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# First inverse moments of heavy-hadron distribution amplitudes

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Heavy-quark symmetry (HQS), despite being approximate, allows to relate dynamically many hadron systems. In the HQS-limit, heavy mesons and doubly-heavy baryons are very similar as their dynamics is determined by a light quark moving in a color field of a static source. As in the meson case, matrix elements of non-local interpolation currents between the baryon state and vacuum are determined by light-cone distribution amplitudes (LCDAs). The first inverse moment of the leading twist $B$-meson distribution amplitude is a hadronic parameter needed for an accurate theoretical description of B-meson exclusive decays. It is quite natural that a similar moment of doubly-heavy baryon is of importance in exclusive doubly-heavy baryons' decays. We obtain HQET sum rules for the first inverse moment based on the correlation functions containing nonlocal heavy-light operator of the doubly-heavy baryon and its local interpolating current.

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Two heavy $c$- or $b$-quarks and one light $u$-, $d$- or $s$-quark can couple together due to the strong interactions and the resulting system is known as the Doubly Heavy Baryon (DHB). The quantum numbers of the baryon like isospin or strangeness are because of the light quark. The isospin-1/2 DHBs are $\Xi_{cc}(ccq)$, $\Xi_{bc}(bcq)$, and $\Xi_{bb}(bbq)$, where $q = u, d$, and differ by their third projection. The strange DHB family is naming as $\Omega_{cc}(ccs)$, $\Omega_{cb}(cbs)$, and $\Omega_{bb}(bbs)$. For a long time, DHBs were a matter of intense searches but their existence was either contradictory as for the $\Xi_{cc}^{+}(ccd)$-baryon or not confirmed experimentally. The situation has been changed in 2017 when one of them, $\Xi_{cc}^{++}(ccu)$, was observed by the LHCb Collaboration in the $\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^-$ decay [1] and confirmed in the $\Xi_{cc}^{++} \to \Xi_c^+ \pi^+$ mode [2]. Its experimental mass and width are $M_{\Xi_{cc}^{++}} = (3621.6 \pm 0.4)$ MeV and $\Gamma_{\Xi_{cc}^{++}} = (2.56 \pm 0.27) \times 10^{-13}$ s [3]. An existence of its isospin partner $\Xi_{cc}^{+}$ still remains controversial and experimental observation of this state is one of the main goal of the LHCb. It was searched in the decays $\Xi_{cc}^{+} \to \Lambda_c^+ K^- \pi^+$ [4] and $\Xi_{cc}^{+} \to \Xi_c^+ \pi^- \pi^+$ [5] and the present combined global significance is reached 2.9 standard deviations including systematic uncertainties. While the existence of doubly charmed baryons, at least $\Xi_{cc}^{++}$, is not debatable and an evidence of its isospin partner seems to be a matter of statistics collected by the LHCb, searches of bottom-charmed baryons by the LHCb [6–9] is a next step in completing the flavor sector of ordinary baryons.

Dynamics of a doubly heavy baryon is similar to a heavy meson and both are determined by a light quark moving in the color external field. In the heavy meson, the source of the field is the heavy antiquark while the doubly heavy diquark (DHD) is the source in the DHB. Under the assumption that the DHD is a point-like static source like heavy antiquark in the heavy meson, one can apply the light-cone approach suggested for the heavy-meson wave-function [10, 11] to the DHB wave-function. Let the origin of the hadron rest frame is coincides with the position the heavy constituent (the frame for the heavy meson is presented in Fig. 1). In this frame, it is desirable to introduce the light cone determined by two light-like vectors $n_\pm^\mu = (1, 0, 0, \mp 1)/\sqrt{2}$ ($n_\pm^2 = 0$ and $(n_+ n_-) = 1$), and an arbitrary four-vector has the following decomposition: $V^\mu = V_+ n_-^\mu + V_- n_+^\mu + V_\perp^\mu$, where $V_\pm = n_\pm^\mu V_\mu$. The heavy antiquark or doubly heavy diquark is connected with the light quark by the Wilson line $E(0, z)$ which becomes trivial, $E(0, z) = 1$, in the Fock-Schwinger gauge of the gluonic field, $A_+(z) = 0$.

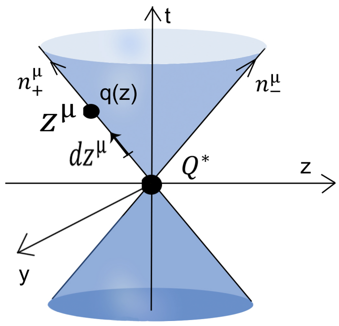


**Figure 1:** The heavy-quark rest frame related with the $B$-meson. The light quark $q(z)$, being massless, is situated on the light cone.

Let us consider the $B$-meson as a representative heavy meson and restrict ourselves by the bottom-charmed doubly heavy baryon, $\Xi_{bc}$, as an example of DHBs. For the $\Omega_{bc}$-baryon, the $s$-quark mass should be taken into account but, as far as $s$-quark mass corrections are neglected, this baryon is completely coincides with $\Xi_{bc}$.

Let us demonstrate the heavy-antiquark — heavy-diquark symmetry between $B$-meson and $\Xi_{bc}$-baryon explicitly. The matrix element of the $B$-meson axial-vector current has the form:

$$\langle 0 \mid \bar{b}(0) \gamma^\mu \gamma_5 q(0) \mid B(p_B) \rangle = i f_B p_B^\mu, \tag{1}$$

where $q(x)$ and $b(x)$ are the light and heavy quark fields, $p_B^\mu$ is the four-momentum of the $B$-meson, and $f_B \simeq 190$ MeV [3] is the $B$-meson decay constant. The general structure of the $\Xi_{bc}$-baryon interpolation current is as follows:

$$J_{\Xi_{bc}}(x) = \epsilon_{\alpha\beta\gamma} \left[ b(x)^{\alpha T} C\gamma_5 c(x)^\beta \right] \Gamma' q(x)^\gamma, \tag{2}$$

where $\alpha$, $\beta$, and $\gamma$ are color indices, $\Gamma^{(\prime)} = \left\{ I, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2 \right\}$, and $C$ is the charged conjugation matrix. After the current (3) is projected onto the state with $J^P = 1/2^+$, the transition matrix element between the $\Xi_{bc}$-baryon and vacuum states can be written in terms of the bispinor, $U(v)$:

$$\langle 0 | J_{\Xi_{bc}}(0) | \Xi_{bc}(v) \rangle = i\, f_{\Xi_{bc}}\, U(v), \tag{3}$$

where $f_{\Xi_{bc}}$ is the $\Xi_{bc}$-baryon decay constant and $v^\mu$ is the DHB velocity. For the baryon with the spin-parity $J^P = 3/2^+$, $U(v)$ should be replaced by a vector-spinor $U^\mu(v)$.

A generalization of the transition matrix elements (1) and (3) of local interpolation currents to the case of non-local currents can be done following the analysis in [10]. In the heavy meson, the antiquark $Q^*$ is infinitely heavy and, hence, static. The light quark $q$ is separated from it by a distance $z$ ($z^2 = 0$), as shown in Fig. 1, and determines the meson dynamics. In the DHB considered the light quark is at the distance $z$ from the center of the doubly heavy diquark $QQ'$:

$$\tilde{O}_0(t) = d(0)\, q(z), \qquad \tilde{O}_1^\mu(t) = \varepsilon^\mu(0)\, q(z), \tag{4}$$

where $d(0)$ and $\varepsilon^\mu(0)$ are spin $S = 0$ or $S = 1$ doubly heavy diquark, being at rest, $t = (vz)$, and the Wilson line $E(0, z)$ is suppressed. The scalar spin diquark is assumed for the $\Xi_{bc}$-baryon, while its counterpart with the axial-vector diquark will be considered elsewhere. Because of the heavy-quark symmetry, there are two Light-Cone Distribution Amplitudes (LCDAs) only entering the heavy meson wave-function [10]. The application of the heavy-quark — heavy-diquark symmetry results the similar decomposition of the heavy baryon wave-function with two LCDAs $\tilde{\varphi}_+(t)$ and $\tilde{\varphi}_-(t)$:

$$\left\langle 0 \left| \tilde{O}_0(t) \right| \Xi_{bc} \right\rangle = i f_{\Xi_{bc}} \left\{ \tilde{\varphi}_+(t) + \left[ \tilde{\varphi}_-(t) - \tilde{\varphi}_+(t) \right] \frac{\hat{z}}{2t} \right\} U(v), \tag{5}$$

where $\hat{z} = z_\mu \gamma^\mu$. In general, the doubly heavy diquark should be considered as a state formed by two heavy quarks which are spatially separated by a distance $R$. To work out LCDAs or their moments theoretically, matrix elements of time-ordered products of local and non-local currents are required. This makes matrix elements dependent on two variables $t$ and $R$ which must reduce to (5) in the case of the local diquarks ($R = 0$). This analysis will be left for further publications.

In QCD Sum Rules (QCD-SRs), one starts from a vacuum average of two or more currents [10], for example local and non-local $\Xi_{bc}$-baryon interpolation currents: $\left\langle 0 \left| \tilde{O}^{\Xi_{bc}}(t)\, J_{\Xi_{bc}}(-x) \right| \Xi_{bc} \right\rangle$. Following the same procedure of getting the QCD Sum Rules as for the $B$-meson [10], we arrive to the sum rules for the leading twist LCDA $\varphi_+(\omega)$ which is the Fourier transform of $\tilde{\varphi}_+(t)$:

$$f_{\Xi_{bc}}^2 \varphi_+(\omega) = \frac{3\omega}{8\pi^2 \tau}\, e^{(\bar{\Lambda} - \omega/2)\tau} \left[ 1 - e^{-(\varepsilon_c - \omega/2)\tau} \right] - \frac{< \bar{q}q >}{8\tau}\, \tilde{f}_S \left( \frac{\omega}{2\tau} \right) e^{(\bar{\Lambda} - \omega/2)\tau}, \tag{6}$$

where $\bar{\Lambda} = M_{\Xi_{bc}} - m_b - m_c$ is the effective baryon mass, $m_b$ and $m_c$ are the $b$- and $c$-quark masses, $< \bar{q}q >$ is the local condensate of light quarks, $\tau$ is the Borel parameter, $\varepsilon_c$ is an effective

QCD-SRs threshold. The function $\tilde{f}_S(\nu)$, being a shape of a non-local quark condensate, is defined as follows [10]:

$$f_S(x^2) = \int d\nu\, \tilde{f}_S(\nu)\, e^{\nu x^2}. \tag{7}$$

There are two known models for $\tilde{f}_S(\nu)$ [12, 13]:

$$\tilde{f}_S^{\rm I}(\nu) = \delta\left(\nu - m_0^2/4\right), \qquad \tilde{f}_S^{\rm II}(\nu) = \frac{\lambda^{p-2}}{\Gamma(p-2)}\, \nu^{1-p}\, e^{-\lambda/\nu}, \tag{8}$$

where $p = 3 + 4\lambda/m_0^2$, $m_0^2 = \langle \bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle/\langle\bar{q}q\rangle$ is the parameter characterized the quark-gluon condensate, and $\lambda$ is a free parameter.

As in the case of heavy mesons [10], the first inverse moment of the leading-twist LCDA:

$$\lambda_{\Xi_{bc}}^{-1} = \int_0^\infty \frac{d\omega}{\omega}\, \varphi_+(\omega), \tag{9}$$

is of interest in physical applications where a doubly heavy baryon is involved. These sum rules can be obtained from the QCD-SRs for $\varphi_+(\omega)$ by integrating them over the variable $\omega$ with the $1/\omega$ weight factor. Such sum rules for the $B_{(s)}$-meson were derived in [14]. The analytical form of these sum rules obtained for the $\Xi_{bc}$-baryon to leading order in $\alpha_{\rm st}$ for the second type, $\tilde{f}_S^{\rm II}(\nu)$, of the non-local quark condensate is as follows:

$$\lambda_{\Xi_{bc}}^{-1} = \frac{e^{\bar{\Lambda}\tau}}{4\pi^2 f_{\Xi_{bc}}^2}\left[1 - \frac{\pi^2 < \bar{q}q >}{\tau\,\Gamma(p-2)}\, \lambda^{(p-3)/2} K_{p-1}(2\sqrt{\lambda})\right], \tag{10}$$

where $K_\nu(z)$ is the modified Bessel function, also known as the McDonald function. One can also obtain $s$-quark corrections when apply this analysis to the strange $\Omega_{bc}$-baryon, similar to the $B_s$-meson [14]. Numerical results and details of the analysis can be found in the forthcoming paper.

In summary, the heavy-quark — heavy-diquark symmetry was applied to the light-cone decomposition of the doubly-heavy baryon wave-function and definition of corresponding LCDAs. Dynamically, these LCDAs are similar to the heavy meson ones as both are determined by one light quark being in the field of a color external source. This source is either the heavy antiquark in the heavy meson or a point-like double-heavy diquark in the DHB. Among two LCDAs allowed by the heavy-quark symmetry, $\varphi_+(\omega)$ is the leading twist one. QCD Sum Rules for the first inverse moment of the DHB leading-twist LCDA are presented.

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