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Estimation of orbital inclination angle for compact binary mergers

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The joint detection of gravitational wave (GW) event GW 170817 and the short-duration gamma-ray burst (GRB) event GRB 170817A, marked the beginning of GW multi-messenger astronomy and confirmed that binary neutron star mergers are progenitors of at least some short GRBs. An estimated joint detection rate of 0.3 - 1.7 per year between the LIGO Hanford, LIGO Livingston and Virgo GW network at design sensitivity, and the Fermi Gamma-ray Burst Monitor was predicted. However, to date, GW 170817/GRB 170817A joint detection has been the only event of its kind so far. Taking into the account that GRBs are narrowly beamed, we propose that previous mergers involving neutron stars, were orientated such that observation of the emitted GRB along the narrow jet was not possible. To support this hypothesis we aim to determine the inclination of previously detected Binary Neutron Star and Black Hole Neutron Star mergers through GW analysis. Here we present a preliminary analysis to estimate the orbital inclination parameter of GW 170817 based on different GW waveform models using BILBY, a Bayesian parameter estimation python library. Using only GW data, the inclination of the progenitor system of GW 170817 was found to be $\approx 152^\circ$, in agreement with constraints on this value through observations of GRB 170817A.

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1. Introduction

Gravitational waves (GWs) are travelling disturbances in spacetime that are caused by the acceleration of massive bodies. These ripples travel away from the source at the speed of light, containing information about the system [1]. Despite being one of the key predictions of General Relativity, the first indirect observational evidence of their existence was from observations of the orbital decay of the Hulse and Taylor binary pulsar PSR 1913+16 [2]. It was only on 14 September 2015 that the first direct observation of GWs by the Laser Interferometer Gravitational wave Observatory (LIGO) was achieved. GW 150914 was detected from the merger of 2 binary black holes marking the beginning of the first observing run (O1) by LIGO [3]. By studying these waves we are then able to study the motion and macroscopic behaviour of these systems.

On 17 August 2017 at 12:41:04 UTC, the first GW event produced from a binary neutron star (BNS) merger, GW 170817, was detected by the LIGO-Hanford and LIGO-Livingston detectors. Approximately 1.7 s later, the short gamma ray burst (GRB) event GRB 170817A was detected independently by the Fermi Gamma-ray Burst Monitor (GBM), and the Anti-Coincidence Shield for the Spectrometer for the International Gamma-Ray Astrophysics Laboratory [4]. This event marked the beginning of GW multi-messenger astronomy. GRBs are bursts of non-thermal gamma radiation (100 keV - 1 MeV photons) which are extra-galactic in origin [5]. First discovered in the late 1960s, these phenomena have puzzled astronomers for a long time. GRB sources are extremely luminous, emitting photons at $10^{51} - 10^{53}$ ergs in a few seconds, making them one of the most luminous EM phenomena in the universe [5]. They can be classified as either long or short GRBs with the two distinct classes arising from distinct physical phenomena [6]

The joint detection of GW 170817 and GRB 170817A confirmed that BNS mergers are the progenitors of short GRBs [4]. From the observations of the events of 17 August 2017, a joint detection rate of 0.1 - 1.4 per year between the LIGO and the Fermi-GBM during the third observing run (O3) of the LIGO-Virgo collaboration was predicted, and at design sensitivity a joint detection rate of 0.3 - 1.7 per year was predicted. However, O3 has been completed with no other joint detection. LIGO detected 1 more BNS merger and possibly 4 neutron star black hole (NSBH) merger events without coincident short GRB detection. This study aims to uncover a possible reason, based on orbital inclination angle, for the lack of short GRB and GW joint detection. We present the preliminary analysis of this study using GW 170817.

2. Numerical analysis method

2.1 Initial hypothesis

If we make the assumption that all merger events involving a neutron star produce an SGRB, two explanations for the lack of joint detections arise. The first being that the merger event was outside the horizon distance (which is the furthest a source can be detected above a signal-to-noise ratio threshold of 8 [7]) of the current ground based GW interferometers. The second possibility has to do with the orientation of the binary system that produces the GW and EM emission. GRBs are narrowly beamed, with a viewing angle of $1/\Gamma$, where Γ is the Lorentz factor of the relativistic

GRB jet. However, matter moves relativistically in the processes that produce GRBs meaning Γ is a very large value[8]. This means the GRB has a very narrow viewing angle. Most GRBs have typical opening angles of $1^\circ < \theta < 20^\circ$ [8]. According to the current models for the production of GRBs, a GRB is formed during the formation of a compact object. An accretion disk forms around this black hole and a funnel forms along the rotational axis [8]. In the case of merging neutron stars, this funnel will be perpendicular to the orbital plane of the system. Assuming mergers involving neutron stars produce GRBs, observation will not be possible if the system is not orientated almost face on with Earth.

2.2 Baye's theorem and parameter inference

In order to infer the waveform parameters describing the GW waveform, some *a priori* knowledge of the noise distribution and general shape of the waveform is required [9]. Since gravitational-wave signals are weak, uncertainties in these parameters may be large and *a priori* assumptions about the amplitudes and phase evolution of GWs have a significant impact on the reconstructed waveform. We therefore perform inference of the physical parameters describing a GW waveform within the framework of Bayesian parameter estimation [9]. To perform the inference we make use of `BILBY`, which is a python based user-friendly parameter estimation infrastructure that provides expert-level parameter estimation with a simplified syntax [10]. For this paper we use GW 170817 as a test to determine how efficient our analysis is in comparison to detailed analysis taking into account constraints placed by the detected EM counterpart. For the analysis we use a similar methodology to that used in [11].

According to General Relativity the GW waveform can be described by 2 tensorial polarisation modes given by

$$h_+(t) = -\frac{1+\cos^2\iota}{2}\left(\frac{G\mathcal{M}}{c^2 D}\right)\left(\frac{t_c - t}{5G\mathcal{M}/c^3}\right)^{-1/4}\cos[2\phi_c + 2\phi(t - t_c; M, \mu)] \tag{1}$$

$$h_\times(t) = -\cos\iota\left(\frac{G\mathcal{M}}{c^2 D}\right)\left(\frac{t_c - t}{5G\mathcal{M}/c^3}\right)^{-1/4}\sin[2\phi_c + 2\phi(t - t_c; M, \mu)] \tag{2}$$

where D is the distance from the source, ι is the angle between the observer's line of sight and the orbital angular momentum axis of the binary, μ is the reduced mass, $\mathcal{M} = \mu^{3/5}M^{2/5}$ where $M = m_1 + m_2$ and $\phi(t - t_c; M, \mu)$ is the orbital phase of the system. ϕ_c and t_c are the coalescence phase and time of the system, respectively, when the waveform is terminated [7]. At $t - t_c = 0$ then $\phi(0; \mu, M) = 0$. The above waveforms are also referred to as the chirp waveforms.

For non-spinning bodies below $12M_\odot$, the waveform can be accurately modelled by the restricted post-Newtonian waveform. Initially LIGO made use of these waveforms, however at higher masses, techniques such as the Effective One Body (EOB) waveforms better reproduce the waveforms that are computed by numerically solving the full set of nonlinear Einstein equations [7].

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2.3 Data

For each detector we assume that the noise is additive, given by the relation $d(t) = h(t, \boldsymbol{\theta}) + n(t)$ where $h(t, \boldsymbol{\theta})$ is the GW signal and $n(t)$ is the noise present in the detector [11]. The noise is Gaussian and stationary characterised by one sided spectral density given by $S_n = (2/T)\langle|\tilde{n}(f)|^2\rangle$, where $\tilde{n}(f)$ is the Fourier transform of the data over duration T [11]. In our case we use data of a duration of 4 seconds with 2 seconds before and 2 seconds after the GW trigger time. The data used in our analysis is from 3 GW detectors, LIGO-Livingston, LIGO-Hanford and the Virgo detector. Present in the Livingston data is a short instrumental transient (or glitch) 1.1 s before GW 170817's coalescence time [12]. The glitch produced a very brief (less than 5 ms) saturation in the digital-to-analog converter of the feedback signal controlling the position of the test masses [12]. Generally analysis of this GW waveform would require the removal of the glitch from the data. The details of this procedure is provided in [12]. This proved difficult to replicate using BILBY and so for the purposes of this paper, the glitch was not removed.

2.4 Waveform models for BNS

For the purposes of this study we make use of 3 frequency domain waveform models for our parameter estimation. The waveform models used are `TaylorF2`, `IMRPhenomD-NRTidal` and `IMRPhenomP-NRTidal`. These waveforms are computationally fast enough to be used as templates for inference and incorporate point-particle, spin and tidal effects in different ways [11]. These waveform models are higher order corrections in the GW perturbation to the quadrupole formalism presented in equations 1 and 2. At speeds close to the speed of light and in strong gravitational fields, these higher order terms contribute more to the GW waveform. For more details on the waveform models used see [11].

2.5 Source parameters and choice of priors

We use the same set of priors used in [11] (except for the sky location and luminosity distance) to factor in the uncertainty stemming from the lack of concrete localisation with other signal events. The aim is to produce a repeatable set of analysis that can be used on other GW signals that perhaps have larger uncertainties in the sky location of the wave. The GW signal is parameterised by 2 different sets of parameters: intrinsic parameters which describe the binary's components and extrinsic parameters which are the sky location and orientation of the binary with respect to the observer [11]. Beginning with the intrinsic parameters, we chose the convention that $m_1 \geq m_2$. The chirp mass is given by $\mathcal{M} = (m_1 m_2)^{3/5}/(m_1 + m_2)^{2/5}$. Note, ground-based detectors measure redshifted (detector frame) masses $m^{det} = m(1 + z)$, where z is the redshift of the binary system. We assume a uniform prior distribution in the detector frame masses and place the constraints $m_1^{det} \geq 0.5\ \mathrm{M}_\odot$; $m_2^{det} \leq 7.7\ \mathrm{M}_\odot$ and $1.184\mathrm{M}_\odot \leq \mathcal{M} \leq 2.168\mathrm{M}_\odot$

These priors were chosen to mimic [11]. The spin angular momenta of the two merging stars contribute six intrinsic parameters. They are usually represented in their dimensionless forms as $\boldsymbol{\chi}_i = c\boldsymbol{S}_i$ [11]. We separate our priors into two separate cases for differing magnitudes of the dimensionless spins $|\boldsymbol{\chi}| = \chi$ of the two bodies: a low spin prior case and a high spin prior case. For both cases use a uniform distribution assuming that the spins are isotropic and uncorrelated in their orientations. The maximum spins for the two cases are given by $\chi \leq 0.891$ and $\chi \leq 0.05$. Priors are grouped into a low-spin and a high-spin with the high-spin prior being the prior including $\chi \leq 0.891$ and the low-spin referring to $\chi \leq 0.05$. The high-spin prior case allows us to explore the possibility of more exotic BNS systems while the low-spin prior is consistent with the population of BNS systems [11]. For the dimensionless tidal deformation parameters Λ_i $(i = 1, 2)$ we assume a uniform distribution restricted between $0 \leq \Lambda_i \leq 5000$ with no correlation between Λ_1 and Λ_2. This implies we assume the 2 stars have the same Equation of State (EOS).

The remaining signal parameters are the extrinsic parameters. For GW 170817 we restrict the right ascension to that of SSS17a/AT 2017gfo [11] and assume the declination is uniform in cosine. For GW 170817 the luminosity distance is constrained between 40^{+7}_{-15} Mpc. Given that $\cos\theta_{JN} = \hat{\boldsymbol{J}} \cdot \hat{\boldsymbol{N}}$, where $\boldsymbol{J}$ is the total angular momentum and $\boldsymbol{N}$ is the line of sight vector, we assume a uniform distribution in $\cos\theta_{JN}$.

3. Results

The key results of the analysis for GW 170817 from each waveform model using the high and low spin priors are given in table 1. Note, we exclude the extrinsic parameters such as the sky location and the tidal deformation parameter. This is because the only extrinsic parameter of interest in this study is the binary inclination.

Parameter	`IMRPhenomP`		`IMRPhenomD`		`TaylorF2`	
	Low spin	High spin	low spin	high Spin	low spin	high spin
Inclination θ (°)	$155.28^{+15.99}_{-18.57}$	$152.6^{+18.65}_{-16.01}$	$155.21^{+15.98}_{-18.56}$	$155.57^{+15.62}_{-18.82}$	$142.88^{+0.9}_{-0.8}$	$152.41^{+18.64}_{-15.82}$
Chirp mass $\mathcal{M}$ ($M_\odot$)	1.20 ± 0.00	1.19 ± 0.00	1.20 ± 0.00	1.21 ± 0.00	1.19 ± 0.00	1.19 ± 0.00
Mass ratio q	$0.83^{+0.11}_{-0.11}$	$0.65^{+0.24}_{-0.19}$	$0.83^{+0.11}_{-0.11}$	$0.65^{+0.19}_{-0.21}$	$0.42^{+0.17}_{-0.03}$	$0.61^{+0.26}_{-0.23}$
Primary spin χ_1	$0.00^{+0.01}_{-0.01}$	$-0.03^{+0.13}_{-0.15}$	$0.00^{+0.02}_{-0.01}$	$0.30^{+0.15}_{-0.25}$	$0.00^{+0.02}_{-0.02}$	$-0.02^{+0.13}_{-0.14}$
Secondary spin χ_2	$0.00^{+0.01}_{-0.01}$	$-0.05^{+0.18}_{-0.22}$	$0.00^{+0.01}_{-0.01}$	$0.16^{+0.32}_{-0.25}$	$0.00^{+0.02}_{-0.02}$	$-0.06^{+0.18}_{-0.20}$

Table 1: Results for the low-spin and high-spin priors for GW 170817 data analyses with different waveforms.

4. Discussion and conclusion

The results obtained have some interesting differences between the results obtained in [11] and also between the waveforms in the implementation of the code. The results obtained by [11] are given below. In [11] the deviations between the waveform models used are negligible. The results presented in the table below are generated using `IMRPhenomP`.

From this initial set of analysis performed on GW170817, we find some agreement between the results obtained in this study and those found by [11]. The chirp mass found between all the waveform models used, is in strong agreement with that found in [11]. The mass ratios obtained for waveform models `IMRPhenomP` and `IMRPhenomD` for both the high- and low-spin cases are within the bounds found in [11]. The major differences however, lie in the inclination angles obtained and some deviations in the results obtained with the waveform model `TaylorF2`. We see that the

Parameter	Low spin	High spin
Inclination θ (°)	146^{+25}_{-27}	152^{+21}_{-27}
Chirp mass $\mathcal{M}$ ($M_\odot$)	1.1975 ± 0.0001	$1.1976^{+0.0004}_{-0.0002}$
Mass ratio q	0.73 – 1.00	0.53 – 1.00
Primary spin χ_1	0.00 – 0.04	0.00 – 0.50
Secondary spin χ_2	0.00 – 0.04	0.00 – 0.61

Table 2: Results obtained for GW 170817 evcent by LIGO Collaboration [11]

inclination angles found using models `IMRPhenomP` and `IMRPhenomD` for both the high-spin and low-spin cases are all close to the high spin results found in [11]. It is only the `TaylorF2` model that is largely consistent with the result in [11]. This is a rather interesting result as the waveform models `IMRPhenomP` and `IMRPhenomD` are more complete waveform models than the `TaylorF2` model as they include the inspiral, merger and ringdown (IMR) whereas the `TaylorF2` model does not include those effects [13]. We still make use of this model as it is not computationally as expensive as the other two models and for lighter systems (as in the case of BNS mergers) this waveform model is still largely valid [13]. Considering the fact that the `IMRPhenomP` and `IMRPhenomD` waveform models are closer to reality, these results seem to suggest the system had an inclination of $\approx 152°$. However, this is in contrast with the results found in [11] which make use of the same waveform models. The deviation then might be attributed to the sampling technique used in `BILBY` as compared to `LALInference` which is the inference software suite used in [11]. Comparing the results obtained for spins of the component masses, we see again some deviation between the results in [11] and those in table 1. The negative spin values found in this study are most likely due to an error in the analysis, the cause of which is still unknown. It's important to note however, the spin parameter does not affect the measurement of the inclination of the binary. Degeneracy between parameters of the GW waveform model exist, most notably between the luminosity distance and orbital inclination degeneracy [14]. There is no spin and inclination degeneracy. Due to the luminosity distance and inclination degeneracy, it becomes difficult to determine the inclination of the binary. It is more difficult to constrain the inclination accurately, especially for low inclinations as the contributions made by the two polarisation modes are almost identical [14]. For angles below 45° this large uncertainty is commonplace [14]. We see this in the uncertainty present in the results of [11]. Taking into account measurements of the GRB (EM counterpart) to GW170817, this degeneracy can be broken to get a well constrained measurement of the inclination. With the EM constraints on the luminosity distance [11] finds that the inclination of the binary is 152° (+15°, −11°). This clearly shows the viability of this study and `BILBY` for measuring the orbital inclination of these binaries as no constraints from any observations of GRB 170817A were used.

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