

Theoretical analysis of $B^0 \rightarrow \phi \ell^+ \ell^-$ decay

Alexander Parkhomenko^a and Irina Parnova^{a,*}

^aDepartment of Theoretical Physics, P. G. Demidov Yaroslavl State University, Sovietskaya 14, 150003 Yaroslavl, Russia

E-mail: parkh@uniyar.ac.ru, parnova.irina@yandex.ru

In the Standard Model (SM), the $b \to s(d)$ flavor-changing neutral currents, being loop-induced, are standard experimental channels for testing the SM precisely and searching for possible physics beyond the SM. Pure annihilation *B*-meson decays originating by these currents are of significant interest as they are extremely suppressed in the SM and New Physics effects can increase substantially their decay widths. Typical examples of these annihilation processes are radiative and semileptonic decays with ρ^0 -, ω -, and ϕ -production like $B_s^0 \to \rho^0(\omega) \gamma$, $B_s^0 \to \rho^0(\omega) \ell^+ \ell^-$, $B^0 \to \phi \gamma$, and $B^0 \to \phi \ell^+ \ell^-$, where $\ell = e, \mu$ is the charged lepton. At the beginning of 2022, the LHCb Collaboration presented the upper limit on the $B^0 \to \phi \mu^+ \mu^-$ decay branching fraction $\mathcal{B}_{exp}(B^0 \to \phi \mu^+ \mu^-) < 3.2 \times 10^{-9}$, and it is important to have a precise SM prediction for this decay. Here, we present theoretical predictions for $B^0 \to \phi \ell^+ \ell^-$ branching fraction in the lepton-pair invariant mass range 1 GeV² < $q^2 < 8$ GeV², so far without taking into account $\omega - \phi$ mixing effect. The main goal is to study a dependence of the branching fraction on the choice of the *B*-meson distribution amplitude model. Theoretical prediction for the total branching fraction $\mathcal{B}_{th}(B^0 \to \phi \ell^+ \ell^-) \sim 10^{-12}$, being an order of magnitude estimate, is far below the LHCb experimental limit.

The Tenth Annual Conference on Large Hadron Collider Physics - LHCP2022 16-20 May 2022 online

*Speaker

[©] Copyright owned by the author(s) under the terms of the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License (CC BY-NC-ND 4.0).

Irina Parnova

1. Introduction

Rare radiative and semileptonic *B*-decays are very interesting to study theoretically and experimentally, since they provide an excellent possibility to constrain Standard Model parameters, to test New Physics models, and to understand QCD factorization of the decay amplitudes. At present a majority of experimental data on *B*-meson decays is coming from the Belle II and LHCb collaborations. There are only two experimental limits on the branching fraction of pure-annihilation semileptonic *B*-meson decays: $\mathcal{B}(B^0 \to \phi\gamma) < 1.0 \times 10^{-7}$ by the Belle collaboration of 2016 [1] and $\mathcal{B}(B^0 \to \phi\mu^+\mu^-) < 3.2 \times 10^{-9}$ by the LHCb of 2022 [2].

Theoretical analysis of radiative annihilation-type *B*-meson decays $B^0 \rightarrow \phi \gamma$ and $B_s \rightarrow \rho^0(\omega)\gamma$, including the $\omega - \phi$ mixing effect was undertaken in [3], where predictions for the B_s -meson indicate a significant contribution from the mixing. Here, we consider its semileptonic counterpart, $B^0 \rightarrow \phi \ell^+ \ell^-$ decay, and present SM predictions, so far, without taking into account $\omega - \phi$ mixing. We estimate also a dependence on the choice of theoretical models for the *B*-meson distribution amplitudes entering the decay width through their first inverse moments.

2. Theoretical analysis of $B^0 \rightarrow \phi \ell^+ \ell^-$ decay

Analysis is done in the Effective Electroweak Hamiltonian approach [4, 5]. The effective Lagrangian density for $b \rightarrow d$ flavor-changing neutral current (FCNC) is derived from the SM by integrating out heavy particles — the top quark, W-, Z- and Higgs bosons. This effective theory does not contain also photons and gluons with energies of the order of heavy particle masses. The effective Lagrangian density includes the QED and QCD Lagrangians and Effective Electroweak Hamiltonian:

$$\mathcal{L}_{\text{eff}}(x) = \mathcal{L}_{\text{QED}}(x) + \mathcal{L}_{\text{QCD}}(x) - \mathcal{H}_{\text{weak}}^{b \to d}(x).$$
(1)

The FCNC term, $\mathcal{H}_{\text{weak}}^{b \to d}(x)$, describes the $b \to d$ transition:

$$\mathcal{H}_{\text{weak}}^{b \to d} = -\frac{4G_F}{\sqrt{2}} \sum_{p=u,c} V_{pd}^* V_{pb} \sum_j C_j(\mu) \mathcal{P}_j(\mu) + \text{h. c.}, \qquad (2)$$

where G_F is the Fermi constant, $C_j(\mu)$ are Wilson coefficients, and $\mathcal{P}_j(\mu)$ are local $b \to d$ transition operators. The $\mathcal{P}_j(\mu)$ operator basis includes 10 operators [5]. The leading-order contribution to the $B^0 \to \phi \ell^+ \ell^-$ decay amplitude is given by the following penguin operators:

$$\mathcal{P}_{3} = (\bar{d}\gamma_{\mu}Lb)\sum_{q}(\bar{q}\gamma^{\mu}q), \qquad \mathcal{P}_{5} = (\bar{d}\gamma_{\mu}\gamma_{\nu}\gamma_{\rho}Lb)\sum_{q}(\bar{q}\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}q), \qquad (3)$$

where $L = (1 - \gamma_5)/2$ is the fermionic left-handed projector and summation is over all the quark flavors except the *t*-quark. On the tree level, the $B^0 \rightarrow \phi \ell^+ \ell^-$ amplitude is described by 8 diagrams. The largest contribution is from the diagram with the ϕ -meson emission from the light quark line in the *B*-meson (left diagram in Fig. 1). The diagram with the meson emission from the *b*-quark (right diagram in Fig. 1) is $1/m_b$ suppressed. Contributions of other 6 diagrams are also suppressed either by the fine-structure constant, α , or its power-suppressed combination, α/m_b .

As follows from the diagrams in Fig. 1, the current with the *b*- and *d*-quarks determines the initial *B*-meson, and the other, with *s*-quarks, is hadronized into the final ϕ -meson. Consequently,



Figure 1: Diagrams given the largest contribution to the $B^0 \rightarrow \phi \ell^+ \ell^-$ amplitude on the tree level

the amplitude of the process is factorized into the product of the ϕ -meson decay constant, f_{ϕ} , and the *B*-meson wave-function defined on the light cone. In lowest Fock state, this wave function is determined by two *B*-meson distribution amplitudes (DAs), $\tilde{\varphi}^B_+(t)$ and $\tilde{\varphi}^B_-(t)$, through the transition matrix element from the meson state to the vacuum [6, 7]:

$$\langle 0|q_{\alpha}(z) E(0,z) h_{\nu,\beta}(0)|\bar{B}(\nu)\rangle = -\frac{if_B m_B}{4} \left[(1+\hat{\nu}) \left\{ \tilde{\varphi}^B_+(t) - \left[\tilde{\varphi}^B_+(t) - \tilde{\varphi}^B_-(t) \right] \frac{\hat{z}}{2t} \right\} \gamma_5 \right]_{\beta\alpha}, \quad (4)$$

where m_B and f_B are the *B*-meson mass and decay constant, respectively, z^{μ} is a light-like separation between quarks under the assumption of the massless *d*-quark, $v^{\mu} = (1, 0, 0, 0)$ is the *B*-meson fourvelocity in its rest frame, t = (vz) is the *B*-meson proper time, $\hat{v} = \gamma_{\mu}v^{\mu}$, $\hat{z} = \gamma_{\mu}z^{\mu}$, and E(0, z) is the Wilson line which ensures the gauge invariance of the operator. The amplitudes of the process considered include Fourier transforms of the DAs [6, 8]. In the Wandzura-Wilczek approximation, the subleading DA, $\phi_{-}^{B}(\omega)$, is related to the leading one, $\phi_{+}^{B}(\omega)$, by the relation [6, 7]:

$$\tilde{\varphi}^{B}_{\pm}(t) = \int_{0}^{\infty} d\omega \,\mathrm{e}^{-i\,\omega t} \phi^{B}_{\pm}(\omega), \qquad \phi^{B}_{-}(\omega) = \int_{\omega}^{\infty} \frac{\phi^{B}_{\pm}(\omega')}{\omega'} \,d\omega'. \tag{5}$$

So, if the leading DA is known, the subleading one can be easily calculated. The $B^0 \rightarrow \phi \ell^+ \ell^-$ differential branching fraction can be written as follows:

$$\frac{d\mathcal{B}}{dq^2} = \tau_B \frac{G_F^2 |V_{td}^* V_{tb}|^2 \alpha^2}{216\pi} m_B f_B^2 f_{\phi}^2 Q_d^2 \lambda^3 (1, m_{\phi}/m_B, \sqrt{q^2}/m_B) \\ \times |C_3 + 4C_5|^2 \left[\left| \lambda_{B,-}^{-1}(q^2) \right|^2 + \frac{m_{\phi}^2}{q^2 \left(1 - q^2/m_B^2\right)^2} \left| \lambda_{B,+}^{-1}(q^2) \right|^2 \right], \tag{6}$$

where τ_B is the *B*-meson mean life, m_{ϕ} is the ϕ -meson mass, $Q_d = -1/3$ is the *d*-quark relative charge, $\lambda(a, b, c)$ is the kinematical function. The differential branching fraction (6) also depends on the first inverse moments, $\lambda_{B,\pm}^{-1}(q^2)$, of the *B*-meson DAs, $\phi_{\pm}^B(\omega)$:

$$\lambda_{B,\pm}^{-1}(q^2) = \int_0^\infty \frac{\phi_{\pm}^B(\omega)d\omega}{\omega - q^2/m_B - i\epsilon},\tag{7}$$

where q^2 is the momentum squared of the lepton pair. Their q^2 -dependence is determined by a choice of DA theoretical models. Here, we use two DA models — the Exponential model [6], which is also known as the Grozin-Neubert (GN) model, and Linear model [9], sometimes called the KKQT-model. The corresponding q^2 -dependence of the first inverse moments (7) for each type of the model is shown in Fig. 2.



Figure 2: First inverse moments of DAs in the Exponential (left plot) and Linear (right plot) models

3. Numerical results

Experimentally, partially integrated branching fractions are measured:

$$\Delta \mathcal{B}(q_{\min}^2 < q^2 < q_{\max}^2) = \int_{q_{\min}^2}^{q_{\max}^2} \frac{d\mathcal{B}}{dq^2} dq^2.$$
(8)

The differential branching fraction (6) includes the q^2 -dependent first inverse moments of the *B*-meson, the explicit form of which is fixed by the choice of the DA model. For exponential (GN) and linear (KKQT) models, the partially integrated branching fractions in the range $q^2 \in [1.0, 8.0]$ GeV², which is restricted by the light vector mesons from below and J/ψ -meson by above, are as follows:

$$\Delta \mathcal{B}^{\text{GN}}(1 \text{ GeV}^2 < q^2 < 8 \text{ GeV}^2) = \left(2.14^{+1.57}_{-0.96}\right) \times 10^{-13},$$

$$\Delta \mathcal{B}^{\text{KKQT}}(1 \text{ GeV}^2 < q^2 < 8 \text{ GeV}^2) = \left(3.88^{+2.85}_{-1.75}\right) \times 10^{-13}.$$
 (9)

One can see that the uncertainties in both predictions are quite large. The central values demonstrate a strong dependence on the choice of the DA model but well coincide with each other within the present errors. The difference in the model predictions is of the order of the factorization scale uncertainty. The estimation of the total branching fraction due to the perturbative contribution only is as follows:

$$\mathcal{B}_{\rm th}(B^0 \to \phi \, \ell^+ \ell^-) \sim 10^{-12},$$
 (10)

which is three orders of magnitude lower than the upper limit obtained by the LHCb [2].

4. Summary and outlook

In the paper, rare annihilation-type semileptonic decays $B^0 \rightarrow \phi \ell^+ \ell^-$ are considered, where $\ell = e, \mu$, within the framework of the Effective Electroweak Hamiltonian approach and in the approximation of the hadron factorization in the decay amplitude. For the models of the *B*-meson distribution amplitudes, the two simplest — exponential and linear are selected, and their first inverse moments are calculated. Theoretical predictions for the branching fraction are presented in the leading order in α_s and $1/m_b$. The total branching fraction $Br(B^0 \rightarrow \phi \ell^+ \ell^-) \sim 10^{-12}$ is three orders of magnitude lower than the limit obtained by the LHCb collaboration. However, there are mechanisms like $\omega - \phi$ mixing that allow to increase the branching fraction but its numerical impact requires an additional study.

Irina Parnova

Acknowledgments

This research is supported by the Russian Science Foundation (Project № 22-22-00877, https://rscf.ru/project/22-22-00877/).

References

- [1] Belle collaboration, *Search for the decay* $B^0 \rightarrow \phi \gamma$, *Phys. Rev. D* **93** (2016) 111101 [1603.06546].
- [2] LHCB collaboration, Search for the decay $B^0 \rightarrow \phi \mu^+ \mu^-$, JHEP 05 (2022) 067 [2201.10167].
- [3] H. Deng et al., *Study on pure annihilation type* $B \rightarrow V\gamma$ *decays, Phys. Rev. D* **103** (2021) 076004 [2101.01344].
- [4] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Weak decays beyond leading logarithms, *Rev. Mod. Phys.* 68 (1996) 1125 [hep-ph/9512380].
- [5] K. G. Chetyrkin, M. Misiak and M. Munz, Weak radiative B-meson decay beyond leading logarithms, Phys. Lett. B 400 (1997) 206 [hep-ph/9612313].
- [6] A. G. Grozin and M. Neubert, Asymptotics of heavy-meson form factors, Phys. Rev. D 55 (1997) 272 [hep-ph/9607366].
- [7] M. Beneke and T. Feldmann, Symmetry breaking corrections to heavy to light B-meson form-factors at large recoil, Nucl. Phys. B 592 (2001) 3 [hep-ph/0008255].
- [8] M. Beneke, T. Feldmann and D. Seidel, *Systematic approach to exclusive* $B \rightarrow V\ell^+\ell^-$, $V\gamma$ *decays*, *Nucl. Phys. B* **612** (2001) 25 [hep-ph/0106067].
- [9] H. Kawamura, J. Kodaira, Q. Cong-Feng and K. Tanaka, B-Meson Light-Cone Distribution Amplitudes in the Heavy-Quark Limit, Phys. Lett. B 523 (2001) 111 [hep-ph/0109181].