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Intrinsic quantum mechanics behind the Standard Model? - predictions in the baryon and Higgs sectors

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I introduce quantum mechanics on an intrinsic configuration space for baryons, the Lie group U(3), which carries the three gauge groups of the standard model of particle physics as subgroups SU(3), SU(2) and U(1). The strong and electroweak interactions become related via the Higgs mechanism. I namely settle the electroweak energy scale by the neutron to proton decay where both sectors are involved through quark flavour changes. Predictions of neutral pentaquark resonances reachable at LHCb follow in the baryon sector as does an accurate expression in the electroweak sector for the Higgs mass (yielding 125.104(14) GeV) and predictions on the couplings of the Higgs to itself and to the gauge bosons with signal strengths deviating by the presence of the up down quark mixing matrix element. The intrinsic view means that quantum fields are generated by the momentum form on intrinsic wavefunctions and local gauge transformations in laboratory space equate translations in the intrinsic configuration space which may be likened to a generalised spin space. Further insight is gained for the Cabibbo and Weinberg angles expressed in traces of u and d flavour quark generators.

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1. Introduction

The present proceedings contribution sums up work by the author to go behind the Standard Model of particle physics. It wants to understand wherefrom the structure, the gauge groups, the energy scales and other fundamental parameters originate. It does not want to go beyond the Standard Model in the sense of predicting brand new particles. Rather it wants to understand the structure of the Standard Model [1] as a quantum field theory with its specific gauge groups originating in an intrinsic configuration space for baryon mass eigenstates.

It offers results on the baryon spectrum [2], the Higgs mass [3, 4], its self coupling and coupling to gauge bosons [5]. It gives insight to the Cabibbo and Weinberg angles [6] and possibly an explanation for the values of the u and d quark masses [2]. These results cannot be derived within the Standard Model, where they all rely on experimental input. The experimental inputs for the present work, however, are limited to the electron mass and the sliding scale fine structure coupling α (depending though on fermion mass inputs). The major results then more or less follow from the specific choice of the Lie group U(3) as configuration space for baryonic mass eigenstates. This configuration space contains all three gauge groups SU(3), SU(2) and U(1) as subspaces. The Higgs mechanism is linked to a symmetry break in the strong interaction sector which selects the subspace U(2) spontaneously and shapes the Higgs potential from within the U(3) configuration in the baryon sector. This link settles the electroweak energy scale by the neutron to proton decay where both strong and electroweak sectors are involved through quark flavour changes.

2. The baryon sector

We consider baryons to be eigenstates of the following mass Hamiltonian [2, 7, 8, 9] with $mc^2 = \mathscr{E}$

$$\frac{\hbar c}{a}\left[-\frac{1}{2}\Delta+\frac{1}{2}\mathrm{Tr}\,\chi^2\right]\Psi(u)=\mathscr{E}\Psi(u),\quad u=e^{i\chi}\in U(3). \tag{2.1}$$

The configuration variable u is generated by nine kinematic generators T_j, S_j, M_j, thus

$$\chi=\left(a\theta_j p_j+\alpha_j S_j+\beta_j M_j\right)/\hbar,\qquad p_j\equiv -i\hbar\frac{1}{a}\frac{\partial}{\partial\theta_j}=\frac{\hbar}{a}T_j,\quad \theta_j,\alpha_j,\beta_j\in\mathbb{R},\, j=1,2,3 \tag{2.2}$$

where $e^{i\theta_j}$ are the three eigenvalues of u and θ_j are dynamical angular variables and a is a length scale. We interpret the three toroidal dimensions as colour degrees of freedom. The toroidal generators T_j for the abelian subspace $diag\left(e^{i\theta_1}, e^{i\theta_2}, e^{i\theta_3}\right)$ are conjugate to the angular variables, i.e.

$$[iT_j,\theta_i]=\delta_{ij}\sim \mathrm{d}\theta_i(\partial_j)=\delta_{ij},\qquad \partial_j|_u=uiT_j, \tag{2.3}$$

where ∂_j are left invariant coordinate fields on $U(3)$ and $\mathrm{d}\theta_j$ are corresponding coordinate forms, see e.g. pp. 84 in [10]. We see the T_js as generated from laboratory space by the three momentum operators p_j. Likewise the spin degrees of freedom are generated by the set of three S_js, e.g. $S_3 = a\theta_1 p_2 - a\theta_2 p_1$, and the mixing operators M_j contain flavour degrees of freedom mixed with spin. The configuration variable $u \in U(3)$ may thus be seen as a generalised intrinsic spin variable [11]. Equation (2.1) is solved accurately in [2] for neutral electric charge states yielding an accurate neutron mass value, and solved approximately for charged states with a promising neutral flavour baryon spectrum and a promising neutron to proton mass shift together with a proton spin structure function that matches the experimental data over four orders of magnitude, see fig. 1. Further, scarce neutral flavour neutral charge resonances are predicted that might be reachable at LHCb.

3. Neutron mass and decay - Higgs mass and couplings

The neutral charge ground state of (2.1) is identified with the neutron [7]. In [2] we present in details the solution by a Rayleigh-Ritz method (diagonalisation of the Hamiltonian) that yields

$$m_n c^2 = \mathrm{E}\Lambda = \mathrm{E}\frac{\pi}{\alpha(m_n)} m_e c^2 = 939.9(5)\ \mathrm{MeV} \approx 939.56542052(54)\ \mathrm{MeV}\ [1]. \tag{3.1}$$

The energy scale $\Lambda = \hbar c/a = \frac{\pi}{\alpha} m_e c^2 \approx 214$ MeV in (2.1) is set by the length scale a (toroidal radius) that projects to the classical electron radius r_e [12, 13] as $\pi a = r_e$ [7]. The dimensionless eigenvalue $\mathrm{E} \equiv \mathscr{E}/\Lambda$ is found quite accurately as $\mathrm{E} = 4.382(2)$ since the integrals needed for diagonalising the Hamiltonian can be found analytically. The same length scale a is involved in the relation from laboratory space coordinates x_j to intrinsic configuration space toroidal angles

$$\theta_j = x_j/a \tag{3.2}$$

and thus sets the scale of the neutral flavour baryon spectrum.

The neutron decay

$$n \rightarrow p + e + \bar{\nu}_e \tag{3.3}$$

offers a common ground for the strong and electroweak interactions since the baryonic state undergoes a change when a d-quark is transformed into a u-quark. We interpret the charged ground state of (2.1) as the proton. It involves period doublings in the toroidal angles interpreted as topological charge. The topological change in the wavefunction is carried by Bloch wave phase factors [14] allowed by the periodicity of the potential in (2.1) and by degrees of freedom opened by the Higgs mechanism. The Higgs potential V_H is scaled by an exchange of one unit of (space) action hc from the strong interaction sector to the electroweak sector [3, 4] with Higgs field vacuum expectation value φ_0 during the period doubling [7] from 2π periodicity to 4π periodicity in the strong sector

$$2\pi\Lambda = \alpha\varphi_0 \sim hc = \alpha\varphi_0 a. \tag{3.4}$$

The neutron decay involves a virtual W^- boson for the d to u quark transformation and we choose the fine structure coupling scale of α in (3.4) accordingly. We then have

$$v/\sqrt{2} \equiv \varphi_0 = \frac{2\pi}{\alpha(m_w)}\Lambda = \frac{2\pi}{\alpha(m_w)}\frac{\pi}{\alpha_e} m_e c^2 = 176.924(20)\ \mathrm{GeV}. \tag{3.5}$$

With Fermi constants $G_{F\beta} = G_{F\mu}|V_{ud}|$, the electroweak energy scale of the Standard Model $v_{\mathrm{SM}} \approx v\sqrt{|V_{ud}|} = 246.86(5)$ GeV $\approx 246.21964(6)$ GeV [1]. Shaping V_H troughs by $\frac{1}{2}\mathrm{Tr}\,\chi^2$ in (2.1) [3, 4]

$$\mu^2 = \frac{1}{2}\varphi_0^2 \rightarrow m_H c^2 = \mu = \frac{1}{\sqrt{2}}\varphi_0 = 125.104(14)\ \mathrm{GeV} \approx 125.25(17)\ \mathrm{GeV}\ [1]. \tag{3.6}$$

From the Higgs potential fitting to $\frac{1}{2}\mathrm{Tr}\,\chi^2$ and from the Higgs mechanism we further predict [5, 15] the Higgs to gauge boson $V = W, Z$ couplings and the Higgs self couplings to expect signal strengths

$$\mu_{HVV} = \left(\frac{g_{HVV}}{g_{HVV,SM}}\right)^2 = \left(\frac{v}{v_{SM}}\right)^2 = \frac{1}{|V_{ud}|} \approx 1.03,\ \ \mu_{HHHH} = |V_{ud}|^2 \approx \frac{1}{1.06},\ \ \mu_{HHH} = |V_{ud}| \approx \frac{1}{1.03}. \tag{3.7}$$

4. Quark fields and masses, proton spin structure, Cabibbo and Weinberg angles

Quark and gluon fields are created when the momentum form of either the toroidal measure-scaled wavefundtion R acts on combinations of toroidal generators [7] or when the momentum form $\mathrm{d}\Phi$ of the full measure-scaled wavefunction $\Phi = J\Psi \equiv R(\theta_1,\theta_2,\theta_3)\Upsilon(\alpha_1,\alpha_2,\alpha_3,\beta_1,\beta_2,\beta_3)$ acts on the eight Gell-Mann generators λ_k [2, 17]. Here $J(\theta_1,\theta_2,\theta_3)$ is a Jacobian in the Laplacian Δ in (2.1) [16]. Thus, colour fields ψ_j, flavour fields ψ_q and gluon fields G_k are generated as

$$\psi_j(u) = \mathrm{d}R_u(iT_j) \equiv \frac{\mathrm{d}}{\mathrm{d}\theta} R(ue^{\theta iT_j})|_{\theta=0}, \qquad \psi_q(u) = \mathrm{d}R_u(iT_q), \;\; G_k(u) = \mathrm{d}\Phi_u(i\lambda_k/2). \tag{4.1}$$

The flavour quark generators T_q of the first two generations are

$$T_u = \frac{2}{3}T_1 - T_3, \quad T_d = -\frac{1}{3}T_1 - T_3, \quad T_s = -\frac{1}{3}T_1, \quad T_c = \frac{2}{3}T_1. \tag{4.2}$$

Since the wavefunction in (2.1) lives on the compact configuration space $U(3)$, the *quarks and gluons are confined per construction*. Using the u and d flavour quark generators from (4.2) for parton distributions, we get the proton spin structure function shown as the solid curve in figure 1.

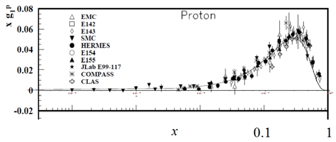

Figure 1: Spin structure function of the proton (solid line) - with no fitting parameters - from an exemplar calculation [2] based on flavour quarks derived from an intrinsic $U(3)$ configuration overlaid on experimental data [18] spanning four orders of magnitude in parton momentum fraction x. Figure from [2].

We suggest u and d masses to reflect intrinsic Gaussian curvatures $K_q = 1/r_q^2$ on the configuration space weighted by the probability density in the intrinsic protonic wavefunction R. Thus - in analogy with $m_e c^2 = \frac{e^2}{4\pi\varepsilon_0}\frac{\hbar c}{r_e}$ - averaging over three colours - we get [2]

$$m_u c^2 = \frac{(g_s/3)^2}{4\pi}\frac{\hbar c}{r_u} = 4.0(2)\ \mathrm{MeV}, \;\; m_d c^2 = \frac{\alpha_s}{9}\frac{\hbar c}{r_d} = 8.7(3)\ \mathrm{MeV} \;\; @\ \alpha_s(2\,\mathrm{GeV}) = 0.305\ [1]. \tag{4.3}$$

When the sum over colour states is made explicit [19] in the amplitude for the strange decay $\Lambda \to p + \pi^-$, we derive for the Cabibbo angle θ_C and the Weinberg angle θ_W respectively [6]

$$\sin\theta_C = \mathrm{Tr}\, T_u^\dagger T_s = -\frac{2}{9} \to \cos\theta_C = \sqrt{\frac{77}{81}} \approx |V_{ud}|, \;\; \cos^2\theta_W = \mathrm{Tr}\, T_u^\dagger T_d = \frac{7}{9} \approx 0.7769(3) = \frac{m_W^2}{m_Z^2}\ [1]. \tag{4.4}$$

Left invariant translations from the *origo e* in the intrinsic configuration space equates local gauge transformations in laboratory space [2, 20, 21] (in [21] less abstractly than here from [2])

$$\psi_j(u) = \partial_j|_u[\Phi] = u\partial_j|_e[\Phi] = u\psi_j(e). \tag{4.5}$$

5. Conclusion

A wide list of results and new interpretations follow from intrinsic quantum mechanics.

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