Zhou, J. Zhao, S. Mukherjee and P. Zhuang, *Heavy quark potential in the quark-gluon plasma: Deep neural network meets lattice quantum chromodynamics*, *Phys. Rev. D* **105** (2022) 014017 [2105.07862].

[8] L. Wang, S. Shi and K. Zhou, *Reconstructing spectral functions via automatic differentiation*, 2111.14760.

[9] S. Soma, L. Wang, S. Shi, H. Stöcker and K. Zhou, *Neural network reconstruction of the dense matter equation of state from neutron star observables*, *JCAP* **08** (2022) 071 [2201.01756].

[10] C.A. Raithel, F. Özel and D. Psaltis, *From Neutron Star Observables to the Equation of State. II. Bayesian Inference of Equation of State Pressures*, *Astrophys. J.* **844** (2017) 156 [1704.00737].

[11] F. Ozel, D. Psaltis, T. Guver, G. Baym, C. Heinke and S. Guillot, *The Dense Matter Equation of State from Neutron Star Radius and Mass Measurements*, *Astrophys. J.* **820** (2016) 28 [1505.05155].

[12] J. Nättilä, M.C. Miller, A.W. Steiner, J.J.E. Kajava, V.F. Suleimanov and J. Poutanen, *Neutron star mass and radius measurements from atmospheric model fits to X-ray burst cooling tail spectra*, *Astron. Astrophys.* **608** (2017) A31 [1709.09120].

[13] D. Gonzalez-Caniulef, S. Guillot and A. Reisenegger, *Neutron star radius measurement from the ultraviolet and soft X-ray thermal emission of PSR J0437−4715*, *Mon. Not. Roy. Astron. Soc.* **490** (2019) 5848 [1904.12114].

[14] Y. Fujimoto, K. Fukushima and K. Murase, *Extensive Studies of the Neutron Star Equation of State from the Deep Learning Inference with the Observational Data Augmentation*, *JHEP* **03** (2021) 273 [2101.08156].

[15] LIGO Scientific, Virgo collaboration, *GW170817: Measurements of neutron star radii and equation of state*, *Phys. Rev. Lett.* **121** (2018) 161101 [1805.11581].

PoS(FAIRness2022)055