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From Students' Misconceptions to Reasoning: Lessons Learned from Physics Education Research

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Abstract The paper reports ongoing research investigating pre-service physics teachers' reasoning when they analyse experimental data in the undergraduate physics lab courses. The focus is on how pre-service physics teachers understand mathematical expressions/equations when they analyse experimental measurements in the undergraduate physics laboratory. In particular, we are interested in how they understand slopes of best fit lines and rates of change at some points of best fit lines. The main sources of data are open-ended written tasks, semi-structured interviews and lab reports (written and submitted after each experiment) in order to understand their reasoning behind the students' adopted approaches. In total, sixteen pre-service physics teachers participated. It was found out that the participants have had particular difficulties when they reason with mathematical expressions as they demonstrate a restricted understanding of them as formulas only. The main argument developed by this research study is that physics instructors need to give much importance to lab reports as a learning tool. In fact, lab report write-up is regarded as an integral part of the experimental process. And then, in research terms, lab reports are seen as a valuable source for pre-service physics teachers' difficulties and full reasoning.

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1 Introduction

The present research study is a part of a larger project concerning the difficulties that pre-service physics teachers and undergraduate physics students experience in the undergraduate physics laboratory. In particular, we are interested in investigating students' reasoning behind the documented difficulties as they are articulated in their lab reports, semi-structured interviews and open-ended written tasks. For this purpose we have followed cohorts of undergraduate students from fall semester 2014.

In the reported study here we are interested in looking at pre-service physics teachers' understanding of mathematical expressions and equations and related reasoning. Our current research aims to investigate pre-service physics teachers' reasoning related to mathematical functions, graphs and rates in the context of physics lab reports.

The research questions are as follows:

- To what extent do physics equations (i.e. $F = ma,\ F = kx,\ PV = nTR,\ h = \frac{1}{2}gt^2$) represent functions (which include variables with trends/ relationships, mathematical relationships between two or more variables and graphs when sketched) or, only formulas for the participants?
- To what extent are pre-service physics teachers able to "translate" graph(s) into mathematical expression(s) and vice versa?
- What is the participants' understanding of best fit line? (What do best fit lines and curves represent and what is their role in the analysis of experimental measurements)?

2 Literature Review

2.1 Misconceptions related to physics concepts and laws

From early 1980s, research focused on primary and secondary students' ideas related to particular concepts, ideas and laws in science education and physics education. The terms of misconceptions", "naive ideas" and "alternative ideas" were introduced [1]. For example, many students believe that air does not have weight. Or, that an object at rest does not have any energy. If the object does not have any velocity (velocity is zero), then it is impossible for it to be accelerated. This is the case of an object which undertakes a free fall motion. Another common misconception is that the heavier objects reach the ground earlier than the lighter ones. In addition, the centripetal force is a difficult concept for both primary and secondary students (it may be difficult even for preservice teachers) who state that the centripetal force is one extra force acting on objects moving on a circle. Or, that a cyclist on a circular track is not accelerating because he is travelling at a constant speed.

Physics education research stressed that it requires much energy and rigorous research skills and methods to identify how students think and reason and then, it takes time to deal with such difficulties. "Conceptual change" is not so straightforward because students may regress to the same misconceptions. The identification of misconceptions had a tremendous influence in education research and teaching because it demonstrated that students are active and creative participants in the learning process and that their ideas and understandings need to be taken into account in instruction.

2.2 Research on students' reasoning

Some decades ago, approximately from 90s onwards, physics education research "shifted" to researching students' thinking and reasoning and how to develop appropriate methods and tools towards that direction. For example, the late Viennot [2] investigated reasoning and introduced the term of "spontaneous reasoning" in mechanics to emphasize that physics experts should identify important aspects of physics students' spontaneous reasoning so that they are able to design instruction to challenge such mistakes.

McDermott and Redish [3] published a paper which presented many research studies focusing on students' difficulties within specific domains of Physics like mechanics, electricity and magnetism, light and optics, waves and sound and even in topics in modern physics (i.e. photoelectric effect, Quantum Mechanics and so on).

In another paper, McDermott [4] underlined the importance of investigating students' reasoning in physics because physics education research should be discipline-based and informed by a sound background of physics. Therefore, she argued that physics education research (PER) is discipline-specific research. Indeed, for McDermott, discipline-based education research is different from traditional educational research in that the emphasis is not on educational theory or methodology but on issues related to the teaching and learning of Physics content and related scientific reasoning. McDermott presented strong research evidence that growth in students' reasoning ability does not usually result from traditional instruction and secondly, that there is a gap between what instructors teach and what students learn [5].

Stephens and Clement [6] described a methodology for identifying evidence for the use of three types of scientific reasoning. By working with high school physics students, they used a specific methodology to identify multiple instances of students when spontaneous reasoning was used. Most of those instances were associated with motion and force concepts. They documented a wide range of scientific reasoning processes.

2.3 Research on students' reasoning when experimenting in the physics laboratory

During the last 15 years, there is a rapid growth of research studies related to laboratory education and students' difficulties and reasoning about specific issues related to laboratory work. For instance, Ivanjek and colleagues [7] investigated students' graph interpretation strategies and difficulties in different contexts of mathematics, physics (kinematics) and other contexts different from physics. Students were asked to provide explanations with their answers and the strategies they followed to work on the tasks. They found out that in physics, students preferred to use the strategy of using formulas, which sometimes seemed to block the use of other more productive strategies which students displayed in other domains. Susac and her team [8] investigated students' understanding of graph slope and area under a graph to compare physics' and non-physics' understanding.

An example of recent research on undergraduate students' reasoning when they evaluate experimental measurements is the study by Van Kampen and colleagues [9] which investigated how first- and second-year university students judge the quality of secondary experimental data consisting of measurements of covarying quantities. With written open-ended questions they investigated students' reasoning and by conducting follow-up interviews they wanted to understand such reasoning in more depth. They found out that students showed a fragmented and primarily qualitative understanding of the concepts of mean, uncertainty and line of best fit. With respect to best fit line, students appeared to consider the best fit line as a tool for eliminating outliers, but much less frequently as a way to determine a derived quantity.

In the following sections, we are going to briefly discuss the important role of the laboratory work in the physics curriculum and related students' difficulties.

2.4 Laboratory in the Physics Curriculum

Many scholars, researchers and curriculum developers emphasized the role of laboratory in learning and teaching physics from different perspectives [10, 11, 12]. Physics, by its very nature, is an experimental science. Laboratory activities and courses play a critical role in the school and university physics undergraduate curriculum. Students have the opportunity to develop experimental skills and practices. The American Association of Physics Teachers [13] emphasized a range of learning goals and outcomes for introductory lab curriculum. In a recent report, the same Association emphasized the role of physics in the 21st Century Science Curriculum [14]. In (school and undergraduate) laboratory courses, students learn to plan experiments, take measurements, analyze and interpret data by taking into account measurement uncertainty, draw conclusions and evaluate the experimental procedure. However, very recent research [15] provided strong evidence that labs focused on developing only experimentation skills improve students' critical thinking skills compared to labs focused on reinforcing concepts taught in lectures. They made suggestions for four major areas of investment: Collaborations between researchers and instructors, the introduction of research-based assessments, the development of accessible and inclusive learning environments and, the provision of professional development opportunities closely related to lab education. Caballero and colleagues [16] based on research evidence and experience, argued that, as labs are necessary, we need to invest in them.

In particular, many studies investigated students' difficulties with measurement and measurement uncertainty [8, 17]. In addition, students' ability to draw conclusions from experimental measurements and related reasoning was investigated extensively [17, 18, 19, 20, 21, 22]. They found out that students tend to take into account only the mean of a data set without considering the uncertainty or the range of the set of measurements. Interviews reveal more sophisticated reasoning and interpretations related to measurement uncertainty and the strategies they use when they analyze experimental measurements. They concluded that students' ability to draw conclusions from measurement data did not improve after instruction [17]. Allie and colleagues [23] showed that students interpret uncertainty as a human error or a mistake made by the experimenter. They introduced the "point" and "set" paradigms to categorize students' approaches to measurement process and related understanding of uncertainty.

Sokolowski [24, 25] investigated how undergraduate students understand mathematical equations and algebraic expressions. In addition, he investigated how students perceive physics formulas in the mathematical sense and to what extent they can understand mathematical expressions as functions with variables. He found out that students' understanding of mathematical expressions as functions is fragmented and that students consider mathematical expressions primarily as calculation tools. This may be due to the fact that they apply them and plug in quantities to formulas to solve arithmetical problems in physics in order to calculate unknown quantities. He concluded that physics instructors should emphasize the function nature of many mathematical expressions and equations in physics learning and teaching. Sokolowski argued that on such a "function" basis, students will be able to understand the relationship between two or more variables, the related graphing and changes of one variable with respect to another one and subsequently, slopes of line graphs and tangents (and rates of change). Further studies by Sokolowski [24, 25] in different physics domains looked at the use and understanding of physics formulas in contrast to covariational structures by physics students. Thus, in the literature and physics education research, formulas as calculation tools [24, 25, 26] are opposed to the concept of a function between two or more variables in a mathematical expression.

2.5 Laboratory reports as a learning and assessment tool

When performing an experiment, it is very important for the experimenter to be able to prepare the lab report from scratch (without instructions or guidelines). Many physics educators acknowledged the importance of such writing skills [27, 28, 29]. In recent decades, the lab write-up is neglected and replaced by a few questions to which students should answer so that grading becomes easier and less time consuming for lab instructors. Keys and colleagues [29] presented research evidence that the use of science writing heuristic facilitated students to generate meaning from data, make connections among procedures, data, evidence and claims. Buffler and colleagues [30] developed an instrument to assess writing-intensive laboratory reports in undergraduate physics in which they combined the aspects of laboratory experimentation and written communication [31, 32]. The coherence of the report was used as an important concept in developing the instrument. In Campbell and colleagues' study [33], laboratory reports were used to identify the ways in which students (university entrants) demonstrated experimental and communication skills. They recommended that the communication of science should be taught explicitly and in parallel with experimental procedures and concepts.

Following such research recommendations, in our study, lab report write-up is regarded as an integral part of the experimental process. Subsequently, the challenge of the instructional context of this study was to teach (and then support) pre-service teachers to write a full lab report from scratch, after performing an experiment.

3 Research Method

3.1 The Context of the study and Participants

The study took place at Boğaziçi University in Istanbul (BOU). Pre-service physics teachers were not given any instructions about how to plan and perform experiments. In other words, they were free to plan their own experiments, make decisions and justify decisions by making explicit the related reasoning. Thus, the challenge was for the participants not to follow any instruction in the form of "recipe". They worked and performed the experiments in pairs. Emphasis was given on the analysis and explanation (by using theory) of experimental evidence, the drawing of best fit line(s) and the use of them in the evaluation of the whole experimental process. On completion of each experiment, they would write a full lab report from scratch (without instructions or questions to answer). On the other hand, students were taught what should be included in each section. In total, five sections should be included; theory, set-up and planning, collection of measurements, analysis and explanation of evidence and, evaluation of the experimental process. In short, the foci are on some experimental skills and the write-up of the lab report.

However, the participants, due to their experience in undergraduate physics lab classes, think that the aim is through each lab experiment to reinforce concepts covered in lectures. They have had a mid-term and final exam. Some open-ended written tasks were administered to students during the mid-term exam. The experiments that they performed were: Hookes' law experiment, free fall motion, simple pendulum motion, insulation experiment, reflection and refraction, Ohm's law and resistance experiment and finally, electromagnetic induction and photoelectric effect. In total, sixteen students participated in the interviews.

3.2 Data Sources and Analysis

All participants were informed about the research purposes of the study, they gave their written consent and ethics guidelines were kept. In addition, all participants' names were removed to be replaced by numbers.

Data collected were from the participants' answers to written open-ended tasks, laboratory reports and semi-structured interviews based on the lab reports. All interviews were audio-recorded and fully transcribed. The interview protocol includes the main questions, probes and prompts (Appendix 1). On average, interviews lasted for 50 minutes to one hour. Nine tasks were developed for the particular research purposes of the study. One of the nine tasks is presented in Appendix 2. The data were analyzed by qualitative methods, with open coding [34].

3.3 Results and Discussion

One main tendency is that for the majority of them, the physics equations are "formulas". When presented with the equation $h = \frac{1}{2}gt^2$ (Task 1), $F = kx$ (Task 2), $F = ma$ (Task 3), most of them talked about them as formulas, which they use to solve problems. In fact, the equations $F = kx$, $PV = nTR$ and so on are primarily formulas for students. The heavy use of those equations in calculations in textbook problems accounts for their restricted understanding of such mathematical expressions in physics.

With much prompting, twelve (out of the sixteen) talked about variables, trends between variables, changes and therefore, they called the same equations "functions". Only two of them never stated that the mathematical expressions $F = ma$ and $F = kx$ represent functions. These two students considered them only as formulas for problem solving and as calculation tools.

On the other hand, those who understood the equations as functions, demonstrated a limited understanding of function when talking about changes in the variables:

"*I see from the graph that force F and acceleration (a) are changing continuously,* $F = ma$" (Student 5, p. 22).

When asked explicitly how they understand "functions" or "what "function" means" to them, they replied:

"*Function" is when the two variables are changing at the same time"* (S. 8, p. 37),

"*When one or more variables change, then, the output changes*" (S. 7, p. 30) and,

"*It is a function because the values of pressure and volume change and the rate changes*". (S. 12, p. 50)

In the simple pendulum experiment, they studied the motion of a simple pendulum and determined the acceleration due to gravity by using a simple pendulum. Many pre-service physics teachers stated in the discussion in the laboratory and then, they wrote in the lab report:

"*In this experiment my aim is to find the gravitational acceleration* g *by using the relationship between period and length* l *of a pendulum. The period* T *of a pendulum of length* l *undergoing simple harmonic motion is given by:* $T = 2\pi\sqrt{\frac{l}{g}}$*. Thus, by measuring the period of a pendulum as well as its length, we can determine the value of g from the formula:* $g = 4\,\pi\,\frac{l}{T^2}$

"*My hypothesis is that if the length of the pendulum increases, the period's square will increase in a linear trend. I thought this because of my prior knowledge about the pendulum period formula. By using this formula, we can also calculate g".* (excerpt from a lab report).

For this group of students, theory is restricted to the mathematical expression which gives the period of oscillation as a function of period related to the length l. However, that mathematical expression or equation is only a "formula" for the participants. According to such reasoning, one needs only one data point (l, T) to plug into the equation and calculate the constant of acceleration g.

The theory section is being written around this formula and therefore, there is no need for them to plan for an experiment so that they change the independent variable l length of the pendulum to take measurements of period T. Subsequently, they do not mention about repeating to eliminate errors.

Another part of students change the values of length to measure the period T of one complete period (or 10 periods). They repeat taking measurements of time so that they plot the data points (l, T) and finally draw the line graph length l vs T squared. Interestingly, they use the line graph to identify and eliminate outliers only. They, then, use and apply the formula to plug in only one data point and thus calculate the constant of acceleration g.

Another group of students argued that for this experiment they should select two quantities of length so that they obtain the corresponding value for the period T of oscillation. They use the line graph to calculate the slope and then, the constant g.

In a second experiment, in the free fall experiment, in which the aim is to calculate the constant g of gravity, when they were asked to plan the experiment and think about how to proceed, they stated that they would need to obtain only one data point (h, t) because they would plug in the two quantities in the formula $h = ½\ gt2$. It seems that the fragmented understanding of the equation or the mathematical expression as a formula only, makes them consider it as a calculation tool only. Instead, they should have thought of h (height) and t (time) as variables and their values. In a few words, they should have identified mathematical expressions as functions between height (h) and time (t). Accordingly, they could think that they need to collect a set of data points (h, t) and then plot the points (h, t^2), draw the graph and calculate its slope which represents the change of height with respect to time squared. Through the slope of the line graph, one can calculate the constant of acceleration (g).

The equations $\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{l}}{\mathrm{g}}}$, $\mathrm{h} = \frac{1}{2}\mathrm{g}\,\mathrm{t}^2$, $F = kx$ and so on are only formulas for the majority of the participants and not mathematical expressions representing functions between two or more variables.

Understanding of the rate at a point of a line graph is fragmented since they apply the formula for $\mathrm{slope} = \frac{\Delta y}{\Delta x}$ without a deep understanding of it. Difficulties become clearer when they are asked to calculate instantaneous rates at some points along a curve. The lab report is a valuable source for such pre-service physics teachers' difficulties.

Theory is being restricted to a few formulas which they understand as calculation tools only. This approach prevents students from a proper planning of the experiment so that they change the independent variable to take measurements of the dependent variable and then, prevents them from a proper data analysis of experimental measurements.

Another main tendency demonstrated by the participants is that when they are asked to describe or identify a trend/ relationship between two variables, by being presented with a graph or a data table, they reply by recalling the relevant theory. They make it clear that they "remember", or "recall" or, that they "cannot be sure" or they do not "remember" the related equation (s). For example, when the student was asked to state the physics "behind" the graph of volume V versus temperature T, she replied:

"*Volume V is proportional to volume V, under the condition that P is fixed*" (S. 1, p. 6). Then the student added: "I remember that $PV = nTR$". With much prompting she wrote an equation to represent such a linear relationship: $V = cT$. To the same task, another student stated:

"*When temperature increases, also volume increases. And, I remember* $PV = nTR$" (student's emphasis) (S. 2, p. 8).

Although the second student correctly described the trend, she did not write down a statement showing the trend between the two variables only, but she wrote the equation $\mathrm{PV} = \mathrm{nTR}$, which she could remember and was taught. To the same task, another student replied that he could not remember the theory, as follows:

"I: *Can you write a mathematical expression to show how the two variables are associated with each other? Between temperature and volume?*

S: *I do not remember the theory."*

I: *You do not need to remember any theory. Please just think and write down a mathematical expression for the relationship between the two variables of temperature and volume.*

S: $PV = nTR$??? (The student is asking). *This formula is for an ideal gas"* (S. 3, p.13). The student wrote down an equation he struggled to recall, but not a statement focusing on the simple linear relationship between the two variables (as presented in the written tasks).

Having said that, we need to be cautious with students' answers to tasks, in which the questions are within second Newton's context, a familiar context for all of them. In two tasks, students are presented with a graph of force F versus acceleration (a). Almost all the participants stated: "When force F increases, acceleration increases" by writing $F = ma$, an expression with which all of them were familiar and could be recalled. We cannot be sure to what extent, students understand the linear relationship between the two variables.

In a similar way, when working on Task 9 (Appendix 2), one student replied:

S: *I do not remember anything from theory here. I see that when pressure increases, volume decreases* (S. 5, p. 25). The same tendency to "remember" occurs when students are asked to work out the slope of the line graph or derivatives.

"*As far as I remember, acceleration (a) was the slope*" (S. 8, p. 33).

In addition, when they were asked to calculate one derivative:

I: *From your Calculus classes, if the function is:* $P(V) = \frac{c}{V}$, *how much is the derivative* $\frac{\mathrm{dP}}{\mathrm{dV}}$**?**

S: *I do not remember the calculation from Mathematics. I do not remember the equation* (S. 8, p. 36). It seems that familiarity restricts them from thinking. Interestingly, pre-service physics teachers also talked about how they distinguish between formulas and functions.

S: …. *"So, formula and equation are something different.*
I: *How are the two (formula and mathematical equation) different?*
S: *Equation actually means something to be equal. But formula actually describes some phenomena. This does not describe any phenomena. This just makes equations. Two things equal or it shows the direct proportionality between the two variables*". (S 7, p. 32).
Another student commented on the usual practice in physics teaching by stating:
"*Students know the formula, but they get confused when they are asked to read the graph and interpret the graph F (force) vs a (acceleration). They work with the formula to solve problems in physics and exams. But we are not taught that $F = ma$ is a function. Physics teachers do not emphasize that F and acceleration (a) are variables. In turn, students do not understand the graph. They try to recall theory. They also respond intuitively. I think the issue, here, is how we connect Mathematics and Physics in our teaching"* (S 14, p. 60).

All students applied the same formula to calculate the $\mathbf{slope} = \frac{y2-y1}{x2-x1}$ for line graphs

When asked to calculate the slope of a line graph, all students explained that they apply one formula which gives the slope. For the participants, it is "the formula for $\mathbf{slope} = \frac{y_2-y_1}{x_2-x_2}$". In order to apply the formula for the slope of a line graph they need two points (x, y). For any line graph, they know that the slope is constant, without being able to account for the constant value of the slope. It seems like a rule of thumb that for line graphs the slope is constant given by the formula.

Almost all participants stated that when force F increases, acceleration increases and the $\mathbf{slope} = \frac{\Delta F}{\Delta a}$ is constant and equal to mass. Students stated that the slope of the line graph (F versus acceleration) is constant, almost without thinking and therefore, they could not explain why. Thus, when the interviewer asked them to explain how it is possible for the slope to be constant while the two variables change continuously, the student failed to give an answer explaining the constant slope or work out the derivative for a linear relationship $y(x) = ax$. The following excerpt is representative of such difficulty:
"I: *How do you determine the constant?*
S: *They give it to us. It is the mass.*
I: *But you have said that force and acceleration are variables. You have said that the values of the variables are changing continuously. What is constant, then?*
S: *The rate of change for each pair of values of the variables* $\frac{F}{a} = c$ .... *The rate is constant.*
I: *What does "constant rate" mean to you, because you have said that your variables are changing continuously?*
S: *When F changes, acceleration changes at a constant rate which means ...*" (S. 1, p. 3-4)
The student did not give a full answer. At the same time, it was difficult for them, after writing the expression for the function $V = cT$, **to** give an expression for the rate or the slope as $\frac{dV}{dT} = c = \mathbf{constant}$. For instance, the following conversation took place with one student:
I: *Can you write a mathematical expression which includes the two variables to show the relationship between the variables?*
S: *This is a general formula, which gives general values from the graph data points (x, y), (T, V). We need to make mathematical calculations, here, for example, how we calculate* $a = \frac{y_2-y_1}{x_2-x_2}$ *for two data points* (S 4, p. 19). The student finally did not write any mathematical expression for the slope or rate of change. All of them struggled to write $V = cT$ **and then** $\frac{dV}{dT} = c$. In addition, they could not connect the slope of the line graph with the derivative of the function for the graph.

In the case of a curve, they could not calculate any rate of change, because simply the known formula could be used only for line graphs. In other words, they did not know any formula to apply for the calculation of an instantaneous rate of change for a best fit curve.

Limited understanding of the linear relationship between two variables.

When students asked to describe trends between two or more variables by looking at the graphs, they stated or wrote down:
"*When volume is increasing, temperature is increasing, too*" (S. 8, p. 34) and,
"*There is a directly proportional relationship between force and acceleration… When acceleration (a) increases, then, force F increases*" (for Tasks 2 and 3).

Such statements do not make sense because in physics we would expect students to have said that if force increases, acceleration increases, by distinguishing between the dependent and independent variables or by distinguishing what the reason is (the application of force) and then, the result which is acceleration.

Another tendency, when asked to read and talk about the relationships between variables was to simply read the variables in each equation, like the following:
"*Force is equal to mass times acceleration"* (S. 8, p. 33) and,
"*It is about pressure and volume, when pressure is constant*" (S. 8, p. 35). Such student's tendency is similar with what Sherin [35] found out in her study.

In another task concerning the linear relationship between volume V and temperature T, they have stated that volume V is proportional to temperature T, but they were not able to write the equation $V = cT$ (under the condition that P is fixed/constant). Students struggled with linear relationships and the interviewer helped them to make the connection between two tasks, one (in which they easily stated that there was a linear relationship between force and acceleration) and in another task (in which they were presented with a line graph of volume versus temperature). The following excerpt demonstrates such difficulties, for a student who had already written the equation $F = ma$ to show the linear relationship between force and acceleration (a).
I: "*Can you write a mathematical expression to show how the two quantities are related to each other?*
S: *Volume* V *is proportional to temperature* T, $V \sim T$.
I: *Earlier on, you wrote that acceleration is directly proportional to Force and now I would like you to write that* V *is directly proportional to temperature* (S. 12, p. 48).
Only a few of them referred to the words "continuously" and "simultaneously" but none of them used the term "smoothly". (Is it a matter of language?). They never mentioned the term of "coordination"; that changes in two or more variables are co-ordinated. In fact, only one student, a PhD student in Mathematics Education, did so. For the term "simultaneously":
I: …. *What does a line graph mean to you?*
S: *It means that both temperature T and volume V change simultaneously* (S. 3, p. 13).
More students talked about the "continuous" change of two variables like the following:
S: *Function. These two are variables; they are changing continuously and this is the graph of the function* (S 14, p. 56)

In addition, the participants experienced difficulties when a graph involves more than two variables, as, for example in Task 9 (Appendix 2). Familiarity with the equations for problem-solving at the level only of a formula (as a tool for calculations) prevented them from thinking. We are not sure how they understand the linear relationship between two variables and the constant rate in the task with the second Newton's law. On the other hand, a sound understanding of a linear relationship and function as $V\,(T) = cT$ may enhance their understanding of slope as the derivative of the function with respect to temperature; $\mathbf{\frac{dV}{dT} = c.}$

Limited Understanding of the best fit line

Students' understanding of the best fit line was probed with a few tasks. For the participants, the role of the best fit line in the analysis of experimental measurements is to show the "wrong" measurements which should be excluded. Then, the best fit line should be used to calculate the slope, which is close to the "real" value of the slope. The following excerpt gives a strong flavor of this idea:

S: "*We cannot trust only one measurement because there can be wrong measurements. We should use all our measurements to decide how to draw the best fit line. With the best fit line we can reach the best solution. We are looking for the average of all data points"* (S. 1, Task 8, p. 5).
And then excerpts from interviews with more students:
S: …. *Our graph will show us the anomalies. We cannot trust one point, because that one maybe the wrong point (anomaly), so we need to take many points, plot them and draw the best fit line. If we do not draw the best line, there should be anomalies, then, the acceleration which we will calculate maybe far away from the "real" value g* (S. 14, p. 59). The same student carried on:
S*: I do not trust just one data point, but I trust the best fit line, the pattern and we want to be careful to draw the proper best fit line. And, then, for the calculation of the slope, we select two points (which are not experimental measurements). We should take two points, again for the same reason, because as soon as you get the best fit line, you forget the set of experimental measurements and you work with the best fit line.*
I: *What is a "proper line" for you?*
S: *It shows the average of all the data points* (S. 14, p. 59).
The best fit shows the "average value" and helps to identify wrong measurements. Then, the use of the best fit line is to calculate the slope.
S: *I want to trust the best fit, not just one experimental measurement. I need to draw the best fit and calculate the slope* (S. 5, p. 25).

One may argue that pre-service teachers have a limited understanding of the nature of measurements and the uncertainty related to measurement, as Allie and colleagues found out. Furthermore, this study has provided strong evidence that is much more behind such students' writing in the lab reports.

The main argument is that physics instructors need to develop open-ended tasks in which they ask students to demonstrate their full reasoning. The aim is for instructors to make sense of particular reasoning and difficulties in order to address them in follow-up instruction. Yet, laboratory instructors need to work on the long process of learning and teaching how to write a lab report.

We want to make the argument that supporting our students to write high quality lab reports will help them understand and improve experimental practices in the undergraduate physics lab. Learning how to write high quality physics lab reports should be one of the priorities in physics education and, in particular, in laboratory education. Writing makes it easier to identify such difficulties and reasoning. As lab instructors and physics teacher educators we should not neglect lab report writing. In research terms, lab reports are seen as a major source of the participants' difficulties and reasoning related to the investigated research focus. The challenge is how best to prepare prospective physics teachers to teach in the physics laboratory [36, 37].

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Appendix 1 Semi-structured interviews (based on the lab reports)

Why have you drawn a graph when analyzing experimental measurements (of two or more variables)?

How many data points did you take? Why?

How have you drawn the best fit line?

What does a best fit line mean to you?

How have you calculated the slope?

Appendix 2 (Task 9)

A group of students wanted to investigate the dependence of volume of an ideal gas to its pressure. To do this, they kept all the other variables (except pressure and volume) constant. They increased the pressure, as it is shown in the Table and, they then measured the volume of the gas. Their data is shown in the data table below. (Ideal gas law is given as $PV = nTR$, units of the variables are: P: kPa, V: cm^3).

Pressure (kPa)	Volume (cm^3)
80	55
100	44
120	37
150	29
180	24
220	20
260	17

Plot the data points and draw an appropriate graph.

Write a simple comment about what the experiment has shown.

Write a comment about the trend shown by the results in the Table.

Explain what information the shape of the graph provides about the possible relationship between pressure and volume.

Analyze the information provided by your graph using values obtained from your best fit. The aim is to present this data clearly to prove any relationship shown by the graph.