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Some mathematical and physical aspects of the gravitational collapse of massive stars

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In this work we consider a new coordinate system in Schwarzschild spacetime. This coordinate frame describes the exterior geometry of a black hole but the black hole itself is a non-singular point. From the observational point of view we show that there is no difference between a usual black hole and our model.

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1. Introduction

At the end of its life cycle a massive star undergoes continuous gravitational collapse. The result of such a process should be a black hole [1–3]. Within a short appropriate time period the star's surface crosses its gravitational radius and continues to collapse to a spacetime singularity. However, for a distant observer, the star's surface will never cross its gravitational radius because the coordinate time t tends to infinity whereas the coordinate r tends to the gravitational radius r_g. The event horizon will never form for a distant observer. So we can't get any information from the region $r < r_g$. Thus, from a practical point of view, it makes no sense to consider the region inside the gravitational radius. The question is what happens if we exclude the region $0 \leq r < r_g$ and consider only the region $r_g \leq r < +\infty$? In this case a black hole represents a non-singular point. We show that for a distant observer there is no difference between these models.

The system $G = c = 1$ will be used throughout the paper.

2. A black hole as a point

The exterior geometry of a spherically symmetric black hole is described by the well-known Schwarzschild solution which, in coordinates $\{t, r, \theta, \varphi\}$, is given by:

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 ,$$
$$d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi . \tag{1}$$

Here M is the mass of the black hole. One can see that the metric components g_{rr} and g_{tt} change their sign when $r = r_g$. So one can say that space become time and vice versa. Also the Kretschmann scalar diverges only at $r = 0$ and has a finite value at $r = r_g$. So to exclude the singular point from consideration we perform the following coordinate transformation:

$$x = r - 2M = r - r_g \tag{2}$$

and obtain:

$$ds^2 = -\left(1 + \frac{r_g}{x}\right)^{-1} dt^2 + \left(1 + \frac{r_g}{x}\right) dx^2 + (x + r_g)^2 d\Omega^2 . \tag{3}$$

And the final step is to consider only positive values of x. This model represents a point at $x = 0$. This point is not singular because the Kretschmann scalar is finite at $x = 0$. Now we find out what difference the distant observer sees.

3. A shadow

In this paper we consider the question of a shadow which might be cast by a point in our model and will show that the shadow is the same. We can't consider precession of the perihelion and light rays deflection [4] within this paper but for a distant observer there is no difference between these two models.

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To consider the question of a shadow which might be cast by the object at $x = 0$ one should, first of all, find the energy E, the angular momentum L, and the impact parameter $b = \frac{L}{E}$[5, 6]. The energy and the angular momentum for metric (3) are given by:

$$E = \left(1 + \frac{r_g}{x}\right)^{-1} \left(\frac{dt}{d\lambda}\right) ,$$
$$L = \left(x + r_g\right)^2 \frac{d\varphi}{d\lambda} , \tag{4}$$

here λ is the affine parameter. We can also consider only the equatorial plane motion $\theta = \frac{\pi}{2}$ due to the spherical symmetry.

Substituting (4) into the condition $g_{ik} \frac{dx^i}{d\lambda} \frac{dx^k}{d\lambda}$ and solving it for $\frac{dr}{d\lambda}$ one can obtain:

$$\dot{r}^2 + V_{eff} = 0 ,$$
$$V_{eff} = \frac{L^2 x^2}{(x + r_g)^3} - E^2 . \tag{5}$$

Here V_{eff} is the effective potential.

To calculate the shadow one should know the turning point x_{tp} i.e. $V'_{eff}(x_{tp}) = 0$ ($V' \equiv \frac{dV}{dx}$). From the definition (5) one can obtain:

$$V'_{eff} = \frac{r_g - 2x}{(x + r_g)^4} = 0 ,$$
$$x = \frac{r_g}{2} . \tag{6}$$

Now we should substitute the turning point x_{tp} into (5) to obtain the size of the shadow:

$$(x + r_g)^3 = x b^2 ,$$
$$b = \sqrt{27}\frac{r_g}{2} = 3\sqrt{3}M . \tag{7}$$

And this is exactly the same result as in the Schwarzschild model. So for a distant observer there is no difference if the central object is a black hole or a point.

4. Discussion

We have found that for a distant observer there is no difference between the two models i.e. the standard black hole model or a point. However, if we consider a black hole as a spacetime point at $x = 0$ then we exclude the singularity because the Kretschmann scalar:

$$K \equiv R_{iklm} R^{iklm} = \frac{12 r_g}{(x + r_g^6)} , \tag{8}$$

is finite at $x = 0$:

$$K|_{x \to 0} = \frac{12}{r_g^4} . \tag{9}$$

This is because the point $x = 0$ corresponds to the event horizon $r = r_g$ in the Schwarzschild model and the Kretschmann scalar is also finite at $r = r_g$.

However, for a distant observer the star's surface will never cross its gravitational radius. Thus, one can't consider a spacetime point. For a realistic physical situation one should consider a ball of matter of radius ε around the point $x = 0$. It is worth noting that this ball is regular everywhere i.e. this ball is not singular. For example, a magnetic field could exist whose lines can pass through this ball. So observing the manifestations of these lines could be possible, and these possible observations distinguish our model from the standard Schwarzschild one.

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