PROCEEDINGS OF SCIENCE

Cosmic ray modulation error for 2D SDE SOLARPROP model

Viacheslav Mykhailenko[a,*] **and Pavol Bobik**[b]

[a] *Institute of Physics, Faculty of Science, P. J. Safarik University, Park Angelinum 9, 041 54 Košice, Slovakia*

[b] *Institute of Experimental Physics, Slovak academy of sciences, Watsonova 47, Košice, Slovakia*

E-mail: viacheslav.mykhailenko@student.upjs.sk, bobik@saske.sk

For comprehensive global modeling of cosmic rays modulation in the heliosphere, it is essential to have a sound transport theory, and reliable numerical schemes with appropriate boundary conditions. For the description of the solar modulation process, and the propagation of the particles inside the heliosphere, Parkers transport equation is widely used. The correct and precise solution of this equation also must take into consideration errors. That's why the presented work particularly focused on the estimation of the errors of the SOLARPROP model, based on the input parameters range, and statistical errors for these numerical solutions of 2D Parkers equation by stochastic differential equation method to suggest the safe simulation strategy for spectra evaluation at 1 AU.

**** 27th European Cosmic Ray Symposium - ECRS ****
**** 25-29 July 2022 ****
**** Nijmegen, the Netherlands ****

*Speaker

© Copyright owned by the author(s) under the terms of the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License (CC BY-NC-ND 4.0).

PoS(ECRS)042

1. Introduction

Cosmic rays entering the heliosphere from interstellar space change their energy and distribution in a process called modulation. The process is described by the Parker equation [1], which is currently solved mainly by Stochastic Differential Equations (SDE hereafter) backward in time method [2]. The SDE backward in time method is precisely described in [3]. The method was used to describe modulation processes in the heliosphere in many articles, to name some we could refer to publications [4][5][6][7][8][9][10][11][12]. As a widely used method, it needs a well-described method to evaluate statistical error and suggested strategies for the number of needed simulated particles, to reach an acceptable error level. However, except for publication [13] analyzing SDE backward method for the 1D model, there is no article with a statistical error analysis for the 2D and 3D models of the heliosphere. There are only relatively short mentions of statistical error analysis of models used in articles [14][15][16]. We chose SOLARPROP [17] as the first 2D model to evaluate a statistical error by the method introduced in [13]. The reason to use SOLARPROP is, that SOLARPOP is the first and in the last years only one, open-source public 2D model of cosmic rays heliospheric modulation available (see related discussion in [13]).

PoS(ECRS)042

2. Statistical error in 1D SDE model

In the previous work [13], we describe a method to estimate statistical error for the 1D backward SDE model statistical error. To illustrate how an error in the backward in time method change with the number of injected particles we simulated ten separate sets of one million quasiparticles with energy 5 GeV at 1AU. The result is presented in Figure 1. in [13] and shows how with increasing values of injected particles statistical error of normalized intensity decrease.

To show statistical error, we divided obtained set of 10 million quasiparticles into subsets with N_s quasiparticles. From every subset with $N_s = 1000$ quasiparticles (i.e. 10000 subsets) we evaluate the mean value of intensity I_{Ns}. The histogram of the distribution of intensities in mean values units is shown in red color in Figure 1. taken from [13]. The distribution for $N_s = 100$ is shown in blue color. The normal distribution of intensities is visible (for details see [13]).

The dependency of σ at N_s or sets with a normal distribution (a consequence of the central limit theorem) should have a power law shape with slope 1 / 2. To show it we evaluated a σ_{Ns} set of different N_s with a logarithmical step. In Figure 2. (figure taken from [13]) we show the dependency of σ on N_s for the evaluation of intensities for two energies T_{kin} equal 5 GeV and 1 GeV. The slope of -1 / 2 is visible from a comparison of red lines, with this slope, following points from the simulation shown in the figure (see article [13] for details). This allows us to evaluate σ for any N_s if we have it for one specific N_s.

Consequently, we could rigorously define a one percent statistical error criterium. The one percent error criterium shows a number of quasiparticles needed, for selected energy, to have results within standard deviation equal to one percent of mean intensity value, i.e. $Ns_{\sigma=1\%}$ with the distribution of evaluated intensities, with standard deviation equal to 1 percent of mean intensity (i.e. with 68% of results inside 1 percent of mean intensity).

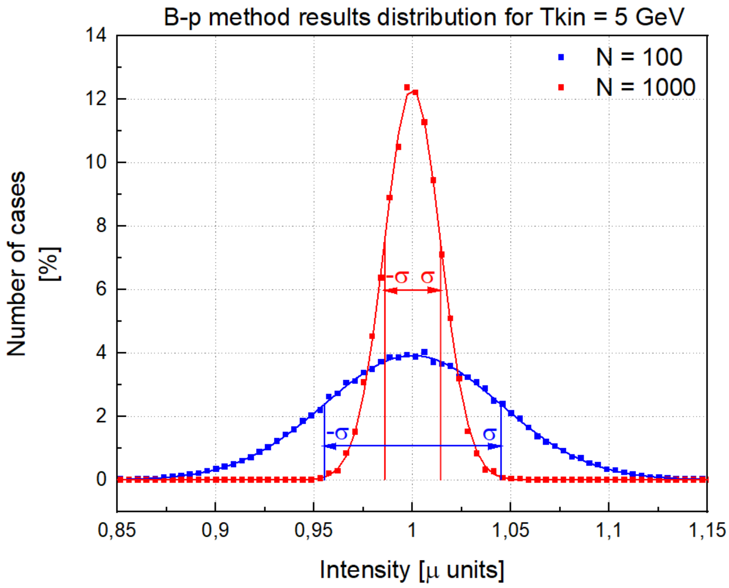

Figure 1: Comparison of I_{N_s} values histogram with probability distribution function with μ_{Ns} and σ_{Ns} of histogram. Red points sign case for $N_s = 1000$, blue for $N_s = 100$. Ranges $\pm 1\sigma$ are shown for both cases.

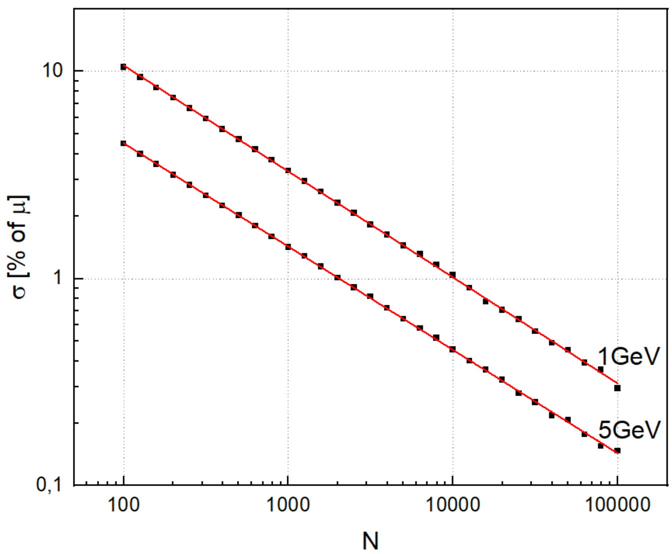

Figure 2: Dependency of σ on N_s for energies $Tkin$ equal 5 GeV and 1 GeV.

3. Results 2D SOLARPROP model

We used the SOLARPROP model [17] to investigate statistical error dependency on the number of injected quasiparticles for 2D models. The procedure described in the previous section for the 1D backward in time model was used. Thus by using the so-called Standard 2D SOLARPROP model, sets of one thousand injected quasiparticles for every energy bin (i.e. for the whole spectrum) were

PoS(ECRS)042

simulated, for every month for years from 1991 – 1999 (i.e. 108 months). Firstly, we evaluated the standard deviation σ distribution in the percent of mean value μ units for every month, and for four different energy bins at 1 AU (with $N_s = 1000$). Results are presented in Figure 3.

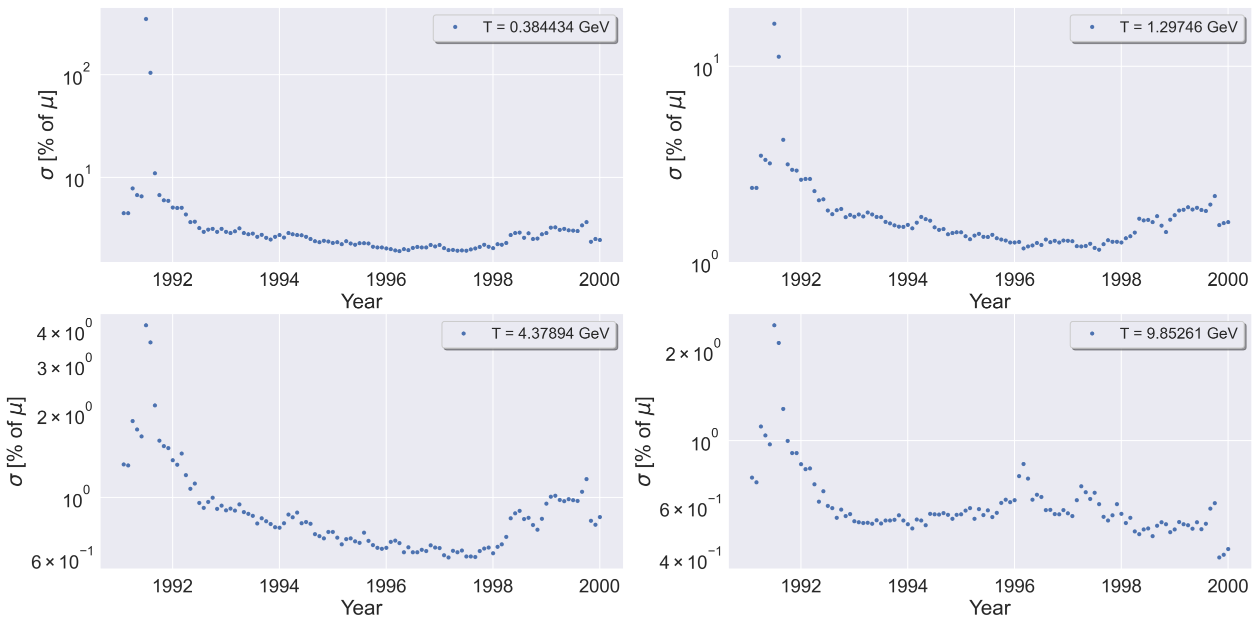

Figure 3: Standard deviation distribution in percent of mean value units with respect to different months, for four energy bins at 1 AU.

Further, with the knowledge that statistical error has a power law shape with slope $1/2$, the $Ns_{\sigma=1\%}$, number of injected quasiparticles needed to reach 1 percent statistical error (i.e. σ equal to 1 in the percent of μ), was calculated for every month, for four selected SOLARPROP energy bins.

In Figure 4 the red line represents the expected value of the needed number of injected particles to reach a one percent statistical error. The red line values were calculated by the method estimated for the 1D model (i.e. $10^4/T_{kin}$ $[in\ GeV]$). One could notice that the previously suggested strategy of a number of injected particles in the backward 1D method [13] is still valid for the 2D SOLARPROP model for years without extreme solar modulation, i.e. 1993-2000. Statistical error is much higher during the period of extreme solar modulation, for example, in June 1991, when for energy $0.384 GeV$ is needed 1.23 million injected particles which is 46 times more than the suggested strategy. However, this moth has extremely strong solar modulation conditions. For a tested period of 108 months suggested strategy gives a good estimation for 91.7 percent of the time for energies 0.384GeV and 1.297GeV. The percentage of estimations when the suggested strategy gives results with a statistical error smaller than 1 percent is shown in Table 1. It shows the number of points in percent that lie under and above the estimated line marked by red color in Figure 4.

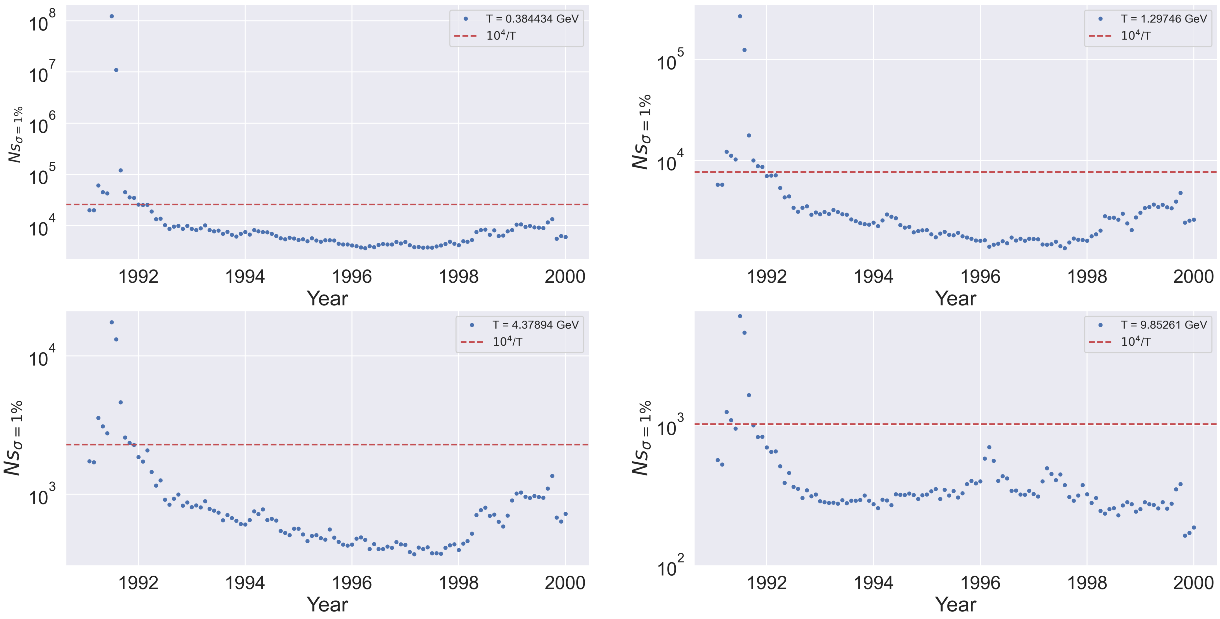

Figure 4: Calculated number of injected needed to reach 1 percent statistical error with respect to different months, for four energy bins at 1 AU.

Table 1: Percent of the points above and under the suggested number of injected particles to reach 1 percent statistical error

Kinetic energy [GeV]	Percent of points above	Percent of points under
0.384434	8.3	91.7
1.29746	8.3	91.7
4.37894	7.4	92.6
9.85261	4.6	95.4

4. Conclusions

Statistical error is much higher during the period of extreme solar modulation (for example June 1991). The previously suggested strategy $10^4/T_{kin}[in\ GeV]$ of injected particles in backward 1D method [13] is still valid for 2D SOLARPROP model for years without extreme solar modulation, i.e. 1993-2000.

5. Acknowledgment

The Slovak VEGA grant agency, project 2/0077/20 is acknowledged, for the support.

)042

References

[1] Parker, E. The passage of energetic charged particles through interplanetary space. Planet. Space Sci. 1965, 13, 9–49.

[2] Kota, J. Energy loss in the solar system and modulation of cosmic radiation. In Proceedings of the 15th International Cosmic Ray Conference, Plovdiv, Blgaria, 13–26 August 1977; pp. 186–191.

[3] Du Toit Strauss, R.; Effenberger, F. A Hitch-hiker's Guide to Stochastic Differential Equations. Solution Methods for Energetic Particle Transport in Space Physics and Astrophysics. Space Sci. Rev. 2017, 212, 151–192.

[4] Florinski, V.; Pogorelov, N.V. Four-dimensional transport of galactic cosmic rays in the outer heliosphere and heliosheath. Astrophys. J. Lett. 2009, 701, 642–651.

[5] Strauss, D.T.; Potgieter, M.; Kopp, A.; Büsching, I. On the propagation times and energy losses of cosmic rays in the heliosphere. J. Geophys. Res. Earth Surf. 2011, 116.

[6] Strauss, R.D.; Potgieter, M.S.; Büsching, I.; Kopp, A. Modelling heliospheric current sheet drift in stochastic cosmic ray transport models. Astrophys. Space Sci. 2012, 339, 223–236.

[7] Effenberger, F.; Fichtner, H.; Scherer, K.; Büsching, I. Anisotropic diffusion of Galactic cosmic ray protons and their steady-state azimuthal distribution. Astron. Astrophys. 2012, 547, A120.

[8] Strauss, D.T.; Potgieter, M.; Ferreira, S.; Fichtner, H.; Scherer, K. Cosmic ray modulation beyond the heliopause: A hybrid modeling approach. Astrophys. J. 2013, 765, L18.

[9] Zhao, L.-L.; Qin, G.; Zhang, M.; Heber, B. Modulation of galactic cosmic rays during the unusual solar minimum between cycles 23 and 24. J. Geophys. Res. Space Phys. 2014, 119, 1493–1506.

[10] Engelbrecht, N.E.; Burger, R. Sensitivity of cosmic-ray proton spectra to the low-wavenumber behavior of the 2d turbulence power spectrum. Astrophys. J. Lett. 2015, 814, 152.

[11] Moloto, K.D.; Engelbrecht, N.E.; Burger, R. A Simplified Ab Initio Cosmic-ray Modulation Model with Simulated Time Dependence and Predictive Capability. Astrophys. J. Lett. 2018, 859, 107.

[12] Shen, Z.-N.; Qin, G.; Zuo, P.; Wei, F. Modulation of Galactic Cosmic Rays from Helium to Nickel in the Inner Heliosphere. Astrophys. J. 2019, 887, 132.

[13] Mykhailenko, V.; Bobik, P. Statistical Error for Cosmic Rays Modulation Evaluated by SDE Backward in Time Method for 1D Model.Fluids 2022,7,46

[14] Moloto, K.; Engelbrecht, N.; Strauss, R.; Moeketsi, D.; Berg, J.V.D. Numerical integration of stochastic differential equations: A parallel cosmic ray modulation implementation on Africa's fastest computer. Adv. Space Res. 2018, 63, 626–639.

PoS(ECRS)042

[15] Zhang, M. A Stochastic Differential Equation Approach to Cosmic Ray Transport. In Proceedings of the Numerical Modeling of Space Plasma Flows: Astronum 2007 ASP Conference Series, Paris, France, 10–15 June 2007; Volume 385, p. 63.

[16] Fiandrini, E.; Tomassetti, N.; Bertucci, B.; Donnini, F.; Graziani, M.; Khiali, B.; Conde, A.R. Numerical modeling of cosmic rays in the heliosphere: Analysis of proton data from AMS-02 and PAMELA. Phys. Rev. D 2021, 104, 023012.

[17] Rolf Kappl, SOLARPROP: Charge-sign dependent solar modulation for everyone, Computer Physics Communications, Volume 207, 2016, Pages 386-399, ISSN 0010-4655.

PoS(ECRS)042