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# Virtual QCD corrections to $gg \to ZH$ via a transverse momentum expansion

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Associated $ZH$ production plays a special role in the determination of Higgs properties at the LHC. An improved theoretical control over the $gg \to ZH$ subprocess is important to reduce the scale uncertainties in the SM prediction. We present the calculation of the virtual QCD corrections to $gg \to ZH$ using an analytic approximation, based on the expansion of the amplitude in terms of a small transverse momentum of the final particles. We also report on the combination of these results with those obtained from a complementary approach, based on the expansion of the amplitude in the high-energy limit. When the results of both expansions are improved using Padé approximants, their combination provides an accurate approximation over the whole phase space.

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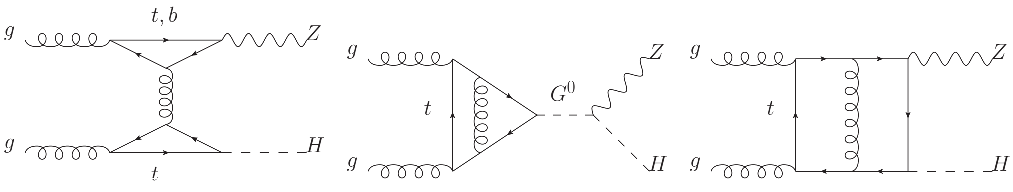
## 1. Introduction

The process of associated $VH$ production ($V = W, Z$) is of particular importance at the LHC not only because it is one of the main production modes of the Higgs boson but also because it provides the best sensitivity to the measurement of the $H \to b\bar{b}$ decay [1]. The experimental uncertainties on $VH$ measurements are expected to reach the 5% level after the High-Luminosity phase, and in order to have a consistent comparison with the future measurements a constant effort is undertaken in reducing the uncertainty in the theory prediction within the Standard Model (SM).

The various sources of theoretical uncertainty have a different impact on the SM prediction for $WH$ and $ZH$ production. In particular, the contribution from the the scale uncertainty due to the missing higher-order terms in the perturbative calculations is moderate in the $WH$ case, whereas it is the largest one in the $ZH$ prediction. This is because $ZH$ production can occur not only via a quark-initiated process (as in $WH$ production) but also via the partonic channel $gg \to ZH$. The latter is suppressed with respect to $q\bar{q} \to ZH$, but nonetheless it provides an $O(10\%)$ contribution to the hadronic cross section. In order to reduce the impact of the related scale uncertainties, it is crucial to improve the accuracy of the current prediction for $gg \to ZH$.

The leading-order (LO) term is given by one-loop diagrams, with loops of top quarks providing the dominant contribution [2, 3]. The calculation of the next-to-leading-order (NLO) corrections in QCD requires the computation of two-loop diagrams in the virtual part. In Fig. 1 diagrams related to different topologies are shown: while the double triangles of Fig. 1a can be computed with standard techniques and the results for the two-loop triangles in Fig. 1b can be obtained from previous calculations [4–6], a full analytic result for the two-loop boxes in Fig. 1c is not available, as it requires the computation of two-loop integrals depending on five energy scales: the external masses $m_Z$ and $m_H$, the partonic Mandelstam variables $\hat{s}$ and $\hat{t}$ and the mass of the top quark running in the loop, $m_t$.

Several approaches have been considered to circumvent this problem, based on the numeric evaluation [7] or on the analytic approximation of the integrals [6, 8], or on the combination of these two methods [9]. In the analytic approximations the complexity of the two-loop integrals is reduced by expanding them after the assumption of a given hierarchy between the various energy scales. In this contribution we discuss the calculation of the virtual QCD corrections to $gg \to ZH$ by expanding the NLO amplitude in the limit of a small transverse momentum of the final-state particles, $p_T$. The calculation has been presented in Ref. [10], while the method has been applied for the first time to $gg \to HH$ [11].



**(a)** **(b)** **(c)**

**Figure 1:** Representative diagrams for the main topologies occuring in the virtual NLO corrections to $gg \to ZH$: (a) double-triangles; (b) two-loop triangles; (c) two-loop boxes.

## 2. Transverse momentum expansion for $gg \to ZH$ at NLO

The method is based on the decomposition of the amplitude as a linear combination of six Lorentz projectors, which multiply scalar form factors

$$\mathcal{A} = i\sqrt{2}\frac{m_Z G_F \alpha_S(\mu_R)}{\pi}\delta_{ab}\epsilon^a_\mu(p_1)\epsilon^b_\nu(p_2)\epsilon^*_\rho(p_3)\hat{\mathcal{A}}^{\mu\nu\rho}(p_1,p_2,p_3), \tag{1}$$

$$\hat{\mathcal{A}}^{\mu\nu\rho}(p_1,p_2,p_3) = \sum_{i=1}^{6}\mathcal{P}_i^{\mu\nu\rho}\mathcal{A}_i(\hat{s},\hat{t},m_t,m_Z,m_H). \tag{2}$$

The form factors are chosen to be symmetric or antisymmetric under the exchange $\hat{t} \leftrightarrow \hat{u}$, and they are expressed as linear combinations of scalar two-loop integrals. We can decompose the NLO form factors $\mathcal{A}_i^{(1)}$ according to the three contributing topologies of Fig. 1 , and in the following we discuss the application of the $p_T$ expansion to compute the box contribution $\mathcal{A}_i^{(1,\Box)}$.

In the $p_T$ expansion the form factors are expanded in the limit of a forward kinematics[1], in which the following quantities are taken as expansion parameters

$$\frac{m_Z^2}{\hat{s}}, \frac{m_H^2}{\hat{s}}, \frac{p_T^2}{\hat{s}} < 1.$$

However, with this expansion one is also implicitly assuming that $p_T < 2m_t$, and this assumption prevents the $p_T$ expansion to be accurate everywere in the phase space. The scalar integrals involved in the expanded form factors have a simpler structure than the original ones, as they depend on three scales: $m_t^2, \hat{s}, p_T^2$. The new scalar integrals are in turn decomposed along a basis of 52 master integrals (MIs) using *integration-by-parts* identities. The MIs depend on a single scale, namely the ratio $\hat{s}/m_t^2$, and they have been already computed in the literature [5, 12–17].

After UV renormalization and IR subtraction we compared our results for the finite part of the virtual corrections with the full numeric evaluation of Ref. [7], finding an agreement at the permille level in the region of validity of the $p_T$ expansion, which corresponds to invariant masses $M_{ZH} \lesssim 700$ GeV. From Fig. 2a it can be observed that the results obtained from the latter approach provide a better accuracy compared to other analytic approximations. Finally, the results of the $p_T$ expansion allow to cover accurately about 98% of the hadronic cross section.

## 3. Merging the transverse momentum and the high-energy expansions

As stated before, the validity of the $p_T$ expansion is restricted to a limited region of the phase space. However, the results obtained in the so-called High-Energy (HE) expansion [8] are valid in an almost complementary kinematical region. This can be understood by considering the different scale hierarchies assumed in the two approaches

$$\underbrace{m_Z^2, m_H^2, p_T^2 \ll m_t^2, \hat{s}}_{p_T\ \text{expansion}} \qquad \underbrace{m_Z^2, m_H^2 \ll m_t^2 \ll |\hat{t}|, \hat{s}}_{\text{HE expansion}} \tag{3}$$

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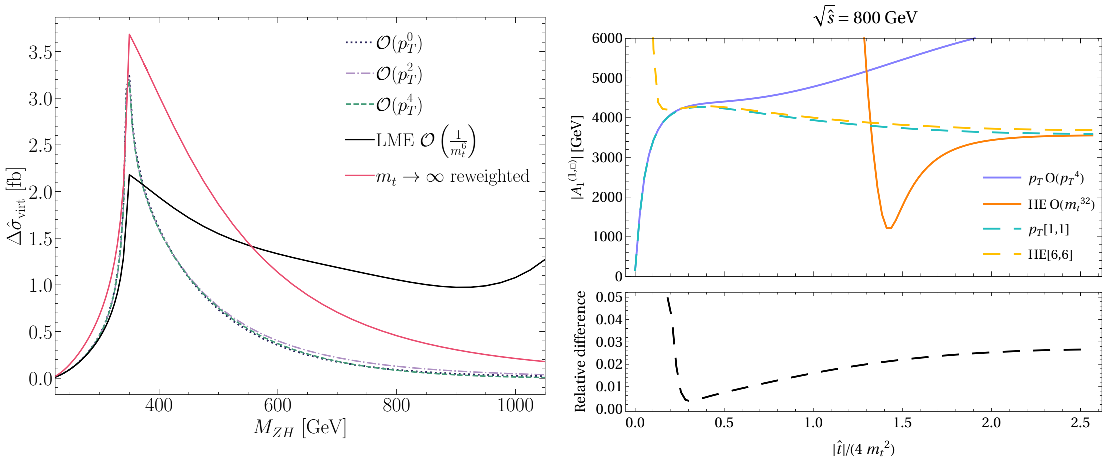
[1] The results for an expansion in the backward limit can be straightforwardly obtained exploiting the (anti)symmetry of the form factors



**(a)** **(b)**

**Figure 2:** **(a)** Contribution to the partonic cross section from the finite part of the virtual NLO corrections, obtained using the $p_T$ expansion (dashed lines) and compared to other analytic approximations (solid lines). **(b)** Absolute value of one box form factor at NLO, as a function of the ratio $|\hat{t}|/(4m_t^2)$ for fixed $\sqrt{\hat{s}} = 800$ GeV. The solid lines show the simple $p_T$-expanded (blue) and HE-expanded (orange) results, respectively; the dashed lines represent the [1/1] $p_T$-Padé (light-blue) and [6/6] HE-Padé (yellow). The bottom part shows the relative difference of the Padé results.

and by noting that $p_T^2 \simeq |\hat{t}|$ for sufficiently high values of $\hat{s}$.

A naive combination of the results from the two expansions does not provide a sufficient accuracy in the $\hat{t} \simeq 4m_t^2$ region of the phase space. Therefore, in Ref. [18] we considered a refined approach based on the use of Padé approximants. The results for a form factor at NLO are illustrated for $\sqrt{\hat{s}} = 800$ GeV in Fig. 2b: the simple $p_T$ and HE expansions are diverging in the region $0.5 \lesssim |\hat{t}|/(4m_t^2) \lesssim 1.5$, whereas in the same region the respective Padé approximants are stable and their relative difference never exceeds 2%. We used our findings to merge the Padé results into a single prediction for the virtual corrections to $gg \to ZH$ that is accurate everywhere in phase space.

## 4. Conclusions

We showed that the virtual NLO corrections in QCD to $gg \to ZH$ can be completely and accurately approximated by merging the results of the $p_T$ and HE expansions, provided that these results are improved using Padé approximants. This method allows to have analytic results that are flexible (e.g. different top-mass renormalization schemes can be used) and fast enough to be implemented in a Monte Carlo code. In a subsequent paper [19], these results have been used to calculate the full NLO QCD corrections to $gg \to ZH$, by including the contribution from real-emission diagrams.

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