Propagtion of CR secondary species and gamma-ray emissions in MHD simulations of galaxies

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We develop a new algorithm for the production and propagation of cosmic ray (CR) secondary elements within the framework of Cosmic Ray Energy SPectrum (CRESP) module of Piernik MHD code (Ogrodnik et al. ApJS 253, 18, 2021). CRESP is based on the piece-wise power-law (coarse-grained) method for self-consistent and numerically efficient cosmic ray (CR) propagation in the magnetized ISM of galaxies. We demonstrate the capability of our code to model spectral evolution of multiple CR species, including primary ones, as secondary products obtained by spallation process. We perform a series of stratified box test simulation which reproduce the physical conditions of the interstellar medium of galaxies such as the Milky Way. We show preliminary results concerning secondary $^{11}$B to primary $^{12}$C ratio, suggesting that this observable depends on parameters such as diffusion of CR or density of the gas. Future extension of the present work towards modelling of hadronic gamma-ray emission is also discussed.

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1. Introduction and Motivation

So far, two approaches have been developed to implement CR propagation in galaxies numerically: GALPROP [1], where the structure of the interstellar medium (ISM) deduced from observational data is stationary, and the self-consistent approach, which combines the propagation of CRs with the system of MHD equations that include the dynamical coupling of CRs with thermal gas and magnetic fields (see eg. [2]). Some recent papers combine both approaches [3–5], with spectrally resolved CR propagation in the MHD context. The CRESP algorithm [4] added recently to PIERNIK MHD code implemented the momentum-dependent transport of spectrally resolved CR electrons on Eulerian grids. Our goal is to extend the CRESP algorithm with the capability to model the propagation of multiple spectrally resolved CR species, including the production and propagation of secondary CR elements.

Within the present project, we extend the data structure of PIERNIK code with the capability to propagate an arbitrary number of spectrally resolved CR species, including primaries and secondaries. We assume that primary CRs are accelerated in astrophysical MHD shocks. At the same time, the secondary CR particles result from hadronic collisions of CR primaries against the kernels of the interstellar gas.

2. Model and method

We assume that CR species, labelled by $a$, are represented by a distribution function $f_a(\vec{r}, \vec{p}, t)$ in phase space and are described by a system of Fokker-Planck equations [6]:

$$
\left( \partial_t + \vec{v} \cdot \nabla \right) f_a(\vec{r}, \vec{p}, t) = \frac{1}{3} \nabla \cdot \vec{v} \partial_p f_a(\vec{r}, \vec{p}, t) + \frac{1}{p^2} \partial_p (p^2 b + D_{pp} \partial_p) f_a(\vec{r}, \vec{p}, t) + q_a(\vec{r}, \vec{p}, t),
$$

(1)

where $D$ and $D_{pp}$ are diffusion coefficients in real and momentum space, $b = dp/dt$ is the energy loss term of processes such as synchrotron, inverse Compton or adiabatic expansion, and $q(\vec{r}, \vec{p}, t)$ represents the source, from astrophysical phenomena, or secondary production processes, as we will discuss here. Following [2–5, 7], we assume the distribution in the phase space is a power-law in momentum, then in the bin $l$ of range $[p_{l-1/2}, p_{l+1/2}]$ it is:

$$
f_a(p) = f_{a,l-1/2} \left( \frac{p}{p_{l-1/2}} \right)^{-q_l},
$$

(2)

where $q_l$ is the power index of the bin, $f_{a,l-1/2}$ is the distribution function at $p_{l-1/2}$. We aim to resolve equation 1 for primary and secondary species with the implementation of secondary nuclei particle production through hadronic processes. For the number density per unit of momentum $dn_a/dp$, the source term for secondary nuclei is:

$$
q_{a,N}(\vec{r}, p, t) = \sum_i n_i \sum_j \beta \int dp' \frac{dn_j}{dp'} (p') \frac{d\sigma_{ij}}{dp}(p, p').
$$

(3)

where $n_i$ represents the density of the interstellar medium (ISM) species $i$ (mostly hydrogen), $n_j$ is the number density of primary species $j$, $p'$ is the momentum of primary particles, $p$ the total
momentum, \(d\sigma_{ij}/dp\) is the cross section per unit of momentum, \(\beta\) is the velocity of CR. The sources \(Q_{a,N}\) and \(S_{a,N}\) are source terms of equation 1 after integration to get the equations for number and energy density. We end up with the following source terms in bin \(l\) [eg. 5, 8]

\[
Q^l_{a,N} = \sum_i n_i \sum_j \beta \sigma_j n^l_j, \tag{4}
\]

\[
S^l_{a,N} = \sum_i n_i \sum_j \beta \sigma_j e^l_j, \tag{5}
\]

where \(n^l_j\) and \(e^l_j\) are number and energy density of species \(j\) in bin \(l\). \(\sigma_j\) are total cross sections for respective spallation processes. Actual values of these are similar to the data from the textbook by Longair [9]. Equations 4 and 5 will correspond to loss terms for primaries, while it will be an injection source for secondaries. Due to the difference of atomic numbers \(A_j\) and \(A_S\), the spallation process shifts the energy spectra by changing the momentum of CRs by a factor of \(A_j/A_S\) in momentum space, and the algorithm has to take into account these effects.

We implement such processes in the Piernik MHD code and CRESP algorithm [4] (Ogrodnik et al, 2020) using a piece-wise power-law method for the distribution function. We consider advection, energy-dependent, magnetic field-aligned diffusion \((D \propto p^{0.5})\), and pressure gradient effects in the dynamical evolution of CR species in the wind-driven by CR protons. 2D simulations are performed to follow evolution of primaries and secondaries. Primaries are initialized with relative abundances to protons and injected in the interstellar medium (ISM) via supernovae (SN) explosions.

3. Galactic plane simulations with primary and secondary nuclei

We assume a local rectangular patch of the ISM stratified by vertical gravity, with the thermal gas density \(n \approx 1 \text{ cm}^{-3}\) at the galactic midplane and temperature \(T \approx 7000 \text{ K}\). We assume an initial magnetic field of 1 \(\mu\text{G}\) along \(y\) axis, and we inject randomly located supernovae as in [10], with surface frequency of 20 \(\text{kpc}^{-2}\text{Myr}^{-1}\).

We present the results obtained from simulations with Piernik MHD code, focusing on the evolution of primary Carbon 12 and secondary Boron 11 over 55 Myr. We compute the evolution of the system in two spatial dimensions \(y, z\), corresponding to the azimuthal and vertical directions in the local rectangular patch of the ISM. The computational box size is 1000 pc \(\times\) 2000 pc with the resolution of 48 \(\times\) 96 grid cells in the horizontal and vertical directions respectively. We assume CR diffusion coefficients of the order of \(3 \times 10^{27} \text{ cm}^2\text{s}^{-1}\).
We observe dynamical effects of CRs in figure 1 related to the pressure gradient ($\nabla p_{\text{CR}}$) of the CR dominant proton component. Interstellar gas becomes strongly buoyant due to the pressure of the nonthermal components: cosmic rays and magnetic fields [see 11, 12, eg.]. Cosmic rays contribute essentially to the structuring of the ISM. In these highly variable conditions the spectra of primaries and secondaries evolve in a position-dependent manner.

Figure 2 presents $^{11}\text{B}$ to $^{12}\text{C}$ ratio around the mid-plane from our simulation and with respect to momentum, taken at different position. Different shapes appear here, what indicates that the ratio of B to C in the computational volume plane strongly depends on the position. These results differ from the previous simulation work and terrestrial experimental data (see [13]). We can therefore investigate if the B to C ratio can be a universal property, or it rather depends on the position in the gaseous disk or galactic halo.

In the future work, we will investigate more realistic environments of the ISM in order to comprehend better the physics of spectrally resolved CRs in a dynamical galactic context, and to compare our simulations with observational data. The important next step is to investigate this dependence in a more complete framework of stratified box and global galactic disks.
Figure 1: 2D simulation of primary $^{12}\text{C}$ and secondary $^{11}\text{B}$ number density in a galactic plane over 55 MyR, with the corresponding spectrum obtained from some points of the plane. The light blue spectrum corresponds to the top marker, the dark blue to the middle marker, and the red one to the bottom marker. We observe stratification of CR by gravity. The blue line in the corner is a numerical artefact to compute the left cutoff.
Figure 2: $^{11}\text{B}$ to $^{12}\text{C}$ ratio with respect to normalized momentum, with H density of $d = 1\text{cm}^{-3}$ and $D = 3.10^{27}\text{cm}^2\text{s}^{-1}$, taken at different positions in the $(y, z)$ plane. We observe a maximum (or a minimum) for $p \approx 10^5$, but the shape is varying, indicating it depends on the varying parameters and the position in the map.

4. Gamma ray modeling as future goal

As a side part of the current project we introduce the algorithm for gamma ray production resulting from hadronic collision of protons in the ISM. We aim to calculate and implement a piece-wise power law method for the gamma-ray spectrum from the reaction (see [14]):

$$p + p \rightarrow p + p + \pi^0,$$  \hspace{1cm} (6)

$$\pi^0 \rightarrow 2\gamma.$$  \hspace{1cm} (7)

As for CR secondaries, we have a source function $q_{\gamma}$:

$$q_{\gamma}(\vec{r}, E_{\gamma}) = 2\int_{E_{\gamma} + m_{\pi^0}c^4}^{+\infty} \frac{dE_{\pi^0}(E_{\pi^0}^2 - m_{\pi^0}c^4)}{dE_{\gamma}} \left(E_{\pi^0}^2 - m_{\pi^0}c^4\right)^{-1/2} q_{\pi^0}(\vec{r}, E_{\pi^0}),$$  \hspace{1cm} (8)

where the pion source $q_{\pi^0}(\vec{r}, E_{\pi^0})$ is obtained from proton distribution (see ref [14, 15] for derivation and context):

$$q_{\pi^0}(\vec{r}, E_{\pi^0}) = 2^3 n_{\text{ISM}}(\vec{r}) \sigma_{pp} c \frac{n_p(\vec{r})}{E_0} \left(\frac{6E_{\pi^0}}{E_0}\right)^{-4/3(q_0-1/2)}.$$  \hspace{1cm} (9)
This depends on the number density of gas and CR protons, the proton-proton cross-section $\sigma_{pp}$ and the power index $q_p$. Once the distribution function of protons is known, we can access gamma-ray distribution in the ISM. Preliminary tests have already been performed outside the CRESP algorithm (see figure 3).

![Gamma ray spectrum and Relative error graphs](image)

**Figure 3:** Comparison between the exact analytical formula of equation 8 with beta function, and the numerical solution, for $\alpha_p = 2$. Here, our figure contains 20 bins. The relative error, which is of maximum $\approx 10\%$, is acceptable.

5. **Summary and Conclusion**

We presented a general model to compute the CR evolution of nuclei with secondary production from spallation processes. We described how to numerically implement these processes using the piece-wise power law method for the distribution function in phase space. We also showed preliminary test simulations with Piernik MHD code and its CRESP algorithm to compute CR spectral evolution of multiple (primary and secondary) components of CRs in a MHD environment. We presented some important observables from our setup related to CR physics: CR energy spectra and primary $^{12}$C to secondary $^{11}$B ratio. The future of this study, concerning the investigation of a local stratified box, a global galactic disk, production of secondary CR species and the gamma-ray emission spectrum, has been discussed.

6. **Acknowledgements**

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7. References


