PROCEEDINGS OF SCIENCE

Electroweak input schemes

Mauro Chiesa[a,*]

[a] *Dipartimento di Fisica, Università di Pavia, and INFN, Sezione di Pavia, Via A. Bassi 6, 27100, Pavia, Italy*

E-mail: mauro.chiesa@unipv.it

In the Standard Model, there are three independent parameters for the electroweak sector. In the context of the electroweak corrections for LHC processes, the most common choices of independent electroweak parameters are (G_μ, M_W, M_Z) and $(\alpha(M_Z), M_W, M_Z)$, but many other choices are possible. Though, at a given order in perturbation theory, all schemes are formally equivalent, the corresponding numerical predictions differ because of the truncation of the perturbative expansion. It is thus important to know the strengths and the weaknesses of the possible input parameter schemes in order to choose the most suited for the calculation at hand.

The Tenth Annual Conference on Large Hadron Collider Physics - LHCP2022
16-20 May 2022
online

*Speaker

https://pos.sissa.it/

PoS(LHCP2022)174

Many parameters appear in the Lagrangian of the Standard Model (SM), ranging from the electric charge (often traded for α_0, $\alpha(M_Z)$, or G_μ when performing calculations), to the gauge-boson and Higgs masses, from the sine and cosine of the weak-mixing angle, to the fermion masses. It turns out that only three of these parameters in the electroweak sector are actually independent: when their numerical values are known (together with the fermion masses or Yukawas and, for instance, the Higgs mass), the other parameters can be computed from the independent ones.

From a theoretical point of view, all input parameter schemes are completely equivalent at a given order in perturbation theory, however, the numerical predictions computed in the different schemes will differ because of the truncation of the perturbative expansion. On the one hand, the fact that the differences in the predictions obtained with different schemes are higher-order effects tells us that they can be taken as estimates of the theory uncertainties coming from missing higher-order corrections (though this is likely to be an overestimate). Another possibility is to study the main features and drawbacks of the possible schemes in order to select the ones which are the most suited depending on the calculation at hand and depending on the use that we want to make of the results of such calculation. Adopting this second approach, the following points might be useful criteria to consider for the choice of an electroweak (EW) input parameter scheme.

- Precise experimental knowledge of the three independent parameters, as the experimental uncertainties on the input parameters become parametric theoretical uncertainties in the predictions computed with the input scheme under consideration.
- Rate of the perturbative convergence, since those schemes in which the EW corrections up to a given order are large will reasonably develop large higher-order corrections and thus the corresponding predictions will be affected by large theoretical uncertainties from missing higher-order effects.
- Other parametric uncertainties: typical examples are the top-quark mass dependence or the treatment of light-quark masses.
- In some contexts, like the one of template fits, one needs to vary a specific parameter in the theoretical predictions and thus a scheme where that parameter is among the three independent ones should be used.

The most commonly used input parameter schemes for theory predictions at LHC, in particular for the calculation of NLO EW corrections in the SM, have the W and Z boson masses among the independent parameters, while the third one is some variant of the electric charge (usually rewritten in terms of G_μ or $\alpha(M_Z)$, or also α_0) [1]. It is instructive to analyze these schemes in the light of the criteria listed above. First of all, the choice of the variant of α has an impact on the size of the corrections (for concreteness, in the following we refer to the NLO EW corrections in the on-shell scheme). The predictions in the α_0 scheme are affected by large logarithmic corrections $\sim \log \frac{m_f^2}{M_Z^2}$ (f being a light fermion) related to the electric-charge counterterm δZ_e and arising from the running of α from the Thomson limit to the weak scale. When $\alpha(M_Z)$ or G_μ are used, these logarithmic corrections cancel in the shifts $\delta Z_e \to \delta Z_e - \frac{1}{2}\Delta\alpha$ and $\delta Z_e \to \delta Z_e - \frac{1}{2}\Delta r$ which basically account

[1] A discussion on electroweak input schemes in the context of the SM effective field theory can be found in [1].

for the fact that these couplings are already defined at the weak scale (see, for instance, [2, 3]). For processes involving external photons at LO, fermion-mass logarithms cancel if one uses one factor of α_0 for each external γ and G_μ or $\alpha(M_Z)$ for the remaining vertices ([4], Sect. 0.2).

Concerning the experimental accuracy on the input parameters, predictions in the $(\alpha_0/\alpha(M_Z)/G_\mu, M_W, M_Z)$ schemes have relatively large parametric uncertainties related to the W-boson mass, given the relative accuracy on $M_W \sim 10^{-4}$, compared to the ones on $M_Z \sim 10^{-5}$, $G_\mu \sim 10^{-7}$, or $\alpha_0 \sim 10^{-10}$.

Having M_W as a free parameter is however crucial in the context of the direct determination of the W-boson mass at hadron colliders by means of template fits. In this approach, one measures kinematic distributions of the W decay products in charged-current Drell-Yan, then generates many Monte Carlo samples with different nominal values of M_W, and the measured value of M_W is the one of the sample in best agreement with the data: it is thus clear that M_W has to be an independent parameter in the generation of the templates.

An interesting example of parametric uncertainties entering at 1-loop level in the (α_0, M_W, M_Z) scheme is the one related to the treatment of light quarks[2]. In fact, in this scheme the corrections are proportional to $\delta Z_e \sim 2\Delta\alpha$, $\Delta\alpha$ being the running of α from $q^2 = 0$ to the weak scale. The hadronic part of $\Delta\alpha$ is related to the hadronic corrections to the derivative of the γ 2-point function at $q^2 = 0$ and thus cannot be computed perturbatively. One possible way-out is to calculate this contribution as if it was perturbative, so that $\Delta\alpha^{\mathrm{hadr}} \sim \log \frac{m_q^2}{M_Z^2}$, where the light-quark masses are just effective parameters tuned in such a way to reproduce the measured value of $\Delta\alpha(M_Z^2)$. Another possibility is to derive $\Delta\alpha^{\mathrm{hadr}}$ from the data for inclusive hadroproduction in e^+e^- collisions via dispersion relations [5–9]. In both cases, the experimental uncertainties on the data propagate as theory uncertainties in the predictions obtained with the (α_0, M_W, M_Z) scheme (in the former case, another source of uncertainty is the use if the quark mass values tuned at $q^2 = M_Z^2$ for scales different from the Z mass squared). For a recent review on the determination of $\Delta\alpha^{\mathrm{hadr}}$ including also the results from lattice calculations, we refer to [10].

Besides the $(\alpha_0/\alpha(M_Z)/G_\mu, M_W, M_Z)$ schemes, there are other possible choices of electroweak input parameters, for instance the ones using two couplings and one mass as independent quantities. One example is the (α_0, G_μ, M_Z) scheme, widely used at LEP I, which is characterized by small parametric uncertainties, due to the fact that α_0, G_μ, and M_Z are measured at high precision. In this scheme, however, the corrections tend to be somewhat large, because of the use of α_0. Other examples are the $(\alpha_0/\alpha(M_Z)/G_\mu, \sin^2\theta^l_{W,\mathrm{eff}}, M_Z)$ schemes [11], which have the sine of the effective leptonic weak-mixing angle as an independent parameter and were specifically designed for the direct determination of $\sin^2\theta^l_{W,\mathrm{eff}}$ at hadron colliders from neutral-current Drell-Yan production by means of template fit procedures.

As a last comment, the electroweak input parameter schemes discussed above use parameters defined in the on-shell (OS) scheme. However, it is also possible to perform calculations in the modified minimal-subtraction ($\overline{\mathrm{MS}}$) or mixed OS-$\overline{\mathrm{MS}}$ schemes and adopt an electroweak input choice where the free parameters are defined according to the $\overline{\mathrm{MS}}$ scheme. In this case, the actual input parameters will be the numerical values of the $\overline{\mathrm{MS}}$ quantities at a given renormalization scale.

[2]Also in this case, the picture for processes involving external photons at LO is different.

Acknowledgements

I would like to thank the organizers for their kind invitation.

References

[1] I. Brivio, S. Dawson, J. de Blas, G. Durieux, P. Savard, A. Denner, A. Freitas, C. Hays, B. Pecjak and A. Vicini, "Electroweak input parameters," [arXiv:2111.12515 [hep-ph]].

[2] S. Dittmaier and M. Huber, "Radiative corrections to the neutral-current Drell-Yan process in the Standard Model and its minimal supersymmetric extension," JHEP **01** (2010), 060 doi:10.1007/JHEP01(2010)060 [arXiv:0911.2329 [hep-ph]].

[3] A. Denner, "Techniques for calculation of electroweak radiative corrections at the one loop level and results for W physics at LEP-200," Fortsch. Phys. **41** (1993), 307-420 doi:10.1002/prop.2190410402 [arXiv:0709.1075 [hep-ph]].

[4] J. R. Andersen, S. Badger, L. Barze, J. Bellm, F. U. Bernlochner, A. Buckley, J. Butterworth, N. Chanon, M. Chiesa and A. Cooper-Sarkar, *et al.* "Les Houches 2013: Physics at TeV Colliders: Standard Model Working Group Report," [arXiv:1405.1067 [hep-ph]].

[5] C. Bouchiat and L. Michel, "La résonance dans la diffusion méson π— méson π et le moment magnétique anormal du méson μ," J. Phys. Radium **22** (1961) no.2, 121-121 doi:10.1051/jphysrad:01961002202012101

[6] L. Durand, "Pionic Contributions to the Magnetic Moment of the Muon," Phys. Rev. **128** (1962), 441-448 doi:10.1103/PhysRev.128.441

[7] M. Gourdin and E. De Rafael, "Hadronic contributions to the muon g-factor," Nucl. Phys. B **10** (1969), 667-674 doi:10.1016/0550-3213(69)90333-2

[8] K. Hagiwara, A. D. Martin, D. Nomura and T. Teubner, "Predictions for g-2 of the muon and alpha(QED) (M**2(Z))," Phys. Rev. D **69** (2004), 093003 doi:10.1103/PhysRevD.69.093003 [arXiv:hep-ph/0312250 [hep-ph]].

[9] F. Jegerlehner, "Hadronic Contributions to Electroweak Parameter Shifts: A Detailed Analysis," Z. Phys. C **32** (1986), 195 doi:10.1007/BF01552495

[10] T. Aoyama, N. Asmussen, M. Benayoun, J. Bijnens, T. Blum, M. Bruno, I. Caprini, C. M. Carloni Calame, M. Cè and G. Colangelo, *et al.* "The anomalous magnetic moment of the muon in the Standard Model," Phys. Rept. **887** (2020), 1-166 doi:10.1016/j.physrep.2020.07.006 [arXiv:2006.04822 [hep-ph]].

[11] M. Chiesa, F. Piccinini and A. Vicini, "Direct determination of $\sin^2\theta^{\ell}_{eff}$ at hadron colliders," Phys. Rev. D **100** (2019) no.7, 071302 doi:10.1103/PhysRevD.100.071302 [arXiv:1906.11569 [hep-ph]].