PROCEEDINGS OF SCIENCE

Analytic model for the gravitational wave emission from neutron star merger remnants

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We construct a new analytic model for the description of the gravitational wave (GW) emission in the post-merger phase of binary neutron star (BNS) mergers. The model consists of exponentially decaying sinusoidal functions, attributed to the various oscillation modes, quasi-linear combination tones, or non-linear features that appear in the post-merger phase. We consider a time dependence of the main post-merger frequency peak which is described by a two-segment linear expression. The effectiveness of the model is assessed, in terms of the fitting factor, along a sequence of equal-mass simulations of varying total binary mass. We identify new spectral features, appearing in high-mass configurations, originating from a non-linear coupling between the quasi-radial oscillation and antipodal tidal deformations. The model achieves high fitting factors, and so can be used for the parameter estimation of detections in upcoming searches with aLIGO+ and aVirgo+, or with future detectors such as Einstein Telescope, Cosmic Explorer, or with high-frequency detectors.

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1. Introduction

There are already two GW events where the sources were identified as BNS mergers, GW170817 and [1] and GW190425 [2]. The detection of GWs from the inspiral phase of GW170817 led to new constraints on the neutron star equation of state (EOS) [1, 3] (see [4, 5] for recent reviews). Further constraints are expected by combining information from a larger number of detections in the future [6–10]. The post-merger phase in GW170817 could not be measured, since the sensitivity of aLIGO and aVirgo detectors was not sufficient [1, 3, 11]. Nevertheless, such detections are likely to be possible in the future with upgraded [12], with dedicated high-frequency [13–17] or with third-generation [18, 19] detectors.

Gravitational wave detections of the post-merger phase rely on robust and sophisticated data analysis techniques. One approach is to use so-called matched-filtering schemes, which require faithful GW post-merger template banks. In recent years, several GW template models have been introduced, in the time domain [20–26] or frequency domain [27–30]. Reference [31] introduced an analytic model in the time domain, which consists of a combination of three exponentially decaying sinusoids, corresponding to the most significant frequency components ($f_{\rm peak}$, $f_{\rm spiral}$, f_{2-0}, see below) in the GW post-merger signal, which correlate with the binary's properties. The authors of [24] introduced an analytic model that incorporates exponentially damped sinusoids (as in [31]) but employs linearly time-dependent frequency components. In our work, we provide an extension to [31] by also including the high-frequency f_{2+0} combination tone [32], and a time dependence of the dominant frequency $f_{\rm peak}$. We perform a sequence of simulations with increasing binary mass, and propose an analytic expression for the time evolution of $f_{\rm peak}$ informed by spectrograms of the post-merger GW signal.

2. Methods

We use the Einstein Toolkit [33] to carry out three-dimensional fully general relativistic simulations of binary neutron star mergers. More specifically, we consider a sequence of equal-mass binaries with increasing total binary mass. We use the MPA1 [34] EOS model, which is compatible with current observational constraints [35, 36]. We simulate eight BNS configurations with $M_{\rm tot} = 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0, 3.1\ M_\odot$ and note that none of these configurations collapses to a black-hole (BH) during the simulated time of up to 25 ms after merging. We also note that the configuration with the highest mass, namely the model with $M_{\rm tot} = 3.1\ M_\odot$, is close to the threshold binary mass for prompt BH formation $M_{\rm thres}$. More information about the numerical setup of these simulations can be found in [37] and references therein.

3. Spectral analysis of the post-merger GW emission

The fundamental quadrupolar oscillation mode produces the dominant frequency component, denoted by $f_{\rm peak}$ or f_2, in the GW post-merger signal (see [5, 20, 31, 32] and references therein). The frequency $f_{\rm peak}$ correlates with the size of the remnant and so, encodes information about the high-density regime of the EOS. As the system evolves and redistribution of angular momentum occurs, combined with the losses from GW radiation, the frequency $f_{\rm peak}$ may shift to lower or higher frequencies.

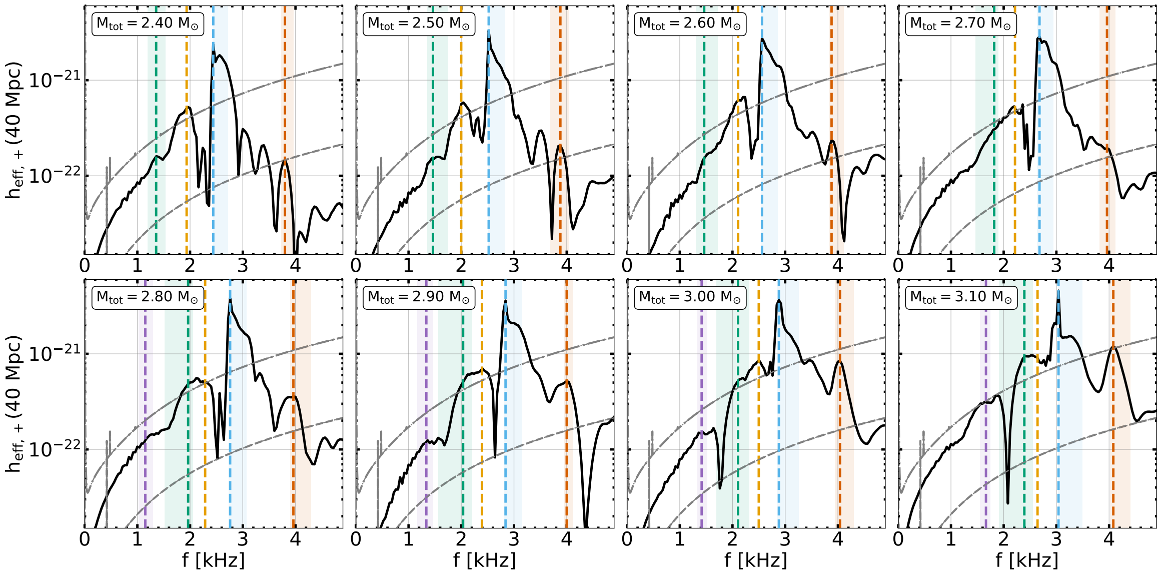


Figure 1: Effective GW spectrum $h_{\rm eff,+}(f)$ for the mass sequence. Colored dashed vertical lines indicate the frequency peaks $f_{\rm peak}$ (cyan), $f_{\rm spiral}$ (yellow), f_{2-0} (green), f_{2+0} (orange), $f_{\rm spiral-0}$ (purple). Shaded areas show the corresponding frequency ranges. The gray curves correspond to the design sensitivity Advanced LIGO [38] and of the Einstein Telescope [39]. Figure taken from [37].

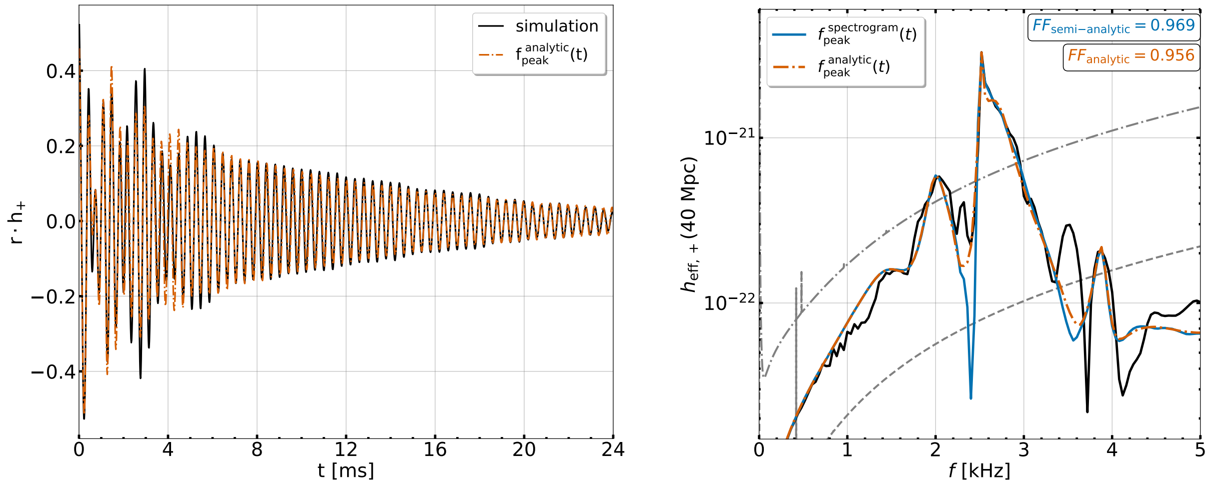
To understand the frequency evolution of $f_{\rm peak}$, we compute the spectrograms of the strain $r \cdot h_+(t)$ using a wavelet-based scheme [40] for all the simulated models in the mass sequence. From the spectrograms, we extract the time-dependent $f_{\rm peak}$, denoted by $f_{\rm peak}(t)$. We find that for each model, $f_{\rm peak}(t)$ can be modeled by an analytic 2-segment piecewise function given by

$$f_{\rm peak}^{\rm analytic}(t) = \begin{cases} \zeta_{\rm drift} \cdot t + f_{\rm peak,0} & \text{for } t \leq t_* \\ f_{\rm peak}(t_*) & \text{for } t > t_* \end{cases} . \tag{1}$$

The analytic function of $f_{\rm peak}(t)$ consists of two phases: a) a term of linear drift; b) and a constant $f_{\rm peak}$. For every configuration, we perform a fit on the extracted $f_{\rm peak}(t)$ using Eq. (1). Thus, we obtain the parameters $\zeta_{\rm drift}$, t_*, $f_{\rm peak,0}$, which describe and quantify the frequency evolution of the quadrupolar mode (see [37] for more details).

Figure 1 displays the effective GW spectra $h_{\rm eff,+}(f)$, defined as $h_{\rm eff,+}(f) = f \cdot \widetilde{h}_+(f)$ where $\widetilde{h}_+(f)$ is the Fourier transform of $h_+(t)$, for all the models in the sequence of simulations. As shown in Fig. 1, for every model, the frequency peak $f_{\rm peak}$ (indicated by cyan color) is not symmetric. It exhibits an one-sided and broad distribution, which is explained by a time-dependent $f_{\rm peak}(t)$. Using the aforementioned fits of $f_{\rm peak}(t)$ extracted from spectrograms, we find that the frequency range of $f_{\rm peak}(t)$ (cyan-shaded area) is in good agreement with the one-sided peak of the dominant mode.

The GW spectra, as can be seen in Fig. 1, contain several secondary components in addition to the dominant $f_{\rm peak}$ mode. A non-linear coupling between $f_{\rm peak}$ and the quasi-radial oscillation mode f_0, explains two of those, namely f_{2-0} and f_{2+0} (see [32]). Another feature, denoted by $f_{\rm spiral}$, originates from the formation of tidal antipodal bulges in the remnant [41]. We have identified these features in the all the GW spectra considered in this work (see Fig. 1 and [37] for more details). We



(a) **(b)**

Figure 2: *Left panel*: Post-merger GW signal $r \cdot h_+(t)$ from the simulation (black) and analytic model fit for the configuration with $M_{\rm tot} = 2.5\ M_\odot$. *Right panel*: Effective post-merger GW spectra $h_{{\rm eff},+}(f)$ for the simulation (black), for the analytic model $h_+^{\rm Fit}(t)$ (orange) and for the semi-analytic model [37] (cyan) for the configuration with $M_{\rm tot} = 2.5\ M_\odot$. Colored boxes indicate the respective fitting factors FFs. Figures taken from [37].

confirm a smooth transition of the spectral features as the total mass increases (see also [41]). In high-mass configurations, we find a new coupling between the $f_{\rm spiral}$ component and the quasi-radial oscillation f_0 which explains the frequency components at roughly $f_{\rm spiral\pm0} \approx f_{\rm spiral} \pm f_0$ (see purple dashed lines in Fig. 1 and [37]).

4. Analytic model

Furthermore, using the aforementioned spectral analysis, we construct an analytic model for the post-merger GW emission that consists of exponentially decaying sinusoidal functions. Our model is an extension of [31], which included fixed $f_{\rm peak}$, $f_{\rm spiral}$, and f_{2-0}, and of [24], which incorporated a linear time evolution of $f_{\rm peak}(t)$. In our work, we use Eq. (1) to model the time evolution of $f_{\rm peak}(t)$, and also include the frequency component f_{2+0}. This model can be easily modified to include additional frequency components, such as $f_{\rm spiral-0}$. The analytic model reads

$$\begin{aligned} h_+(t) &= A_{\rm peak}\ e^{(-t/\tau_{\rm peak})} \cdot \sin(\phi_{\rm peak}(t)) \\ &+ A_{\rm spiral}\ e^{(-t/\tau_{\rm spiral})} \cdot \sin(2\pi f_{\rm spiral} \cdot t + \phi_{\rm spiral}) \\ &+ A_{2-0}\ e^{(-t/\tau_{2-0})} \cdot \sin(2\pi f_{2-0} \cdot t + \phi_{2-0}) \\ &+ A_{2+0}\ e^{(-t/\tau_{2+0})} \cdot \sin(2\pi f_{2+0} \cdot t + \phi_{2+0}), \end{aligned} \tag{2}$$

where the phase of the $f_{\rm peak}$ component, $\phi_{\rm peak}(t)$, is chosen so the frequency $f_{\rm peak}(t) = \frac{1}{2\pi}\frac{d\phi_{\rm peak}(t)}{dt}$ features a time dependence as in Eq. (1). More information on the description of the analytic model, its implementation, and its parameters can be found in [37].

We perform fits using the analytic model for all the configurations of the mass sequence. Figure 2 displays the fit for the model with total binary mass $M_{\rm tot} = 2.5\ M_\odot$ in the time domain (left panel) and frequency domain (right panel). There is a good agreement between the simulation signal and the analytic model. The time-dependent description of $f_{\rm peak}(t)$ is crucial for ensuring that the signal is well described in the early and late phase of the evolution. Moreover, the one-sided structure of $f_{\rm peak}$ is reproduced remarkably well (see Fig. 2).

The quality of the fits is evaluated using the noise-weighted fitting factor (FF) defined in Eq. (7) and (8) in [37]. For most models, the analytic model achieves FF > 0.95, where FF $= 1$ corresponds to a perfect match. Compared to many models in the literature, those are considered good FFs. Furthermore, we consider simplified analytic models that incorporate fewer secondary components (see [37] for definitions). The performance of the model significantly deteriorates if none of the secondary features is included (see Fig. 14 in [37]). Finally, we note that, although not presented in this work, all parameters of the analytic model follow empirical laws (some less tight than others) as a function of the total binary mass $M_{\rm tot}$ (see [37]).

5. Conclusions

We perform a spectral analysis of the GW post-merger emission for a mass sequence of binary neutron star mergers, and then introduce an analytic model for the GW signal, which uses exponentially decaying sinusoids. We find that the strongest spectral feature, $f_{\rm peak}$, shows a time evolution described by an analytic 2-segment piecewise function. We identified a new coupling mechanism, between $f_{\rm spiral}$ and f_0, which explains additional frequency peaks in the GW spectrum. The analytic model performs well, achieving a good agreement with the simulations (FFs > 0.95) for the majority of the models. Finally, we find that faithful post-merger GW templates should include several frequency components.

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