PROCEEDINGS OF SCIENCE

Analytic treatment of Neutrino oscillation and decay in Matter

Srubabati Goswami ,[a,*] Dibya. S. Chattopadhyay,[b] Kaustav Chakraborty[a] and Amol Dighe[b]

[a] *Physical Research Laboratory, Navrangpura, Ahmedabad 380009, India*

[b] *Tata Institute of Fundamental Research, Homi Bhabha Road, Colaba, Mumbai 400005, India*

E-mail: sruba@prl.res.in, d.s.chattopadhyay@theory.tifr.res.in, amol@tifr.res.in, kaustav.chk@gmail.com

We discuss the propagation of decaying neutrinos in matter and outline a procedure to obtain the analytic oscillation probabilities in two and three generation framework. For unstable neutrinos, the Hermitian and anti-Hermitian components of the effective Hamiltonian do not commute and cannot be simultaneously diagonalised by unitary transformations for all matter densities. We present the compact analytic expressions for neutrino probabilities in matter for the two flavour case, obtained using a re-summation of the inverse Baker-Campbell-Hausdorff (BCH) or Zassenhaus expansion, We also outline briefly how the approximate probabilities for the three flavour case can be obtained assuming only the vacuum mass eigenstate ν_3 decays. We show the baselines and energies where the different approximations give good matching with the numerical probabilities.

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*Speaker

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1. Introduction

Neutrino oscillation experiments have unequivocally established that neutrinos have masses, and their flavours mix. However, the possibility of new physics effects at a sub-leading level cannot be eliminated. In this talk, we discuss the scenario of invisible decay and oscillation of neutrinos.

The effective Hamiltonian for neutrino decay can be represented as $H_m = H_m - i\Gamma_m/2$ with Hermitian matrices H_m and Γ_m corresponding to energy and decay respectively. In general, the mass and decay eigenstates may not be the same and hence are not simultaneously diagonalisable by unitary transformations [1]. Even if the eigenstates are same in vacuum, the matter effect makes the mismatch inevitable. In this talk, we will describe the propagation of unstable neutrinos in matter We work in the basis where the Hermitian part of the Hamiltonian is diagonal, which is the same as the basis of neutrino mass eigenstates in matter in the absence of decay.

2. Two Flavour case

The effective Hamiltonian may be written in the basis of neutrino mass eigenstates in matter as

$$H_m = \begin{pmatrix} a_1 - ib_1 & -\frac{1}{2}i\gamma e^{i\chi} \\ -\frac{1}{2}i\gamma e^{-i\chi} & a_2 - ib_2 \end{pmatrix}, \tag{1}$$

where a_i, b_i, γ, χ are real. Since Γ_m needs to be positive semidefinite, $b_i \geq 0$ and $\gamma^2 \leq 4b_1b_2$.

Note that since $[H_m, \Gamma_m] \neq 0$ in general, $\mathcal{H}_m$ is not a normal matrix, and $e^{-i\mathcal{H}_m t} \neq e^{-iH_m t}e^{-\Gamma_m t/2}$. Thus, one has to express $e^{-i\mathcal{H}_m t}$ in terms of a chain of commutators using the inverse Baker-Campbell-Hausdorff (BCH) formula, also known as the Zassenhaus formula [2]. The survival probability of a neutrino of flavor α is [3]

$$P_{\alpha\alpha} = \frac{e^{-(b_1+b_2)t}}{2}\Big[(1+|A|^2)\cosh(\Delta_b t) + (1-|A|^2)\cos(\Delta_a t) - 2\mathrm{Re}(A)\sinh(\Delta_b t) + 2\mathrm{Im}(A)\sin(\Delta_a t)\Big] \tag{2}$$

where, $Re(A) = -\cos 2\theta_m + \bar{\gamma}\bar{\Delta}_b \sin 2\theta_m \cos\chi \quad Im(A) = -\bar{\gamma}\bar{\Delta}_a \sin 2\theta_m \cos\chi$

$|A|^2 = \cos^2 2\theta_m - 2\bar{\gamma}\bar{\Delta}_b \sin 2\theta_m \cos 2\theta_m \cos\chi; \quad |B|^2 = \sin^2 2\theta_m + 2\bar{\gamma}\sin 2\theta_m\left(\bar{\Delta}_a \sin\chi + \bar{\Delta}_b \cos 2\theta_m \cos\chi\right)$

with $\Delta_a \equiv a_2 - a_1, \quad \Delta_b = b_2 - b_1, \bar{\gamma} \equiv \frac{\gamma}{|\Delta_d|}, \quad \bar{\Delta}_a \equiv \frac{\Delta_a}{|\Delta_d|}, \quad \bar{\Delta}_b \equiv \frac{\Delta_b}{|\Delta_d|}$ ..

For the special case where only the mass eigenstate ν_2 in vacuum decays (with lifetime τ_2), the probabilities in matter may be obtained by the following identifications: $a_{1,2} = \frac{\tilde{m}^2_{1,2}}{2E}, b_{1,2} = \frac{\alpha_2}{4E}[1 \mp \cos[2(\theta - \theta_m)], \chi = 0, \gamma = \frac{\alpha_2}{2E}\sin[2(\theta-\theta_m)]$. Here, $\tilde{m}_i(m_i)$ and $\theta_m(\theta)$ are the mass eigenvalues and mixing angle in matter (vacuum) in, and $\alpha_2 = m_2/\tau_2$. Note that, in matter both states decay.

3. Three flavour case

For three flavours the most general Hamiltonian in matter can be written as,

$$\mathcal{H}_m \equiv H_m - \frac{i}{2}\Gamma_m \equiv \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix} - \frac{i}{2}\begin{pmatrix} 2\gamma_1 & \gamma_{12}e^{i\chi_{12}} & \gamma_{13}e^{i\chi_{13}} \\ \gamma_{12}e^{-i\chi_{12}} & 2\gamma_2 & \gamma_{23}e^{i\chi_{23}} \\ \gamma_{13}e^{-i\chi_{13}} & \gamma_{23}e^{-i\chi_{23}} & 2\gamma_3 \end{pmatrix}. \tag{3}$$

[1]Diagonalisation using non-unitary matrices have been discussed in [1] for propagation in vacuum.

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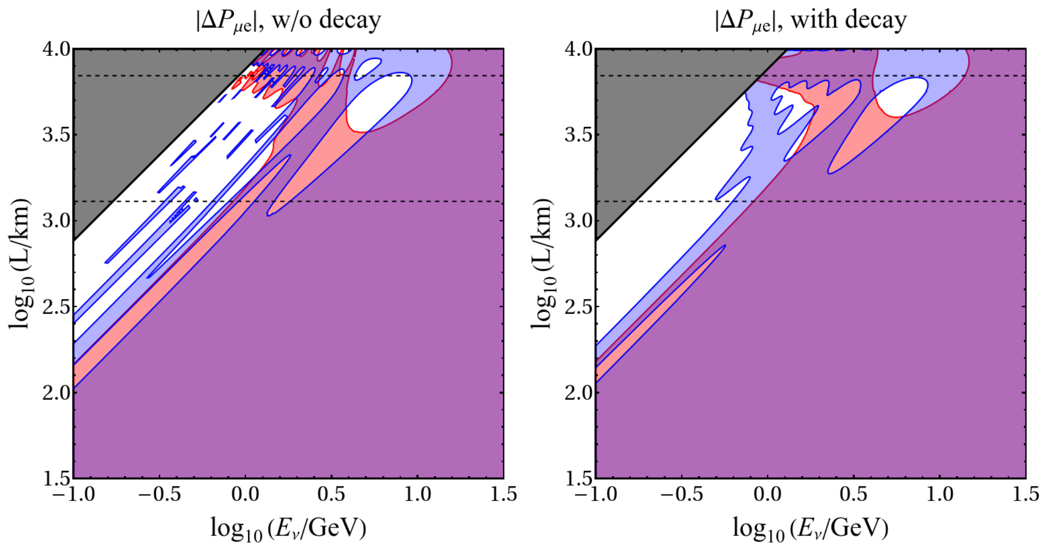

Figure 1: Regions in $(E_\nu - L)$ parameter space where $|\Delta P_{\mu e}| < 1\%$. See text for details.

However, neutrino decay has not been seen yet and it is plausible to take the length-scales governing the decay to be greater by at least a factor of $\sim 1/O(\lambda)$ as compared to the associated oscillation length-scales. In addition, the positive definiteness of the decay matrix Γ would imply $O(\gamma_{ij}^2) \lesssim O(\gamma_i)O(\gamma_j)$. Thus we get, $\gamma_1 \Delta m^2_{31} \lesssim O(\lambda)\Delta m^2_{21}, \gamma_2 \Delta m^2_{31} \lesssim O(\lambda)\Delta m^2_{21}$, $\gamma_3 \Delta m^2_{31} \lesssim O(\lambda)\Delta m^2_{31}$; and therefore, $\gamma_1, \gamma_2 \lesssim O(\lambda^3); \gamma_3 \lesssim O(\lambda), \gamma_{12} \sim O(\lambda^3), \gamma_{13}, \gamma_{23} \sim O(\lambda^2)$; $\Delta m^2_{ij} = m_i^2 - m_j^2$, the difference between the mass squared values.

Thus, it is a good approximation to consider the scenario with only $\gamma_3 \neq 0$ in vacuum. But matter effect can introduce off-diagonal terms. We have obtained analytic expressions of probabilities in matter for this case for – (i) the One Mass Scale Dominance (OMSD) approximation with $\Delta m^2_{21} = 0$; The 2-flavor formalism discussed in the earlier section can be used effectively for this. (ii) Using the Cayley-Hamilton theorem – expanding in terms of $\alpha = \frac{\Delta m^2_{21}}{\Delta m^2_{31}}$, s_{13} and γ_3 as well as those expanded in terms of α, s_{13} and exact in γ_3 [4]. $s_{13} = \sin\theta_{13}$, the 1-3 mixing angle of the PMNS matrix. These probabilities are relevant for long baseline and atmospheric neutrino experiments and can provide new insights. Figure 1 shows regions in the $(E_\nu - L)$ parameter space where $|\Delta P_{\mu e}| < 1\%$ with the OMSD approximation (blue) and for expansion in α, s_{13} and exact in γ_3 (red). In the purple region this condition is true for both methods. In the gray region the analytic approximations are not valid, since $\alpha\Delta > 1$. In the white spaces the analytic approximations are valid but not accurate up to 1%. The horizontal dashed lines denote $L = 1300$ km and $L = 7000$ km.

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