PROCEEDINGS OF SCIENCE

PoS(MUTO2022)052

# Cowling resistivity and Joule dissipation in the solar atmosphere

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Joule dissipation rates in the magnetic flux tubes at different heights of the solar atmosphere, from the photosphere to the corona, are studied. It is assumed that the generation of electric currents is caused by convective motions at the photospheric foot-points of the flux tubes. It is shown that the role of ion–atom collisions in the Joule dissipation and solar plasma heating is decisive. The Cowling resistivity exceeds the classical (Spitzer) resistivity not only in the weakly ionized layers of the atmosphere but also in the corona with the relative density of the neutral atoms of about $10^{-7}$ cm$^{-3}$ if the electric currents are $I \geq 2\times10^{8}$ A. As an illustration of the altitude dependence of the Cowling resistivity for different magnitudes of the electric current, we applied the atmospheric model developed by Avrett and Loeser [1]. We have shown that the maximal dissipation occurs in the transition region. The role of Cowling resistivity in Joule dissipation increases with the filamentation of magnetic flux tubes.

*The Multifaceted Universe: Theory and Observations - 2022 (MUTO2022)*
*23–27 May 2022*
*SAO RAS, Nizhny Arkhyz, Russia*

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## 1. Introduction

It was shown in a number of studies that the neutral component of the plasma plays an important role in the Joule dissipation both in the stellar atmospheres and in the magnetospheres and ionospheres of planets (see, e.g., [2–4]). A number of phenomena on the Sun and stars are associated with the dissipation of the electric currents, for example, flare precursors ([5] and references therein), flare energy release [6–8], heating of magnetic loops up to 10–15 MK [9], heating of the stellar coronae [10, 11]. The flare energy release in the partially ionized plasma of the solar photosphere and chromosphere has been repeatedly considered (see, e.g., [2, 3, 12]). However, a detailed analysis of the Joule dissipation rates in different layers of the solar atmosphere (photosphere — chromosphere — transition region — corona) with allowance for the neutral plasma component has not been performed. In this paper, we use the solar atmosphere model proposed by Avrett and Loeser [1] and show that

1. the Joule dissipation is characterized by a maximum in the transition region;
2. despite the relatively low neutral density, $n_\mathrm{a}/n_\mathrm{e} \approx 10^{-7}$, in the corona with temperature $T \approx$ 1–2 MK, the Cowling resistance exceeds the classical (Spitzer) resistance if the electric currents are $I \geq 2 \times 10^8$ A;
3. even for the strong currents, $I > 10^{11}$ A, the velocity of the relative motion of ions and electrons in the current is much lower than the ion sound speed. This contradicts the popular notion about an important role of anomalous resistance in the Joule dissipation in the solar atmosphere.

## 2. The problem statement

Let us consider the dissipation of longitudinal electric current in a coronal magnetic flux tube (magnetic flux rope) with the foot-point anchored at the photosphere. Slow electric current changes in a flux tube, with a magnetic loop as a specific case, are described by the equation [13]

$$\frac{1}{c^2}\frac{d(LI)}{dt} + RI = \Xi \tag{1}$$

where $L$ is the flux tube inductance, $I$ is the electric current,

$$R(I) \approx \frac{1.5\, l\, F^2 I^2}{\pi r_1^4 c^4 (2-F) m_\mathrm{i} \nu'_\mathrm{ia}} \tag{2}$$

describes the resistance in the electromotive force (EMF) region of the flux tube associated with Cowling resistance and depending on the magnitude of the electric current. The EMF is caused by the photospheric convection at the foot-point of a flux tube:

$$\Xi = \frac{l_1}{\pi c r_1^2}\int_0^{r_1} V_\mathrm{r} B_\varphi 2\pi r dr \approx \frac{|\bar{V}_\mathrm{r}| I l_1}{c^2 r_1} \tag{3}$$

In Equations (2)–(3), $V_\mathrm{r}$ is the radial component of the convective motion speed in the photosphere, $B_\varphi \approx 2I/cr_1$ is the azimuthal component of the magnetic field generated by the longitudinal current

$I$, $l$ and $r$ are, respectively, the length and radius of a flux tube, the subscript "1" here and further indicates correspondence to the area of EMF action, $\nu'_{kj} = \left[m_j/(m_k + m_j)\right]\nu_{kj}$, where $k$ and $j$ determine the type of particles, $F = n_a/(n_a + n_e)$, where $n_a$ and $n_e$ are the electron and atom number densities, respectively, and $\nu'_{ia} \approx 1.6 \times 10^{-11} F(n_e + n_a)\sqrt{T}$ Hz is the effective ion-atom collision frequency [13].

The current generation in the region of EMF has a threshold at the convection velocity $V_r$. Zaitsev et al. [14] have shown that the Reynolds magnetic number for the ionized plasma component should satisfy the condition $Re_M = 4\pi|V_r|r_1\sigma_1/c^2 > 4$. For the solar photospheric convection, this means $V_r > 0.3\ \mathrm{km\,s^{-1}}$. The steady-state current value ($RI = \Xi$) is determined as

$$I \approx 0.8\left[|V_r|\pi r_1^3 c^2 n m_i \nu'_{ia}(2 - F)F^{-2}\right]^{1/2}. \tag{4}$$

The Joule dissipation rate per unit volume of the flux tube is given by the formula [13]

$$q = \frac{j_z^2}{\sigma} + \frac{F^2 B_\varphi^2 j_z^2}{(2 - F)c^2 n m_i \nu'_{ia}}\ \mathrm{erg\,cm^{-3}s^{-1}}. \tag{5}$$

Here $j_z = \frac{I}{\pi r_1^2}$ is the electric current density, subscript $z$ means the projection on the axis of symmetry of the magnetic flux tube, $\sigma = \frac{n e^2}{m_e\left(\nu'_{ei} + \nu'_{ea}\right)}$ is the Spitzer conductivity.

To compare the dissipation rates due to the Spitzer and Cowling conductivities, we represent Equation (5) as

$$q = \frac{j_z^2}{\sigma} + K_{\mathrm{cow}}\frac{j_z^2}{\sigma}, \tag{6}$$

where $K_{\mathrm{cow}}$ depends on the presence of neutral atoms in the plasma, as well as on the magnitude of the electric current flowing in the magnetic tube

$$K_{\mathrm{cow}} = \frac{F^2}{(2 - F)}\omega_e\,\tau_e\,\omega_i\,\tau_{ia}, \tag{7}$$

$$\omega_e = \frac{eB_\varphi}{cm_e}, \quad \omega_i = \frac{eB_\varphi}{cm_i}, \quad \tau_e = \frac{1}{\nu'_{ei}}, \quad \tau_{ia} = \frac{1}{\nu'_{ia}}. \tag{8}$$

The effective frequency of electron-ion collisions is $\nu'_{ei} = \nu_{ei}$ due to the large mass ratio $m_i/m_e \gg 1$, where [15]

$$\nu_{ei} \approx \frac{5.5 n_e}{T^{3/2}}\Lambda, \quad \Lambda(T \le 4 \times 10^5) \approx \ln\left(220\frac{T}{n_e^{1/3}}\right), \quad \Lambda(T > 4 \times 10^5) \approx \ln\left(10^4\frac{T^{2/3}}{n_e^{1/3}}\right). \tag{9}$$

Let us consider the definition of gyro frequencies in Equations (8), which in this case are proportional to the magnitude of the electric current in the flux tube. This means that at sufficiently large longitudinal currents in the magnetic flux tube the condition $B_\varphi \gg B_z$ can be satisfied, which can lead to the development of MHD instabilities.

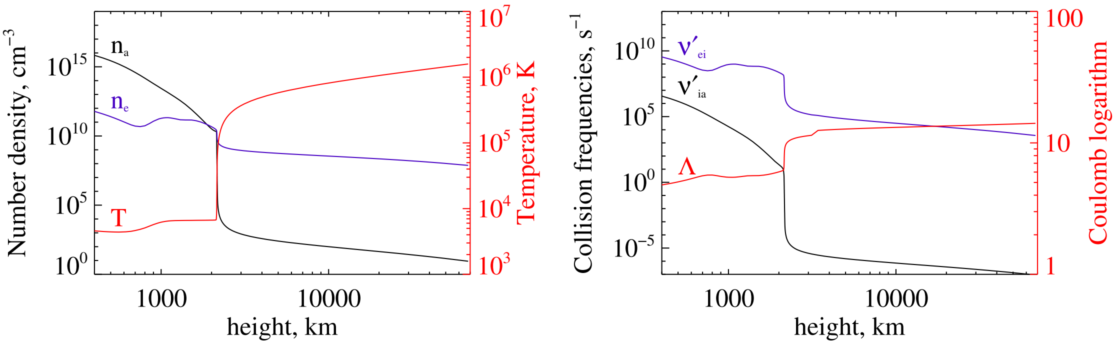

**Figure 1:** Left panel: number density of neutrals $n_a$, electrons $n_e$ and temperature $T$ as a function of height [1] . Right panel: collision frequencies $\nu'_{ei}$, $\nu'_{ia}$ and the Coulomb logarithm $\Lambda$ *vs* height.

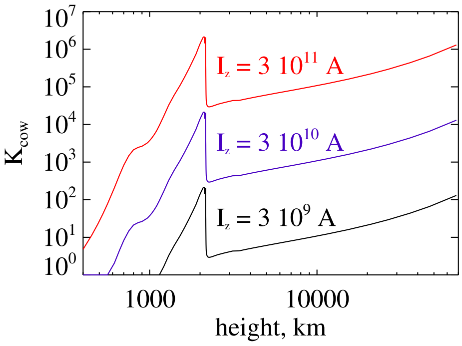

**Figure 2:** $K_{\rm cow}$ values for different electric currents depending on the height above the level $\tau_{5000} = 1$.

## 3. Results

To illustrate the altitude dependence of $K_{\rm cow}$, which determines the Joule dissipation rate, we will use the Avrett and Loeser solar atmosphere model [1] (see left panel in Figure 1). The collision frequencies $\nu'_{ia}$, $\nu'_{ei}$ and the Coulomb logarithm (Equation (9)) are shown in the right panel in Figure 1. The $K_{\rm cow}$ values for typical solar flare currents and the flux tube radius $r_1 = 10^8$ cm are shown in Figure 2. It can be seen that the $K_{\rm cow}$ maximum corresponds to the transition region.

It also follows from Figure 2 that at altitudes less than 1000 km, the Cowling resistance is less than the Spitzer one ($K_{\rm cow} < 1$) at currents $I < 3 \times 10^9$ A. This is due to the increase in the role of inter-particle collisions as the density of the atmosphere increases with decreasing altitude.

At coronal heights for an optically thin medium with temperature $T > 2 \times 10^5$ K, the main contribution to the neutral component comes from hydrogen, while the relative abundance of neutrals is determined by the empirical formula [16]

$$F = 0.32 \times 10^{-3} \frac{1 + \frac{T}{T_{\rm H}}}{\left[\sqrt{\frac{T}{T_1}}\right]^{(2-b)} \left[1 + \sqrt{\frac{T}{T_1}}\right]^{(1+b)}} \exp\left(\frac{T_{\rm H}}{T}\right) \tag{10}$$

where $T_{\rm H} = 1.58 \times 10^5$ K is the hydrogen ionization temperature, and $T_1$ and $b$ are empirically defined constants: $T_1 = 7.036 \times 10^5$ K, $b = 0.738$. Within the temperature interval $T = 10^6$–$10^7$ K, Equation (10) can be represented as $F \approx 0.15/T$ with an accuracy of a few percent. This means that, at the coronal temperatures $T = (1–5) \times 10^6$ K, the relative fraction of neutral atoms $F$ in the coronal plasma varies within $(0.3–1.5) \times 10^{-7}$, i.e. is extremely small. Nevertheless, at sufficiently high current value, the Joule dissipation associated with ion-atom collisions, that is, with the Cowling conductivity, the second term in Equation (6), turns out to be more significant than the contribution of the Spitzer conductivity. This is because at sufficiently high currents, the Ampère force accelerates ions to significantly higher velocities compared to the relative velocity of electrons and ions in an electric current. In this case, accelerated ions give up their energy to neutrals in the process of one collision, while the energy exchange between electrons and ions occurs $m_e/m_i$ times slower [17]. Assuming $\nu'_{ei} \approx 50 n_e/T^{3/2}$ and taking into account the formulas for $\nu'_{ia}$ and $F \approx 0.15/T$ we obtain

$$K_{cow} = 7 \times 10^{-18} \frac{I^2}{n_e^2} \tag{11}$$

For typical values of $n_e = 10^9$ cm$^{-3}$ and $T = 1–5$ MK in the magnetic flux tube, the Cowling dissipation rate is comparable with the Spitzer one ($K_{cow} \approx 1$) at $I \approx 1.26 \times 10^8$ A. For $I \approx 3 \times 10^{10}$ A, $n_e = 10^9$ cm$^{-3}$, $T = 1$ MK the value of $K_{cow} \approx 5.7 \times 10^4$, i.e. despite the smallness of $F \leq 1.5 \times 10^{-7}$, the role of ion-atom collisions in the Joule dissipation and plasma heating is decisive.

## 4. Discussion

*Is the Joule dissipation due to anomalous resistivity real?*

Ion acoustic and ion cyclotron modes, which are usually associated with anomalous (turbulent) current dissipation in magnetic flux tubes (see, e.g., [18, 19]) never become unstable under typical current values because the drift velocity of electrons relative to ions ($u$) in an electric current is significantly smaller than ion acoustic velocity $c_s$.

$$u = \frac{I}{\pi r_1^2 e n} \ll c_s = \sqrt{\frac{\kappa_B T_e}{m_i}} \tag{12}$$

Indeed, assuming $I = 3 \times 10^{10}$ A, $r_1 = 10^8$ cm, $n = 10^9$ cm$^{-3}$, $T = 5 \times 10^6$ K, we obtain $u_{max} \approx 6 \times 10^3$ cm s$^{-1}$ $\ll c_s \approx 3 \times 10^7$ cm s$^{-1}$, i.e. the condition for ion acoustic instability is not fulfilled. The threshold of electrostatic ion cyclotron instability is even higher, $u \approx 0.3\, V_{T_e}(T_i/T_e)^{3/2}$ [19]. Therefore, along with the Cowling conductivity, here we took into account only the Spitzer conductivity $\sigma = e^2 n/(m_e \nu_{ei})$ .

*Joule heating threshold*

The threshold value of the electric current at which the Joule dissipation (Equation (6)) exceeds the losses due to radiation and electron thermal conductivity along the magnetic field can be estimated based on the following relations: $Q[\text{erg/s}] = qV > q_r V, q_T S$ . Here

$$q = K_{cow} \frac{j_z^2}{\sigma} \approx 2.2 \times 10^{-9} I^4 / \left(n_e^2 r^6 T^{3/2}\right) \text{ erg cm}^{-3}\text{s}^{-1}, \tag{13}$$

$$q_r \approx 10^{-19} n_e T^{-1/2} \text{ erg cm}^{-3}\text{s}^{-1} \text{ for } 10^5 < T < 2 \times 10^7 \text{K}, \tag{14}$$

$$q_T \approx 0.9 \times 10^{-6} T^{7/2} (\Delta z)^{-1} \ \ \mathrm{erg\,cm^{-2}s^{-1}}, \qquad (15)$$

where $\Delta z$ is the spatial scale of the temperature gradient, $V = S\,l$, $S = \pi r^2$. Moreover, under coronal parameters $T = 10^6$ K, $\Delta z = 10^9$ cm, $n = 10^8$ cm$^{-3}$, $l \approx H = \kappa_B T/(m_\mathrm{i} g_\odot) \approx 4 \times 10^9$ cm the ratio $q_T S/(q_r V) \approx 10^{13} T^4/(n_\mathrm{e}^2 \Delta z\, l) \approx 2.5 \times 10^2 \gg 1$. Then, from the inequality $Q > q_T S$, taking into account Equation (13) and Equation (15), we obtain $I^4 > 4 \times 10^2 T^5 n_\mathrm{e}^2 r^6/(l \Delta z)$, which gives $I > 10^{10}$ A.

*Application areas*

Our example with the Avrett and Loeser atmosphere model [1] can be used to interpret Joule dissipation not only in the open magnetic flux tubes, but also in coronal magnetic loops whose height is less than the atmosphere scale height, $H \approx 4.24 \times 10^3\, T \approx 4 \times 10^9$ cm. For open magnetic flux tubes, for example, for type II spicules considered now as candidates for solar corona heating and replenishing the corona with hot plasma [11, 20], the Cowling resistance maximum in the transition region means favorable conditions for heating of the current-carrying type II spicules.

*The role of flux tube filamintation*

For $I =$ const, the coefficient $K_\mathrm{cow} \propto B_\varphi^2 \propto I^2/(c^2 r_1^2)$ increases with decreasing flux tube cross-section. For example, brown dwarfs have $r_1 \approx 10^7$ cm, which is about an order of magnitude smaller than for the Sun. In this case, at similar currents $I \approx 3 \times 10^{10}$ A, the excess of the Cowling resistivity with respect to the Spitzer one is $K_\mathrm{cow} \approx 5.67 \times 10^6$, and the Joule dissipation rate at Cowling resistivity is comparable with the dissipation rate in the Spitzer case ($K_\mathrm{cow} \approx 1$) at the current $I \approx 1.26 \times 10^7$ A. Solar flux tube filamentation with $r_1 \leq 10^7$ cm increases the energy release as $dW/dt \propto K_\mathrm{cow} I^2 \propto I^4$.

*Evidence for Joule heating*

Recently, [21], using data of the Geostationary Operational Environmental Satellite (GOES) and the Reuven Ramaty High Energy Solar Spectroscopic Imager (RHESSI) for four solar flares, reported on the 'hot-onset' interval (10–15 MK) at the initial soft X-ray increase and definitely before any detectable hard X-ray emission. From the emission measure they obtain the electron density, $n_\mathrm{e} = (3\text{–}5) \times 10^{11}$ cm$^{-3}$, consistent with the chromospheric region. They also pointed out that the onset of soft X-ray emission occurs within the foot-points of low-lying loop regions rather than in coronal structures. The conclusion was that this phenomenon suggests the existence of a flare heating process that is physically different from that of the impulsive phase of the solar flares. On the other hand, [5] obtained the electric current $I \approx 10^{12}$ A in the GOES M6.5 class flare precursors on June 22, 2015. The phenomena studied in [21] may be a consequence of the Joule heating of the lower solar atmosphere described in this paper.

## Acknowledgments

This study was supported by the Russian Science Foundation projects No. 20-12-00268 (Sections 1 and 2) and No. 22-12-00308 (Sections 3 and 4).

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