

# Fluid properties of hadron gas produced in relativistic hadronic and heavy-ion collisions

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Thermalization is a vital feature that enables the study of hydrodynamic evolution of the medium produced in nuclear collisions at relativistic energies. Here, we obtain the variation of Knudsen number (*Kn*) to study the degree of thermalization in an excluded volume hadron resonance gas model. *Kn* along with other parameters like Reynolds number (*Re*) and Mach number (*Ma*) give insights into the nature of the flow in a system. The dependence of these dimensionless parameters on system size and baryonic chemical potential ( $\mu_B$ ) are studied. The obtained values of the parameters (*Kn* << 1, *Ma* ~ 1 and *Re* >> 1) indicate the occurrence of compressible inviscid flows at high temperatures close to the QCD phase transition region (*T* ~ 150–170 MeV). The degree of thermalization of hadron gas estimated is comparable over different system sizes, indicating the applicability of hydrodynamics in interpreting the results from high multiplicity pp to heavy-ion collisions.

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### 1. Introduction

Relativistic heavy-ion collisions provide various phases of QCD matter under extreme conditions. Quark Gluon Plasma (QGP), a deconfined phase of quarks and gluons, is the first locally thermalized phase usually produced in heavy-ion collisions. The existence of such a medium in pp collisions is unforeseen. Therefore, results from heavy-ion collisions are used to compare with proton-proton (pp) collisions considering it as a baseline. However, recent results from high multiplicity pp and p-Pb collisions show heavy-ion-like behaviour [1, 2]. In this article, we look at the hadronic phase produced in these collisions within the ambit of the Excluded Volume Hadron Resonance Gas (EVHRG) model with the inclusion of Hagedorn mass spectrum for higher mass states. We present results for three dimensionless parameters: Knudsen number (Kn), Reynolds number (Re), and Mach number (Ma) for a hadronic medium. These three dimensionless quantities provide information concerning thermalization and applicability of hydrodynamics, viscosity and compressibility of the medium, respectively.

#### 2. Formalism

Knudsen number relates the mean free path ( $\lambda$ ) of a system to the system size. It indicates the probability of occurance of collisions between the particles of the medium. A small value for Knudsen number,  $Kn \ll 1$  implies the applicability of fluid dynamics through a high degree of thermalization and small gradients of hydrodynamic quantities [3]. Kn is given by

$$Kn = \lambda/D \tag{1}$$

where  $\lambda = \frac{1}{\sqrt{2}n\sigma}$  is the mean free path and *D* is the system size.  $\sigma$  is interaction cross section between particles of the medium.

The nature and characteristics of the flow in a medium with low Kn can be deduced by Reynolds and Mach numbers, respectively. The magnitude of Re helps to classify the flow as either laminar or turbulent. Re being the ratio of inertial force to viscous force, is inversely related to shear viscosity ( $Re \sim 1/\eta$ ). Thus systems with low viscosity tends to show turbulent nature. Re is defined as,

$$Re = \frac{D\langle v \rangle T}{\eta/s},\tag{2}$$

where  $\langle v \rangle = \frac{1}{m} \sum_{i=1}^{m} \frac{\int \frac{d^3 p}{(2\pi)^3} v_i f_i^0}{\int \frac{d^3 p}{(2\pi)^3} f_i^0}$  is the velocity of hadrons averaged over the distribution function  $(f_i^0)$ .

Compressibility of a fluid can be determined using Mach number. Subsonic flows ( $Ma \ll 1$ ) indicate incompressible nature while supersonic flows ( $Ma \gg 1$ ) signify compressible fluids. Ma is given by

$$Ma = \langle v \rangle / c_s, \tag{3}$$

where  $c_s$  is the speed of sound defined by,

$$c_s^2 = \frac{\frac{\partial P}{\partial T} + \frac{\partial P}{\partial \mu_B} \frac{d\mu_B}{dT}}{\frac{\partial \varepsilon}{\partial T} + \frac{\partial \varepsilon}{\partial \mu_B} \frac{d\mu_B}{dT}},\tag{4}$$

which reduces to  $c_s^2 = (\frac{\partial P}{\partial \varepsilon})$  at vanishing baryon chemical potential ( $\mu_B = 0.0$  GeV). Here,

$$\frac{d\mu_B}{dT} = \frac{s\frac{\partial n}{\partial T} - n\frac{\partial s}{\partial T}}{n\frac{\partial s}{\partial \mu_B} - s\frac{\partial n}{\partial \mu_B}}.$$
(5)

Pressure (*P*), energy density ( $\varepsilon$ ), number density (*n*) and entropy (*s*) which we use above are obtained from the EVHRG model (with the inclusion of Hagedorn mass spectrum) as in Ref.[4]. Moreover, the shear viscosity ( $\eta$ ) used in the above equations is obtained using the Boltzmann Transport Equation (BTE) as [5]

$$\eta = \sum_{i} \frac{1}{15T} \int \frac{d^3p}{(2\pi)^3} \frac{p^4}{E_i^2} (\tau_i f_i^0 + \bar{\tau}_i \bar{f}_i^0) \tag{6}$$

where  $\tau_i$  is the relaxation time of  $i^{th}$  hadron. The particle dependent average relaxation time is given by,

$$\tilde{\tau}_i^{-1} = \sum_j n_j \langle \sigma_{ij} v_{ij} \rangle. \tag{7}$$

Here,  $n_i$  is the number density of  $j^{th}$  hadronic species.

# 3. Results and Discussion



**Figure 1:** (Color Online) Knudsen number (first column), Reynolds number (second column), and Mach number (third column) as functions of temperature. The upper row shows the effect of different values of R and lower row shows the effect of different baryon chemical potentials. (Figs. 4-6 [4])

All the hadrons and resonances listed in the particle data book [6] upto a mass of 2.25 GeV have been included in our analysis. Further, higher massive states are introduced via Hagedorn

mass spectrum [4]. The interaction between hadrons are modelled by considering them as hard spheres of radius  $r_h$  giving (geometric) interaction cross section  $\sigma = 4\pi r_h^2$ . Hard sphere radius is chosen as  $r_h = 0.3$  fm considering the arguments detailed in Ref. [4]. The system size effects are included in our calculations by providing a lower momentum cut-off to the integral over momentum space [7, 8]. This is given by  $p_{min} = 0.197\pi/D$ (fm), where D = 2R is the characteristic system size. We have introduced different system radii varying from 3 fm to 10 fm [9–11] to cover the variation between high multiplicity *pp* to heavy-ion collisions.

Fig.1 first column compares the variation of Knudsen number as a function of temperature for different system sizes as well as for different baryon chemical potentials. It can be seen that as the temperature and  $\mu_B$  increases, Kn decreases due to an increase in the particle density thus decreasing the mean free path. Also as expected, increase in system size decreases Kn, indicating more collisions. The low value for Kn at high temperatures indicate a high degree of thermalization and possible hydrodynamic behaviour. The second column of Fig.1 shows the variation of Reynolds number as a function of temperature in similar conditions. It can be seen that Re increases with temperature and  $\mu_B$  hinting towards low viscous nature at these conditions approaching a "perfect fluid" limit. The variation of Mach number with temperatures and  $\mu_B$  considered. This indicates the possibility of the system being compressible. Ma being a characteristic property of the medium can also signify the possibility of formation of Mach cones (shock waves) in the medium. Mach cones have been theorized to be sensitive to the critical point [12] and gathers much interest. The value of the parameters observed hint towards compressible inviscous flows as predicted [13].

#### 4. Summary

A study of Knudsen, Reynolds and Mach numbers in an excluded volume hadron resonance gas is presented. Their dependence on temperature, baryon chemical potential and system size is explored. The obtained values ( $Kn \ll 1$ ,  $Re \gg 1$  and  $Ma \sim 1$ ) at high temperatures indicate compressible inviscid flows in this limit close to the phase transition region ( $T \sim 150 - 170$  MeV). The degree of thermalization consistent over different system sizes raises a question about using *pp* collisions as a baseline for heavy-ion collisions.

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