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Lidov-Kozai perturbation in the motion of Jupiter Trojans

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The Lidov-Kozai perturbation was investigated in the region of Jupiter Trojans. These asteroids move in the mean-motion commensurability 1:1 with Jupiter. The study was carried out by means of numerical integration of equations of motion of real asteroids. The simplest dynamical model was used with Jupiter as an only perturbing body moving in a fixed elliptical orbit. After eliminating classical secular perturbations from osculating elements, it becomes possible to determine the influence of the Lidov-Kozai mechanism on the orbital inclination and eccentricity. As a result, it was found that for L_4-trojans the maximum eccentricity, and, accordingly, the minimum inclination is achieved with perihelion argument values $\omega = 30^\circ$ and 210°. For L_5-trojans the maximum eccentricity is located at the points $\omega = 150^\circ, 330^\circ$.

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1. **Introduction**

The Lidov-Kozai perturbation is a secular one which causes long-period periodic oscillations in the orbital inclination i and eccentricity e depending on the perihelion argument ω. This perturbation was named 'Lidov-Kozai mechanism' (LKM). LKM was first described by Lidov [1] in the analytical theory for artificial Earth satellites with high orbital inclinations to the ecliptic plane. Later, Kozai [2] developed a similar analytical theory for asteroids.

Under the influence of LKM the elements e and i change in concert. When eccentricity reaches its maximum, the inclination reaches the minimum, and vice versa. The consistency of the element change is caused by the relation: $(1\text{-}e^2)\cos^2(i) = \text{const}$. According to the theory, the maximum eccentricity and, accordingly, the minimum inclination of an asteroid's orbit is achieved with perihelion argument values ω = 90º, 270º.

Secular perturbations of asteroids are treated in the analytical theory by assuming that no mean-motion commensurabilities take place. However, there are very strong mean motion resonances in the asteroid belt, where analytical methods cannot be applied. LKM in the region of resonances was studied by Y. Kozai [3] with the use of the semi-analytical method. In his paper Y. Kozai considered the 1:1, 4:3, 3:2, 2:1, 2:3 and 3:1 cases.

T. Vinogradova [4] studied LKM in the region of the Hilda group (resonance 3:2), using two other methods. The first one is the empirical method of proper elements calculation, described in [5]. This method does not use analytical constructions or numerical integration to obtain the long-period terms of secular perturbations, but only the observable distribution of the osculating elements. The second method used numerical integration of the equations of motion. The results obtained for the 3:2 resonant region with the use of three different methods matched.

For the 1:1 resonance, i.e. for Trojans, Y. Kozai [3] showed that the maximum inclination is achieved with 2ω = 120º and its minimum with 2ω = 280º. In Y. Kozai's work the variations of e and i as functions of ω were graphically estimated. Attention must be paid to the fact that the interval between inclination maximum and its minimum is not equal to 90º.

In the present work, to study the LKM effect in the region of Jupiter Trojans, the second method described in T. Vinogradova's [4] work was used.

2. **Method**

The study of the LKM in the resonant region of Jupiter Trojans was carried out by numerical integration of equations of motion of asteroids for a long time. The orbital evolution of real Trojan asteroids moving in the region of Lagrangian points L_4 and L_5 was investigated. As a source of initial osculating elements, the Minor Planet Center (MPC) catalogue version 2022 March was used. A simplified dynamical model was applied for this aim. Jupiter, moving in a fixed elliptical orbit, was taken as an only perturbing body. The elements of Jupiter's orbit were fixed at the moment corresponding to the epoch of the asteroid elements.

Under the influence of perturbations from the major planet, the elements of asteroid orbits change. There are two types of long period, secular perturbations: classical and LKM. Under the influence of these perturbations, the eccentricities and inclinations of the orbits oscillate periodically. Classical secular perturbation forces the inclination to oscillate depending on the longitude of ascending node Ω, and eccentricity - depending on the longitude of perihelion ϖ. These oscillations are characterized by forced elements: i_f, Ω_f, e_f, ϖ_f.

As noted above, the LKM induces coupled oscillations of the inclination and eccentricity depending on the perihelion argument ω. Both types of secular perturbations act simultaneously.

Therefore the movement of asteroids is very complex and confusing. In the region of small inclinations and eccentricities, classical perturbations predominate, but in the region of high inclinations and large eccentricities, the LKM becomes predominant. As a rule, the period of the classical secular perturbations is much greater than the period of LKM.

To reveal the LKM influence, the classical secular perturbations should be eliminated from the osculating elements. As it was shown in the work of T. Vinogradova [5], classical secular perturbations can be excluded from osculating elements using the coordinate transformation formula if the corresponding forced elements are known. In the case of orbital inclination, this means a transformation from the ecliptic plane to a forced plane. For eccentricities the analogous procedure of coordinate transformation can be carried out.

For Jupiter Trojans there is no need to calculate the forced elements, because the corresponding orbital elements of Jupiter can be taken as the forced elements: $i_f = 1^\circ.3$, $\Omega_f = 100^\circ.5$, $e_f = 0.049$. It was shown by T. Vinogradova [4] that the forced perihelion longitude ϖ_f for two populations of Trojans are different. For L4 Trojans $\varpi_f = \varpi_{jup} + 60^\circ = 74^\circ$, whereas for L5 Trojans $\varpi_f = \varpi_{jup} - 60^\circ = 314^\circ$ ($\varpi_{jup} = 14^\circ.0$).

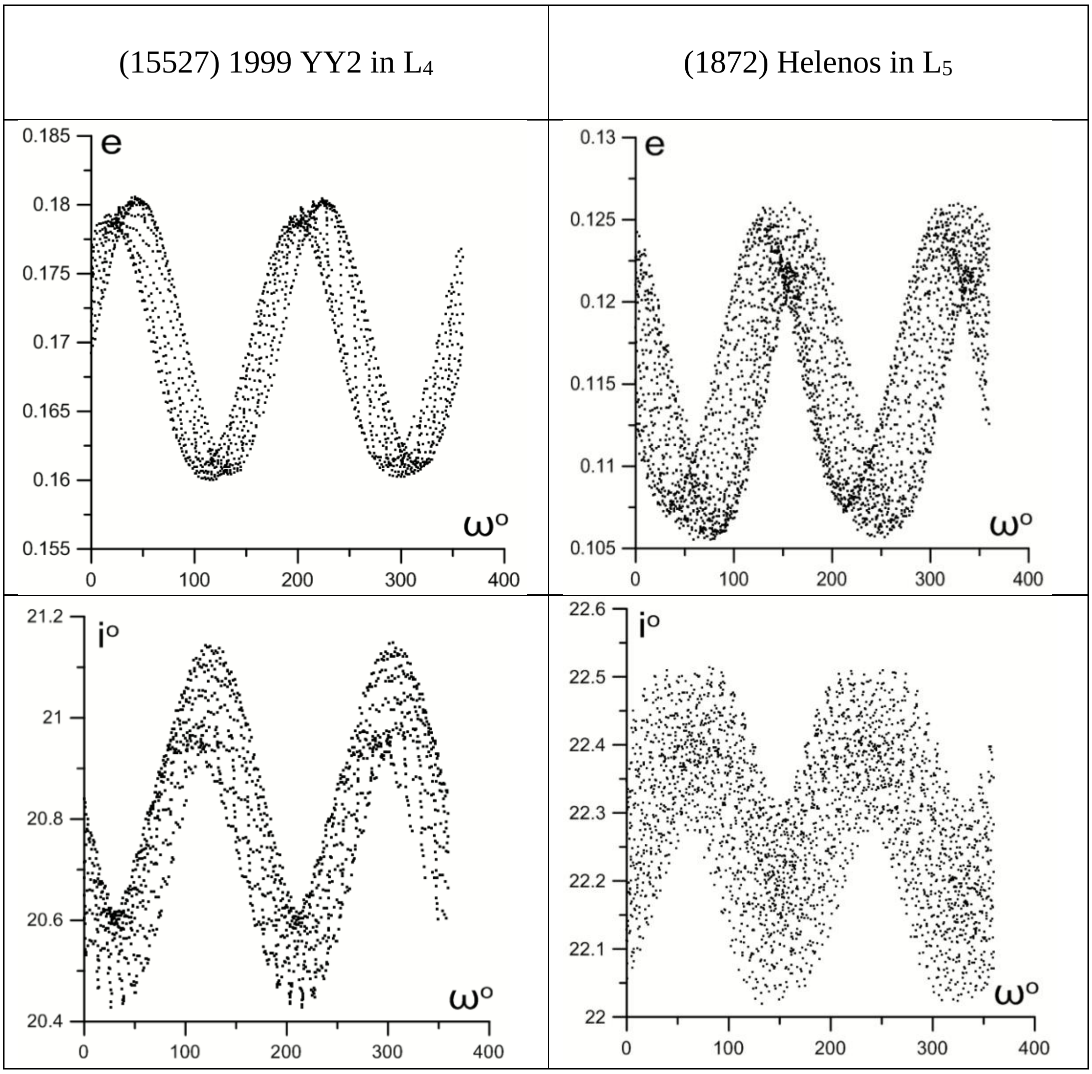
In the process of numerical integration, the classical secular perturbations were excluded from the osculating elements at every step. After this procedure, derived values of i and e were plotted versus ω. A complete picture of the orbital element change can be obtained only after several revolutions of ω. Thereafter, the entire range of possible values of the elements becomes outlined on the plot. The resulting plots were analyzed and the positions of the maxima and minima of the elements were found. Calculations were made for about two dozen asteroids.

3. **Results**

This method allows us to discover the LKM effect in the motion of Jupiter's Trojans. After eliminating the classical secular perturbations, the plot of the inclination and eccentricity against the perihelion argument ω clearly shows the LKM perturbation. Fig. 1 shows well how the orbital elements of asteroid (15527) 1999 YY2 from the L_4 region and asteroid (1872) Helenos from the L_5 region change under the action of the LKM. The choice of these asteroids is not random. Their orbits are characterized by rather large eccentricity and inclination. Therefore, amplitudes of the element oscillations are significant and the oscillations are well visible in the plots. It is difficult to resolve the oscillations of the elements in the plot if the orbital eccentricity and inclination are small.

One can see that the LKM manifests itself in the region of Jupiter's Trojans differently than in the main asteroid belt. The eccentricity maximum (or inclination minimum) is shifted relative to the usual position ($\omega = 90^\circ, 270^\circ$) and, moreover, its position is different for two groups of Trojans. For L_4, the maximum is shifted by -60° and is located at the points $\omega = 30^\circ, 210^\circ$, and for L_5 it is shifted by $+60^\circ$ to the points $\omega = 150^\circ, 330^\circ$. The interval between the positions of the maxima and minima is correct, it is equal to 90º.

The result derived for L5 asteroids is found to agree partially with those by Y. Kozai [3]. In Fig. 1 the maximum inclination for (1872) Helenos is achieved at the points $\omega = 60^\circ, 240^\circ$ (or $2\omega = 120^\circ$), and this coincides with his result. On the other hand, the inclination minimum is achieved with $2\omega = 300^\circ$, and this is inconsistent with Y. Kozai's conclusion ($2\omega = 280^\circ$). The result concerning the L_4 Trojans is missing in his work It can be assumed that Y. Kozai considered only the L_5 case in his paper.



(15527) 1999 YY2 in L_4

(1872) Helenos in L_5

Fig 1. The Lidov-Kozai perturbations in the orbital elements of (15527) 1999 YY2 in the L_4 region (left panels) and (1872) Helenos in the L_5 region (right panels).

References

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