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Study of collective modes of gluon in an anisotropic thermomagnetic medium

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Using the hard thermal loop (HTL) perturbation theory, we investigate the collective modes of gluon in an anisotropic thermal medium in the presence of a constant background magnetic field. The momentum space anisotropy of the medium has been incorporated into the distribution function via the generalized 'Romatschke- Strickland' form. The magnetic modification arises from the quark loop contribution where the lowest Landau level approximation has been considered. We examine two special cases: i) spheroidal anisotropy with an anisotropy vector orthogonal to the external magnetic field, and ii) ellipsoidal anisotropy with two mutually orthogonal vectors describing aniostropies along and orthogonal to the field direction. We use the general structure of gluon self-energy that consists of six independent basis tensors. It is found that the strong background magnetic field has a significant impact on the growth rate of the unstable modes. This could have important effects on the equlibration of magnetized quark-gluon plasma.

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1. Introduction

The diverse findings from different heavy ion research communities indicate that the deconfined quark-gluon plasma (QGP) matter produced in the ultra-relativistic heavy ion collision experiments is most likely to deviate significantly from perfect local isotropic equilibrium. The viscous hydrodynamics which has been proved to be a reliable tool to study heavy-ion collisions, concludes that the quark-gluon plasma (QGP) possess large pressure anisotropy along longitudinal and transverse directions. This happens because of the large local rest frame momentum space anisotropy present in the $p_T - p_L$ plane [1]. According to several studies the momentum space anisotropy plays very important role in isotropization and thermalization of the QCD plasma as it can give rise to plasma instabilities [2–5]. To take into account the momentum space anisotropy within hard-thermal-loop (HTL) perturbation theory, one usually considers the anisotropic momentum distribution function for quarks and gluons which is called 'Romatschke-Strickland' form [1, 6]. In recent years several efforts have been made to study the effect of momentum space anisotropy in bottomonia suppression [7–9], heavy-quark potential [10, 11], photon and dilepton production rates [12, 13] and so on.

On the other hand, the production of a very strong magnetic field [14] in ultrarelativistic heavy-ion collisions and its effect on the QGP properties have recently piqued the interest of heavy-ion collision community. Some studies found that the magnetic field created in such collisions last for a very short time [15, 16]. However, the electric conductivity of the medium can significantly increase the lifetime of the magnetic field [17, 18]. Various research works have been performed to study the effect of the strong magnetic field on the QCD plasma which resulted in several celebrated findings like magnetic catalysis [19], inverse magnetic catalysis [20, 21], chiral magnetic effect [22], photon and dilepton production rates [23, 24], heavy-quark potential [25], thermodynamic properties [20, 26] and so on. The presence of a very strong magnetic field in heavy-ion collision drives one naturally to study the effect of it on anisotropic QGP. We consider the energy scale hierarchy $\sqrt{|eB|} \gg T \gg g_s T$ within strong field approximation where g_s is the strong coupling constant. In the strong field approximation, only the lowest Landau level is taken into account and the quark dynamics becomes restricted to 1+1 dimension. We study the collective modes of gluon in an anisotropic thermomagnetic medium within HTL approximations [27]. The general structure of the gluon self-energy has been used from Ref. [28] and the collective modes are found from the pole of the effective gluon propagator. We also study the instability of the gluon modes in presence of anisotropic thermomagnetic medium. In Sec. 2, we briefly describe the general structure of gluon self-energy that has been used to study the collective modes of gluon. We compute the one-loop gluon self-energy within HTL approximations in Sec. 3. The results have been shown in Sec. 4 and we conclude in Sec. 5.

2. The general structure of gluon self-energy

In this paper, two particular cases have been considered: i) spheroidal anisotropy with an anisotropy vector orthogonal to the external magnetic field, and ii) ellipsoidal anisotropy with two mutually orthogonal vectors describing aniostropies along and orthogonal to the field direction. The general structure of gluon self-energy in the presence of an ellipsoidal anisotropy has been used for both the cases. This consists of six independent basis tensors. The ellipsoidal anisotropy is characterized by two independent four vectors a_1^μ and a_2^μ. These four vectors, along with the heat bath velocity u^μ and the gluon four momentum p^μ are used to construct the independent basis tensors for the general structure of gluon self-energy. The simple choice of basis tensors to express the symmetric gluon polarization tensor is $p^\mu p^\nu$, $u^\mu u^\nu$, $a_1^\mu a_1^\nu$, $a_2^\mu a_2^\nu$, $p^\mu u^\nu + p^\nu u^\mu$, $p^\mu a_1^\nu + p^\nu a_1^\mu$, $p^\mu a_2^\nu + p^\nu a_2^\mu$, $u^\mu a_1^\nu + a_1^\mu u^\nu$, $u^\mu a_2^\nu + u^\nu a_2^\mu$ and $a_1^\mu a_2^\nu + a_2^\mu a_1^\nu$. Here, we did not consider the metric tensor $\eta^{\mu\nu}$ in the set of basis tensors as this no longer remains an independent tensor in this particular case of elipsoidal anisotropy. Now, the number of independent basis tensors is reduced from ten to six by using the constraints from the transversaity condition $p^\mu \Pi_{\mu\nu} = 0$. We obtain the six independent basis tensors systematically in the following way. Firstly, we consider the general structure of gluon self-energy in vacuum

$$\Pi^{\mu\nu} = \left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\Pi(p^2) = V^{\mu\nu}\Pi(p^2). \tag{1}$$

We obtain $\tilde{u}^\mu = V^{\mu\nu}u_\nu$ from $V^{\mu\nu}$ and use it to obtain the first basis tensor

$$A^{\mu\nu} = \frac{\tilde{u}^\mu \tilde{u}^\nu}{\tilde{u}^2}. \tag{2}$$

Now, we define another tensor $U^{\mu\nu} = V^{\mu\nu} - A^{\mu\nu}$ which is used to obtain

$$\tilde{a}_2^\mu = U^{\mu\nu} a_{2\nu}. \tag{3}$$

This $\tilde{a}_2^\mu$ is orthogonal to $\tilde{u}^\mu$ by construction. Now similar to the previous case, we obtain the second basis tensor as

$$B^{\mu\nu} = \frac{\tilde{a}_2^\mu \tilde{a}_2^\nu}{\tilde{a}_2^2}. \tag{4}$$

The next basis tensor can be obtained using $\tilde{a}_2^\mu$ and u^μ as

$$C^{\mu\nu} = \frac{\tilde{u}^\mu \tilde{a}_2^\nu + \tilde{a}_2^\mu \tilde{u}^\nu}{\sqrt{\tilde{u}^2}\sqrt{\tilde{a}_2^2}}. \tag{5}$$

To construct rest of the basis tensors, we define $R^{\mu\nu} = U^{\mu\nu} - B^{\mu\nu}$ and thereafter, $\tilde{a}_1^\mu = R^{\mu\nu} a_{1\nu}$. This newly constructed four vector $\tilde{a}_1^\mu$ is orthogonal to $\tilde{a}_2^\mu$ as well as $\tilde{u}^\mu$. The rest of the basis tensors are constructed as

$$D^{\mu\nu} = \frac{\tilde{a}_1^\mu \tilde{a}_1^\nu}{\tilde{a}_1^2}, \tag{6}$$

$$E^{\mu\nu} = \frac{\tilde{u}^\mu \tilde{a}_1^\nu + \tilde{a}_1^\mu \tilde{u}^\nu}{\sqrt{\tilde{u}^2}\sqrt{\tilde{a}_1^2}}, \tag{7}$$

$$F^{\mu\nu} = \frac{\tilde{a}_1^\mu \tilde{a}_2^\nu + \tilde{a}_2^\mu \tilde{a}_1^\nu}{\sqrt{\tilde{a}_1^2}\sqrt{\tilde{a}_2^2}}. \tag{8}$$

It should be noted that all the four vectors $\tilde{u}^\mu$, $\tilde{a}_1^\mu$ and $\tilde{a}_2^\mu$ are orthogonal to the gluon momentum p^μ. Therefore, all the obtained basis tensors follow the transversality condition. The general structure of gluon self-energy in the presence of an ellipsoidal anisotropic medium is expressed as a linear combination of the six independent basis tensors as

$$\Pi^{\mu\nu} = \alpha A^{\mu\nu} + \beta B^{\mu\nu} + \gamma C^{\mu\nu} + \delta D^{\mu\nu} + \sigma E^{\mu\nu} + \lambda F^{\mu\nu} \,. \tag{9}$$

Here α, β, γ, δ, σ and λ are the gluon self-energy form factors which will be computed from one loop Feynman diagram using HTL approximations in the next section.

3. Collective modes of gluon from one loop diagram

Now, we obtain the one loop gluon self-energy in the presence of an anisotropic thermomagnetic medium within HTL approximations following the Schwinger-Keldysh formalism. The one loop gluon self-energy consists of three different contributions: i) gluon loop ii) ghost loop and iii) quark loop. The gluon and ghost loop remain unaffected by the magnetic field. On the other hand, the presence of anisotropy modifies all the contributions. The anisotropy is introduced to the system by modelling the non-equillibrium distribution function using the 'Romatschke-Strickland' form. The anisotropic distribution function for the gluons and ghosts is given as

$$f^{\rm B}_{\rm aniso}(\mathbf{k}) \equiv f^{\rm B}_{\rm iso}\left(\frac{1}{\Lambda_T'}\sqrt{\mathbf{k}^2 + \xi_x(\mathbf{k}\cdot\hat{\mathbf{x}})^2 + \xi_y(\mathbf{k}\cdot\hat{\mathbf{y}})^2 + \xi_z(\mathbf{k}\cdot\hat{\mathbf{z}})^2}\right). \tag{10}$$

In the present study, this distribution function reduces to the the following.

$$f^{\rm B}_{\rm aniso}(\mathbf{k}) \equiv f^{\rm B}_{\rm iso}\left(\frac{\sqrt{\mathbf{k}^2 + \xi_1(\mathbf{k}\cdot\mathbf{a_1})^2 + \xi_2(\mathbf{k}\cdot\mathbf{a_2})^2}}{\Lambda_T}\right). \tag{11}$$

The gluon and ghost contributions to the retarded gluon self-energy is obtained from the one-loop diagram as

$$\tilde{\Pi}^{\mu\nu}_{ab}(\omega,\mathbf{p},\xi) \;=\; \delta_{ab}\,\tilde{m}_D^2 \int \frac{d\Omega_{\mathbf{v}}}{4\pi} v^\mu \frac{v^l + \xi_1(\mathbf{v}\cdot\mathbf{a_1})a_1^l + \xi_2(\mathbf{v}\cdot\mathbf{a_2})a_2^l}{(1+\xi_1(\mathbf{v}\cdot\mathbf{a_1})^2 + \xi_2(\mathbf{v}\cdot\mathbf{a_2})^2)^2}\left[\eta^{\nu l} - \frac{v^\nu p^l}{\omega - \mathbf{p}\cdot\mathbf{v} + i0^+}\right]\Bigg|_{l\in\{1,2,3\}}, \tag{12}$$

where $\tilde{m}_D^2 = \frac{g_s^2\Lambda_T^2}{3}N_c$ is the squared QCD Debye mass with $N_f = 0$. g_s is the strong coupling constant, whereas Λ_T corresponds to the temperature in the equillibrium limit.

We choose the spatial anisotropy vectors $\mathbf{a_1}$ and $\mathbf{a_2}$ along $\hat{x} = (1,0,0)$ and $\hat{z} = (0,0,1)$ respectively.

In case of quark loop contribution, we construct the nonequllibrium fermion distribution function in the lowest Landau level as

$$f^{\rm F}_{\rm aniso}(k_z) \;\equiv f^{\rm F}_{\rm iso}\left(\sqrt{k_z^2 + \xi_z(\mathbf{k}\cdot\hat{\mathbf{z}})^2}/\Lambda_T\right) = f^{\rm F}_{\rm iso}\left(|k_z|/\lambda_T\right), \tag{13}$$

where $\lambda_T = \Lambda_T/\sqrt{1+\xi_2}$. Now, we obtain the retarded photon self-energy in presence of strong magnetic field and ellipsoidal anisotropy as

$$\Pi^{\mu\nu}_R(p) \;= e^2\frac{|eB|}{\pi}\exp\left(-\frac{p_\perp^2}{2|eB|}\right)\int\frac{dk_z}{2\pi}\frac{f_{\rm F}(k_z)}{|k_z|}\left[\eta_\parallel^{\mu\nu} - \frac{k_\parallel^\mu p_\parallel^\nu + k_\parallel^\nu p_\parallel^\mu}{(k_\parallel\cdot p_\parallel)+i\epsilon} + \frac{p_\parallel^2 k_\parallel^\mu k_\parallel^\nu}{\left[(k_\parallel\cdot p_\parallel)+i\epsilon\right]^2}\right]\Bigg|_{k^0=|k_z|}. \tag{14}$$

This can be easily extended to the gluon self-energy within HTL approximations. By considering the anisotropic fermion distribution function in Eq. (13) we perform the integration in Eq. (14).

$$\Pi_R^{\mu\nu}(p) = \frac{m_{D,e}^2}{2}\exp\left(-\frac{p_\perp^2}{2|eB|}\right)\sum_{\mathrm{sgn}(k_z)=\pm1}\frac{v_\parallel^\mu v_\parallel^l}{1+\xi_2}\left[\eta_\parallel^{\nu l}-\frac{v_\parallel^\nu p^l}{(v_\parallel\cdot p_\parallel+i\epsilon)}\right]\Bigg|_{l=3}, \tag{15}$$

where the Debye mass is given as

$$m_{D,e}^2 = -\frac{e^2}{\pi^2}|eB|\int d|k_z|\frac{\partial f_{\mathrm{F}}^{\mathrm{iso}}(|k_z|)}{\partial|k_z|} = e^2\frac{|eB|}{2\pi^2}. \tag{16}$$

It should be noted that due to dimensional reduction in lowest Landau level, the self-energy in Eq. (15) is independent of the momentum scale Λ_T. However, it has implicit dependence on Λ_T through the running coupling constant. Also, it should be noticed that the anisotropy parameter in Eq. (15) only appears as a multiplicative factor. Now, we obtain the quark contribution to the gluon self-energy by including the flavor sum and the color factor as

$$\bar{\Pi}_{ab}^{\mu\nu}(p) = \delta_{ab}\sum_f g_s^2\frac{|e_fB|}{8\pi^2}\exp\left(-\frac{p_\perp^2}{2|e_fB|}\right)\sum_{\mathrm{sgn}(k_z)=\pm1}\frac{v_\parallel^\mu v_\parallel^l}{1+\xi_2}\left[\eta_\parallel^{\nu l}-\frac{v_\parallel^\nu p^l}{(v_\parallel\cdot p_\parallel+i\epsilon)}\right]\Bigg|_{l=3}. \tag{17}$$

Therefore, the total retarded gluon self-energy can be written by adding the contributions from Eqs. (12) and (17) as

$$\Pi_{ab}^{\mu\nu}(p,eB,\xi,\Lambda_T) = \tilde{\Pi}_{ab}^{\mu\nu}(p,\xi_1,\xi_2,\Lambda_T)+\bar{\Pi}_{ab}^{\mu\nu}(p,eB,\xi_2,\Lambda_T), \tag{18}$$

The collective modes of gluon in an anisotropic thermomagnetic medium can be obtained from the pole of the effective gluon propagator

$$\mathcal{D} = \mathcal{D}_0 - \mathcal{D}_0\Pi\mathcal{D}, \tag{19}$$

where $\mathcal{D}$ is the bare gluon propagator and the inverse of it is given as

$$(\mathcal{D}_0^{-1})^{\mu\nu} = -p^2\eta^{\mu\nu} - \frac{1-\zeta}{\zeta}p^\mu p^\nu. \tag{20}$$

Here ζ is the gauge fixing parameter. We obtain the collective modes of gluon from the pole of Eq. (19) as

$$p^2 - \Omega_{0,\pm}(p) = 0, \tag{21}$$

where the mode functions $\Omega_{0,\pm}$ in terms of the gluon self-energy form factors are given by

$$\Omega_0 = \frac{1}{3}(\alpha+\beta+\delta) - \frac{1}{3}\frac{\varpi}{\left(\frac{\chi+\sqrt{4\varpi^3+\chi^2}}{2}\right)^{\frac{1}{3}}} + \frac{1}{3}\left(\frac{\chi+\sqrt{4\varpi^3+\chi^2}}{2}\right)^{\frac{1}{3}}, \tag{22}$$

$$\Omega_\pm = \frac{1}{3}(\alpha+\beta+\delta) + \frac{1\pm i\sqrt{3}}{6}\frac{\varpi}{\left(\frac{\chi+\sqrt{4\varpi^3+\chi^2}}{2}\right)^{\frac{1}{3}}} - \frac{1\mp i\sqrt{3}}{6}\left(\frac{\chi+\sqrt{4\varpi^3+\chi^2}}{2}\right)^{\frac{1}{3}}, \tag{23}$$

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where, the ϖ and χ in the expression are defined in terms of the form factors as

$$\varpi = \alpha(\beta-\alpha)+\beta(\delta-\beta)+\delta(\alpha-\delta)-3(\gamma^2+\lambda^2+\sigma^2)\,, \tag{24}$$

$$\begin{aligned}\chi &= (2\alpha-\beta-\delta)(2\beta-\delta-\alpha)(2\delta-\alpha-\beta)+54\gamma\lambda\sigma \\ &- 9\left[\alpha(2\lambda^2-\sigma^2-\gamma^2)+\beta(2\sigma^2-\gamma^2-\lambda^2)+\delta(2\gamma^2-\lambda^2-\sigma^2)\right]\,.\end{aligned} \tag{25}$$

We define a mass scale corresponding to each form factors of the gluon self-energy as

$$m_\alpha^2 = \lim_{\omega\to 0}\alpha, \tag{26}$$

where m_α is the mass scale of form factor α. We can also define mass scales corresponding to each gluon dispersive modes as

$$m^2_{\Omega_{0,\pm}} = \lim_{\omega\to 0}\Omega_{0,\pm}(\omega,p,\theta_p,\phi_p). \tag{27}$$

A negative value of the squared mass indicates the presence of instability.

4. Results

To obtain the gluon self-energy form factors in an ellipsoidally anisotropic thermomagnetic medium, we consider the $X-Y$ plane as the reaction plane. The magnetic field is considered along the $\hat{z}$ direction whereas, the two anisotropies $\mathbf{a_1}$ and $\mathbf{a_2}$ are chosen along $\hat{x}$ and $\hat{z}$ respectively as already mentioned before.

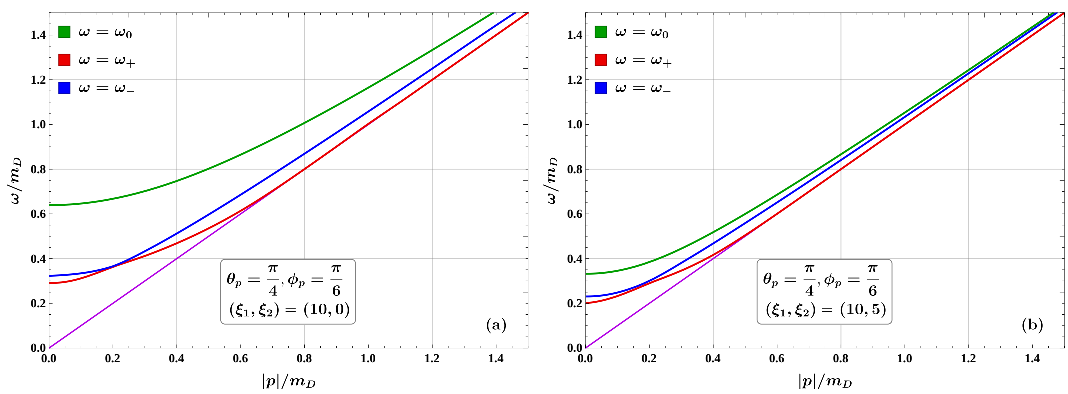

Figure 1: The collective modes of gluon with (a) spheroidal and (b) ellipsoidal anisotropy are shown for $\theta_p = \pi/4$ and $\phi_p = \pi/6$ at fixed momentum scale $\Lambda_T = 0.2$ GeV and magnetic field strength $30m_\pi^2$. The light cone (magenta) is also shown for comparison.

The dispersive modes of gluon can be obtained from the pole of the effective gluon propagator in Eq. (19). Gluon has three collective modes in the presence of momentum space anisotropy and an external magnetic field which has been shown in Fig. 1. In Fig. 1 (a), we show gluon dispersive modes in case of spheroidal anisotropy, whereas, ellipsoidal momentum space anisotropy has been considered in Fig. 1 (b). It can be noticed that all the dispersive modes of gluon possess different plasma frequencies. Moreover, the plasma frequencies decrease when ellipsoidal anisotropy is considered. This happens because the anisotropy parameter ξ_2 appears as an overall suppression factor in Eq. (17) thereby decreasing the quark loop contribution.

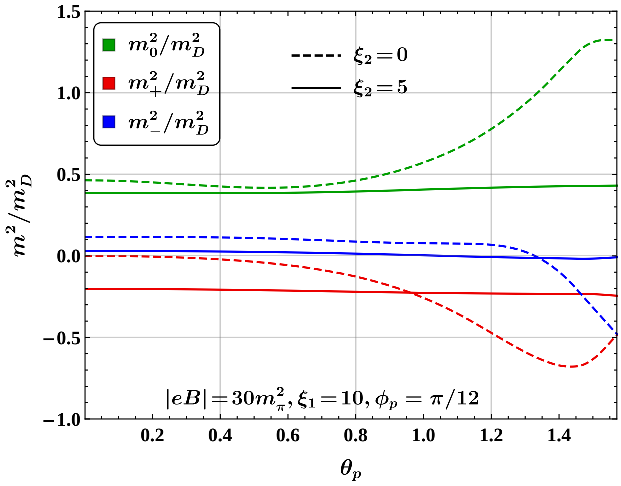

Figure 2: Variation of the squared mass with polar angle θ_p is shown for each mode functions at fixed values of external parameters $\phi_p = \pi/12$, $\xi_1 = 10$, $\Lambda_T = 0.2$ GeV and $eB = 30m_\pi^2$. The continuous and the dashed curves represent $\xi_2 = 5$ and $\xi_2 = 0$ respectively.

The squared mass corresponding to the gluon self-energy modes are plotted as a function of the propagation angle θ_p in Fig. 2. We have shown two scenarios: i) one with $\xi = (10, 0)$ and ii) the other with $\xi = (10, 5)$. It can be noticed from the figure that the squared mass of two modes Ω_+ and Ω_- become negative with increasing θ_p for $\xi = (10, 0)$. On the other hand, $m^2_{\Omega_0}$ remains positive in the whole range of θ_p. However, the θ_p dependence of the squared mass almost vanishes when the momentum anisotropy along the magnetic field is switched on.

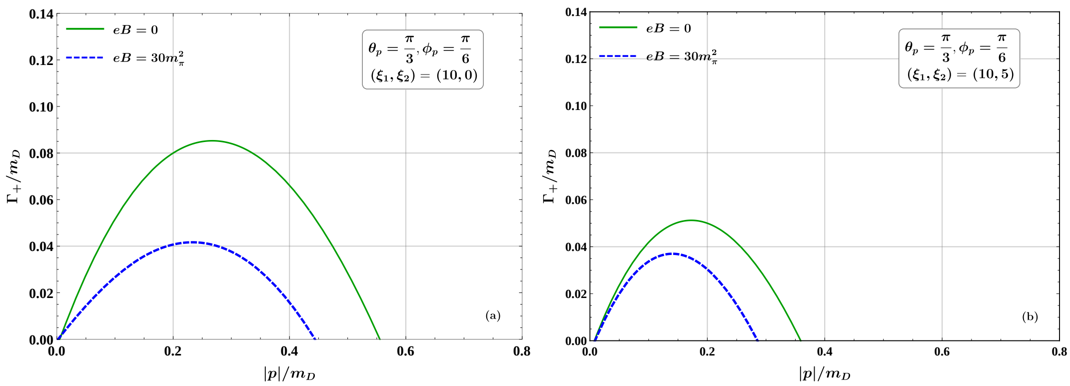

Figure 3: The growth rate corresponding to Ω_+ mode is plotted for (a) $\xi = (10, 0)$ and (b) $\xi = (10, 5)$ at fixed angles $\theta_p = \pi/3$, $\phi_p = \pi/6$. The continuous and the dashed curves correspond to the magnetic field strength 0 and $30m_\pi^2$ respectively.

As previously mentioned, the negative squared mass indicates the presence of unstable modes *i.e.,* the amplitude of the modes grow exponentially with time. This growth rate *i.e.,* the imaginary part of the mode frequency can be found from the pole of the effective propagator in Eq. (19). The growth rate Γ_+ of the unstable '+' mode is shown in Fig. 3 for $(\theta_p, \phi_p) = (\pi/3, \pi/6)$. We show the growth rate Γ_+ for spheroidal anisotropy $\xi = (10, 0)$ in Fig. 3 (a), whereas, in Fig. 3 (b) ellipsoidal momentum space anisotropy $\xi = (10, 5)$ has been considered. It should be noted that the growth rate decreases in the presence of a magnetic field. It is also interesting to note that there exists a critical value of the momentum above which the growth rate of the unstable mode becomes negative *i.e.,* the mode becomes stable. This particular charachertistics of the instability of gluon modes exists for both the spheroidal and ellipsoidal anisotropic system. But the presence of the external magnetic field significantly decreases the critical momentum. Therefore, if the magnetic field is very strong, then the gluon modes may become stable. However, such high magnetic field is unlikely to be produced in heavy-ion collision.

5. Conclusion

We have studied the collective modes of gluon in the presence of an external magnetic field and a momentum space anisotropy within hard-thermal loop perturbation theory. The gluon self-energy consists of gluon, ghost and quark loop contribution among which the gluon and the ghost loop remain unaffected by the external magnetic field. The magnetic field effect is manifested in the quark loop contribution which is computed within the lowest Landau level approximation. The momentum space anisotropy is taken into account using the 'Romatschke-Strickland' form of the anisotropic distribution function. The anisotropic quark distribution function has been carefully constructed for the 1+1 dimensional quark contribution of the gluon self-energy. The general structure of gluon self-energy for ellipsoidally anisotropic medium been used to study the collective modes of gluon. The azimuthal symmetry of the system is lost when the external magnetic field is considered along with the ellipsoidal momentum space anisotropy. Therefore, the three collective modes of the gluon depends on the polar as well as on the azimuthal angle. We found that the anisotropy parameter ξ_2 appears as an overall suppressing factor in the 1+1 dimensional quark loop contribution which counterbalances the magnetic field effects.

We define mass scale corresponding to each gluon modes. The negative value of the squared mass indicates the presence of instability in the system *i.e.,* the modes grow exponentially. The angluar dependence of the squared mass has been analyzed. The magnetic field has significant effect on the growth rate of the unstable gluon modes. The amplitude as well as the critical momentum beyond which unstable modes cease to exist, are reduced in the presence of the external magnetic field. This can be compared to the instability growth rate in collisional plasma where larger collisional frequency suppresses the growth rate.

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