PROCEEDINGS OF SCIENCE

$p \to e^+\gamma$ in LCSR framework

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Proton decay is a baryon number violating process and hence is forbidden in the Standard Model (SM). Baryon number violation is expected to be an important criteria to explain the matter anti-matter asymmetry of the universe. Any detection of the proton decay will serve as a direct evidence of physics beyond the SM. In SMEFT, proton decay is possible via baryon number violating dimension six operators.
In this work, we have considered the proton decay to a positron and a photon, which is expected to be an experimentally cleaner channel because of less nuclear absorption. The gauge invariant amplitude for this process involves two form factors (FFs). We present these FFs in the framework of light cone sum rules (LCSR).

The Tenth Annual Conference on Large Hadron Collider Physics - LHCP2022
16-20 May 2022
Online

[*] Speaker

https://pos.sissa.it/

PoS(LHCP2022)344

1. Introduction

Baryon number is a conserved quantity within the Standard Model (SM). Consequently, one tries to look for baryon number violating (BNV) decays, like proton decay, to probe physics beyond the SM (BSM) which brings interactions that violate baryon number.. Such decays are well motivated in the BSM theories like grand unification theories [1, 2], supersymmetry [3, 4], etc.

The prominent decay mode of the proton decay is $p \to e^+\pi^0$ with the most stringent experimental bound on the lifetime given by $\tau_p > 10^{34}$ years [5]. As there are no clear signature of this decay mode so far, looking for other decay channels becomes important. $p \to e^+\gamma$ is one such channel. Although, this mode will be electromagnetically suppressed, it is still interesting as it is expected to be experimentally cleaner due to less nuclear absorption [6]. The aim of the present work is to compute the form factors (FFs) involved in the amplitude of this decay using the method of Light Cone Sum Rules (LCSR) [7].

2. Amplitude Calculation

The decay, $p \to e^+\gamma$ can be studied theoretically using the BNV dim-6 effective Lagrangian which preserves the gauge group of the SM. This Lagrangian consists of dim-6 operators, $\mathcal{O}_{\Gamma\Gamma'} = \epsilon^{abc}\left(\bar{d}^c_a P_\Gamma u_b\right)\left(\bar{e}^c P_{\Gamma'} u_c\right)$ multiplied by the corresponding Wilson coefficients which incorporates all the flavour effects [8–11]. Here, $\{\Gamma, \Gamma'\} \in \{L, R\}$, superscript c and the subscript $\{a, b, c\}$ denotes the chiral projection operators charge conjugation, and colour indices, respectively. The amplitude of the process can be calculated by computing the matrix element of this Lagrangian taken between the initial and final states. Furthermore, using gauge invariance this amplitude can be parameterised as

$$\mathcal{A}(p(p_p) \to e^+(p_2) + \gamma(k)) = \sum_{\Gamma\Gamma'} c_{\Gamma\Gamma'} \bar{v}^c_e P_{\Gamma'} \left\{ \epsilon_{\alpha^*} A_{\Gamma\Gamma'} \frac{i\sigma^{\alpha\beta} k_\beta}{m_p} \right\} u_p(p_p). \tag{1}$$

Here, $A_{\Gamma\Gamma'}$ represents the four physical form factors (FFs). As a consequence of parity conservation in QCD, it turns out that $A_{LL(LR)} = -A_{RR(RL)}$. Consequently, we are left with only two independent FFs which we choose to be A_{LL} and A_{LR}. These are computed using LCSR [7].

Moreover, the photon emission from the proton state is the only non-trivial contribution to this amplitude and hence, is the focus of this work. This contribution can be factorised in terms of the leptonic and the hadronic parts as $\bar{v}^c_e(p_e) H_{\Gamma\Gamma'}(p_p, p_e) u_p(p_p)$, with $H_{\Gamma\Gamma'}(p_p, p_e)$ being the hadronic matrix element which can be generally parametrised as [12]

$$H_{\Gamma\Gamma'} u_p(p_p) = P_\Gamma \epsilon^*_\mu \left[F^1_{\Gamma\Gamma'} \frac{\not{k} p^\mu_p}{m_p^2} + F^2_{\Gamma\Gamma'} \frac{\not{k} k^\mu}{m_p^2} + F^3_{\Gamma\Gamma'} \gamma^\mu + i F^4_{\Gamma\Gamma'} \frac{\sigma^{\mu\nu} k_\nu}{m_p} + F^5_{\Gamma\Gamma'} \frac{p^\mu_p}{m_p} + F^6_{\Gamma\Gamma'} \frac{k^\mu}{m_p} \right] u_p(p_p). \tag{2}$$

Here, $F^n_{\Gamma\Gamma'}$ for $n = \{1, \ldots, 6\}$ are the functions of p_p and p_e. LCSR allows us to compute these functions which can further be used to calculate $A_{\Gamma\Gamma'}$ using, $A_{\Gamma\Gamma'} = \frac{F^1_{\Gamma\Gamma'}}{2} + F^4_{\Gamma\Gamma'} + \frac{F^5_{\Gamma\Gamma'}}{2}$.

3. Form factors using LCSR

LCSR is a method which uses the analytic properties of the correlation function between the vacuum and an on-shell state to compute the hadronic quantities of interest. The idea is to expand the product of currents near the light-cone and expresses the bilocal operator in terms of the Distribution Amplitude (DAs) as a series in twist (for more details, see [13–16]).

In order to compute the FFs involved, we can proceed in two ways: 1) Using the proton interpolation current and employing the photon DAs [17], and 2) using the photon interpolation current and employing the proton DAs [18].

3.1 Case-1: Interpolating the proton state and using the photon DAs

The interpolation current for the proton state is not unique. We have employed the commonly used current known as the Ioffe current, $\chi(x) = \epsilon^{abc}\left(u^{a^T}(x)C\gamma_\mu u^b(x)\right)\gamma_5\gamma^\mu d^c(x)$ for the present study [19]. Interpolation of the proton state results into the desired correlation function, $\Pi_{\Gamma\Gamma'}(p_p, p_e)$ which can be parameterized in terms of twelve functions represented by $\Pi^{\text{had},r}_{\Gamma\Gamma'}$ with $r = \{PK, KK, V, T, P, K, KPP, KKP, VP, TP, PP, KP\}$ [7]. These functions can be written in terms of the spectral densities, $\rho^{\text{had},r}_{\Gamma\Gamma'}$ using the dispersion relation as

$$\Pi^{\text{had},r}_{\Gamma\Gamma'}(s, P_e^2) = \int_0^\infty ds \frac{\rho^{\text{had},r(s,P_e^2)}_{\Gamma\Gamma'}}{s - p_p^2} = \int_0^\infty ds \frac{1}{\pi}\frac{\text{Im}\Pi^{\text{had},r}_{\Gamma\Gamma'}(s + i\epsilon, P_e^2)}{s - p_p^2} \tag{3}$$

Moreover, the ground state, i.e. proton state contribution can be represented by functions $F^r_{\Gamma\Gamma'}$ multiplied by a delta function, $\delta(s - m_p^2)$. These functions are directly related to the functions $F^n_{\Gamma\Gamma}$ [7] and hence, can be used to compute the FFs by saturating the sum rules with this contribution. Upon assuming the quark hadron quality and performing the Borel transformation, the final sum rules for $F^r_{\Gamma\Gamma'}$ are obtained as a function of $P_e^2 = -p_e^2$ and the LCSR parameters s_0 and M, known as continuum threshold and Borel mass, respectively [7], computed to two particle twist-3 accuracy.

3.1.1 Numerical analysis

The onshellness of photon enables us to put $k^2 = 0$.There are in total eight combinations to compute the FFs using $F^r_{\Gamma\Gamma'}$, however, it has been found that for $\Gamma\Gamma' = LL$, only two combinations i.e. F^T_{LL} and F^{TP}_{LL} with F^{KPP}_{LL} survives and all other turns out to be zero. Furthermore, for $\Gamma\Gamma' = LR$, all the eight combinations survives but the combinations involving $F^{TP}_{\Gamma\Gamma'}$ are found to be more stable. They are shown in Fig.(1) for $s_0 = (1.44\ \text{GeV})^2$ and $M^2 = 2\ \text{GeV}^2$. For $\Gamma\Gamma' = LR$, only one combination out of four as a representative graph. The values of these FFs at $P_e^2 = 0.5\ \text{GeV}^2$ are

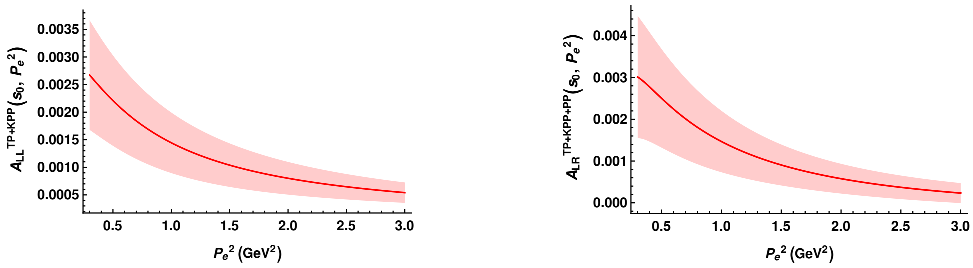


Figure 1: The physical FFs $A_{\Gamma\Gamma'}$ with $\Gamma\Gamma' = \{LL, LR\}$ using the proton interpolation current and the photon distribution amplitudes upto two-particle twist three accuracy. The bands represents the uncertainties.

$A^{TP+KPP}_{LL}(1.44^2, 0.5) = (0.00221 \pm 0.00082)\ \text{GeV}^2$ and $A^{TP+KPP+PP}_{LR}(1.44^2, 0.5) = (0.00250 \pm 0.00118)\ \text{GeV}^2$, respectively with uncertainties associated with the uncertainties in the numerical values of the parameters used for the analysis.

3.2 Case-2: Interpolating the photon state and using the proton DAs

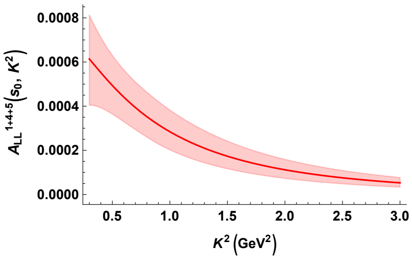
Unlike proton state, the photon state can be uniquely interpolated using the electromagnetic current. Furthermore, in this case, it is better to perform generalised Fierz transformations [20] on the operators $O_{\Gamma\Gamma'}$ in order to obtain the desired correlation function. The correlation function obtained can be parameterised directly in terms of $F^n_{\Gamma\Gamma'}$ after isolating the ground state contribution in the dispersion relation and hence the sum rules obtained in this case are directly in terms of $F^n_{\Gamma\Gamma'}$. The final sum rules after employing the quark hadron duality and the Borel transformation are

$$F^{1,4,5}_{\Gamma\Gamma'}(s_0, K^2) = -\frac{e^{\frac{m_p^2}{M^2}}}{(p_p - k)^2 - m_p^2}\int_0^{s_0} ds e^{\frac{-s}{M^2}} \frac{1}{\pi} \mathrm{Im}\left(F^{\{1,4,5\},QCD}_{\Gamma\Gamma'}(s, K^2)\right) \tag{4}$$

where, $F^{\{1,4,5\},QCD}_{\Gamma\Gamma'}(s, K^2)$ are calculated using proton DAs upto twist-3 accuracy and can be found in the Appendix-C of [7].

3.2.1 Numerical analysis

In this case, the momentum transferred square is $K^2 = -k^2$ and p_e^2 can be set equal to $m_e^2 \approx 0$ as positron is on shell. Both the physical form factors are obtained from the combination of $F^{(1,4,5)}_{\Gamma\Gamma'}$ as described in Section-2. These FFs are shown in Fig.(2) as a function of K^2 for $s_0 = (1.44 \text{ GeV})^2$ and $M^2 = 2 \text{ GeV}^2$. The numerical value for FFs at $K^2 = 0.5 \text{ GeV}^2$ are $A^{1+4+5}_{LL}(1.44^2, 0.5) = (0.00049 \pm 0.00014) \text{ GeV}^2$ and $A^{1+4+5}_{LR}(1.44^2, 0.5) = (0.00174 \pm 0.00027) \text{ GeV}^2$, respectively. Here again the uncertainties are associated with the uncertainties in the values of the used parameters.



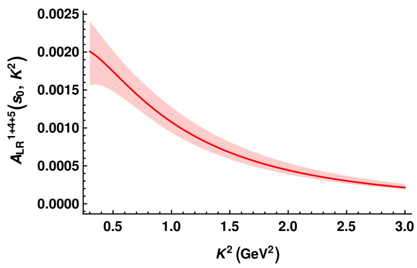


Figure 2: The physical FFs $A_{\Gamma\Gamma'}$ with $\Gamma\Gamma' = \{LL, LR\}$ using the photon interpolation current and the proton distribution amplitudes upto twist three accuracy. The bands represents the uncertainties.

4. Conclusions and Discussions

The process $p \to e^+\gamma$ involves two independent form factors which are computed in the framework of LCSR in the present work. The computation is done for two cases; firstly by interpolating the proton state and employing photon DAs upto two-particle twist-3 accuracy and secondly by interpolating the photon sate and employing proton DAs upto twist-3 accuracy. In case-1, both the FFs are found to be close to each other. However, in the second case, one FF is found to be roughly a factor of three smaller than the other. Moreover, the uncertainties calculated are found to be smaller for case-2 than for case-1. A straightforward comparison of the two cases is not possible due to difference in choice of momentum transfer and hence, further detailed studies are required to shed more light on the issue.

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