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Charmonium-like states with $J^P = 1^+$ and isospin 1

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Many mesons with properties incompatible with a $\bar{c}c$ structure have already been discovered, e.g. the Z_c mesons with isospin 1. We investigate the spectrum of exotic charmonium-like mesons using lattice QCD. The focus is on $\bar{c}c\bar{q}q$ states with $J^{PC} = 1^{+\pm}$ and isospin 1. This is the first study of four-quark states with these quantum numbers, a non-zero total momentum and two different lattice volumes. We extract the energy levels and determine the scattering length for $D\bar{D}^*$ scattering close to the threshold using Lüscher's formalism. Our preliminary results show that the energy shifts for eigenstates dominated by $D\bar{D}^*$ are very small in the 1^{++} channel and consistent with zero in the 1^{+-} channel.

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1. Introduction

The first signal of a nonconventional meson was the discovery of the $\chi_{c1}(3872)$ by Belle in 2003 [1]. Its quantum numbers $I(J^{PC}) = 0(1^{++})$ are compatible with a naive $\bar{c}c$ structure, however, its mass and decay properties point to a more complex nature. The clearest evidence that a resonance containing $\bar{c}c$ cannot be described as a simple quark-antiquark state is when it decays into a charged final state. The first such charged structure in the charmonium sector was discovered in 2013 when the BESIII and Belle collaborations observed the $Z_c^+(3900)$ in the $\pi^+\pi^- J/\psi$ invariant mass spectrum [2, 3]. This observation was confirmed by CLEO-c [4]. The quark content of the $Z_c^+(3900)$ is $\bar{c}c\bar{d}u$ ($I_z = 1$). Its neutral partner $Z_c^0(3900)$ has also been seen [4, 5]. The invariant mass of the $Z_c(3900)$ lies slightly above the $D\bar{D}^*$ threshold suggesting that it could be observed in the decay channel $(D\bar{D}^*)_{I=1}$. This was confirmed by the BESIII collaboration [6, 7]. The current consensus is that $Z_c(3900)$ is a $1(1^{+-})$ state with mass $M = 3887.1 \pm 2.6\,\text{MeV}$ and width $\Gamma = 28.4 \pm 2.6\,\text{MeV}$ [8]. Higher up in the spectrum, the $Z_c(4200)$ [9] and the $Z_c(4430)$ [10–13], have also been seen.

Different binding mechanisms have been suggested for the $Z_c(3900)$: it could be a hadronic molecule, have a compact tetraquark structure or result from a simple kinematic effect linked to the opening of meson-meson thresholds. Many studies involving different effective field theory approaches have been performed. Combining local hidden gauge and heavy quark spin symmetry, ref. [14] finds that the exchange of heavy vector mesons gives the most significant contribution. The resulting scattering amplitude contains information about a state with a mass between 3869 and 3875 MeV and a decay width of around 40 MeV. Another work [15], which studies the invariant mass distribution of the $J/\psi\pi$ and $D\bar{D}^*$ channels suggests that the $Z_c(3900)$ signal may originate from a resonance or a virtual state, depending on whether the $D\bar{D}$ s-wave interaction employed is energy dependent or independent, respectively. If the peak is produced by a virtual state, it must have a hadronic molecular nature. The authors of [16] come to similar conclusions. An analysis of the S-matrix poles in the framework of the constituent quark model involving coupled channels [17] connects the $Z_c(3900)$ signal with the presence of a virtual state that can be seen as a $D\bar{D}^*$ threshold cusp, i.e. a feature caused by the opening of a new threshold. This analysis is consistent with the interpretation that the diagonal interaction between the $D\bar{D}^*$ is too suppressed to develop resonances and that the interaction between different channels is responsible for a peak in the $D\bar{D}^*$, $J/\psi\pi$ invariant mass distributions.

Several lattice studies of the $Z_c(3900)$ have been performed so far: two works by the HAL QCD collaboration [18, 19] suggest the importance of cross-channel interaction, which is consistent with the conclusions of ref. [17]. However, works which employ Lüscher's formalism have not been able to confirm a narrow resonance-like peak close to the threshold. This includes [20–24] and the more recent coupled channel analysis of [25]. In particular, no additional eigenstates are found and the energy shifts with respect to the non-interacting levels turn out to be insignificant. Comparing results from both methods is difficult since the HAL QCD approach does not provide information on the energy shifts.

While charmonium-like states with $1(1^{+-})$ have been discovered in experiment, no states with $1(1^{++})$ and quark content $\bar{c}c\bar{d}u$ have been observed. Such a state would be an isospin partner of the $\chi_{c1}(3872)$. Two lattice QCD studies [26, 27], which find the state $\chi_{c1}(3872)$ slightly below the

$D\bar{D}^*$ threshold, also do not see any new candidates in this spectrum.

In this proceedings, we report on a lattice study of charmonium-like states with quantum numbers $1(1^{+\pm})$. We employ meson-meson interpolating operators that are projected on to two different total momenta. The corresponding two-point correlation functions are calculated on two lattices with different spatial extents. The extraction of the energy levels is challenging since we are interested in the region near the $D\bar{D}^*$ threshold, which lies above several other meson-meson thresholds, e.g., $J/\psi\pi$ and $\eta_c\rho$ in the $1(1^{+-})$ channel and $J/\psi\rho$ in the $1(1^{++})$ channel.

2. Lattice details

We employ two ensembles of gauge field configurations with $N_\mathrm{f} = 2 + 1$ non-perturbatively $O(a)$ improved Wilson dynamical fermions, a lattice spacing $a = 0.08636(98)(40)$ fm and a pion mass $m_\pi = 280(3)$ MeV. The ensembles are provided by the Coordinated Lattice Simulations consortium [28, 29]. The spatial volumes are $N_L^3 = 24^3$ and $N_L^3 = 32^3$, where we utilise 255 and 492 configurations, respectively [30]. Open boundary conditions in time are imposed [31] and the sources of the correlation functions are located in the bulk away from the boundary. The study is performed for a charm quark mass which is slightly larger than the physical quark mass [32].

3. Interpolating operators

The finite-volume energies are determined from the correlation matrices

$$C_{ij}(t) = \langle O_i(t_\mathrm{src} + t) O_j^\dagger(t_\mathrm{src}) \rangle \,, \tag{1}$$

where O_i ($O_j^\dagger$) is an interpolator that annihilates (creates) a state with certain quantum numbers. $\bar{c}c$ interpolators are not considered since we are interested in isospin $I = 1$, while local diquark-antidiquark interpolators are also omitted as they seem to have very little influence, according to [27]. The interpolators used are of two types: charmonium-light meson, $H(|\mathbf{p}_i|^2)L(|\mathbf{p}_j|^2)$, and D-meson-D-meson, $\bar{M}_i(|\mathbf{p}_i|^2)M_j(|\mathbf{p}_j|^2)$, where every $H(\mathbf{p}_i)$, $L(\mathbf{p}_j)$, $\bar{M}_i(\mathbf{p}_i)$ and $M_j(\mathbf{p}_j)$ has an appropriate Dirac structure and is separately projected on to definite momentum $\mathbf{p}_i$, $\mathbf{p}_j$ so that the total momentum is $\mathbf{P} = \mathbf{p}_i + \mathbf{p}_j$. The full set of interpolating operators used are given in Tables 1 and 2. They are constructed for $\Lambda^P = T_1^+$ and $\Lambda = A_2$, which are irreducible representations of the spatial lattice symmetry groups O_h ($|\mathbf{P}| = 0$) and Dic$_4$ ($|\mathbf{P}| = 1 \cdot 2\pi/L$), respectively. The quantum numbers contributing to the chosen irreducible representations are not only $J^P = 1^+$ but also unwanted higher $J = 3, \ldots$ and, in the case of $\Lambda = A_2$, $J^P = 0^-, 2^-$. The Wick contractions are evaluated using the distillation method [33] with 90 (100) Laplacian eigenvectors for $N_L = 24$ (32).

4. Preliminary results

4.1 Energy levels

We extract energy levels E_n^lat from single-exponential fits to the eigenvalues $\lambda^{(n)}(t) \propto e^{-E_n^\mathrm{lat} t}$ of the generalized eigenvalue problem [34]. They are shown in Fig. 1 for 1^{++} and Fig. 2 for 1^{+-}.

$\lvert\mathbf{P}\rvert^2 = 0,\ \Lambda^{PC} = T_1^{+-}$				$\lvert\mathbf{P}\rvert^2 = 1,\ \Lambda^{C} = A_2^{-}$		
$N_L = 24$ 15 interpolators	$N_L = 32$ 21 interp.	$J/\psi(0)\pi(0)$	×2	$J/\psi(1)\pi(0)$	×2	$N_L = 24$ 21 interp.
		$J/\psi(1)\pi(1)$	×2	$J/\psi(0)\pi(1)$	×2	
		$J/\psi(2)\pi(2)$	×3	$J/\psi(2)\pi(1)$	×2	
		$\eta_c(0)\rho(0)$		$J/\psi(1)\pi(2)$	×2	
		$\eta_c(1)\rho(1)$	×2	$J/\psi(4)\pi(1)$		
		$\bar{D}^*(0)D(0)$	×2	$\eta_c(1)\rho(0)$		
		$\bar{D}^*(1)D(1)$	×2	$\eta_c(0)\rho(1)$		
		$\bar{D}^*(0)D^*(0)$		$\eta_c(2)\rho(1)$	×2	
		$J/\psi(3)\pi(3)$	×2	$\bar{D}^*(0)D(1)$	×2	
		$\eta_c(2)\rho(2)$	×3	$\bar{D}^*(1)D(0)$	×2	
		$h_c(1)\pi(1)$		$\bar{D}^*(1)D(2)$	×2	
				$\bar{D}^*(2)D(1)$	×2	

Table 1: Table of interpolators transforming under irreducible representations $\Lambda^{PC} = T_1^{+-}$ and $\Lambda^C = A_2^-$ which correspond to $J^{PC} = 1^{+-}$. All momenta here are in units of $2\pi/L$.

$\lvert\mathbf{P}\rvert^2 = 0,\ \Lambda^{PC} = T_1^{++}$				$\lvert\mathbf{P}\rvert^2 = 1,\ \Lambda^{C} = A_2^{+}$			
$N_L = 24$ 5 interpolators	$N_L = 32$ 10 interp.	$J/\psi(0)\rho(0)$		$\eta_c(1)a_0(0)$		$N_L = 32$ 17 interp.	$N_L = 24$ 13 interp.
		$\bar{D}^*(0)D(0)$	×2	$\chi_{c0}(1)\pi(0)$			
		$\bar{D}^*(1)D(1)$	×2	$\chi_{c0}(0)\pi(1)$			
		$J/\psi(1)\rho(1)$	×3	$J/\psi(1)\rho(0)$			
		$\chi_{c0}(1)\pi(1)$		$J/\psi(0)\rho(1)$			
		$\chi_{c1}(1)\pi(1)$		$\bar{D}^*(0)D(1)$	×2		
				$\bar{D}^*(1)D(0)$	×2		
				$\bar{D}^*(1)D(2)$	×2		
				$\bar{D}^*(2)D(1)$	×2		
				$\eta_c(0)a_0(1)$			
				$\chi_{c0}(2)\pi(1)$			
				$\chi_{c0}(4)\pi(1)$			
				$\chi_{c1}(2)\pi(1)$			

Table 2: Table of interpolators transforming under irreducible representations $\Lambda^{PC} = T_1^{++}$ and $\Lambda^C = A_2^+$ which correspond to $J^{PC} = 1^{++}$. All momenta here are in units of $2\pi/L$.

Many states lie below the lowest $D\bar{D}^*$ levels, in particular, for non-zero total momentum and the larger lattice volume. The energy shifts of states dominated by $D\bar{D}^*$ are very small in the 1^{++} case and negligible within the present uncertainties in the 1^{+-} case. Despite the light mesons ρ and a_0 being resonances, they have been treated as stable particles. In the energy level plots in Figs. 1 and 2, one can see that eigen-energies dominated by interpolators containing the ρ meson have significant uncertainties.

4.2 $D\bar{D}^*$ scattering

To simplify the procedure, we focus on $D\bar{D}^*$ scattering near the threshold. Its coupling to other channels ($J/\psi\pi$, $\eta_c\rho$, $J/\psi\rho$) is neglected when studying the scattering amplitudes. The spectrum is expected to be dominated by the $\ell = 0$ partial wave. The finite volume eigen-energies are connected

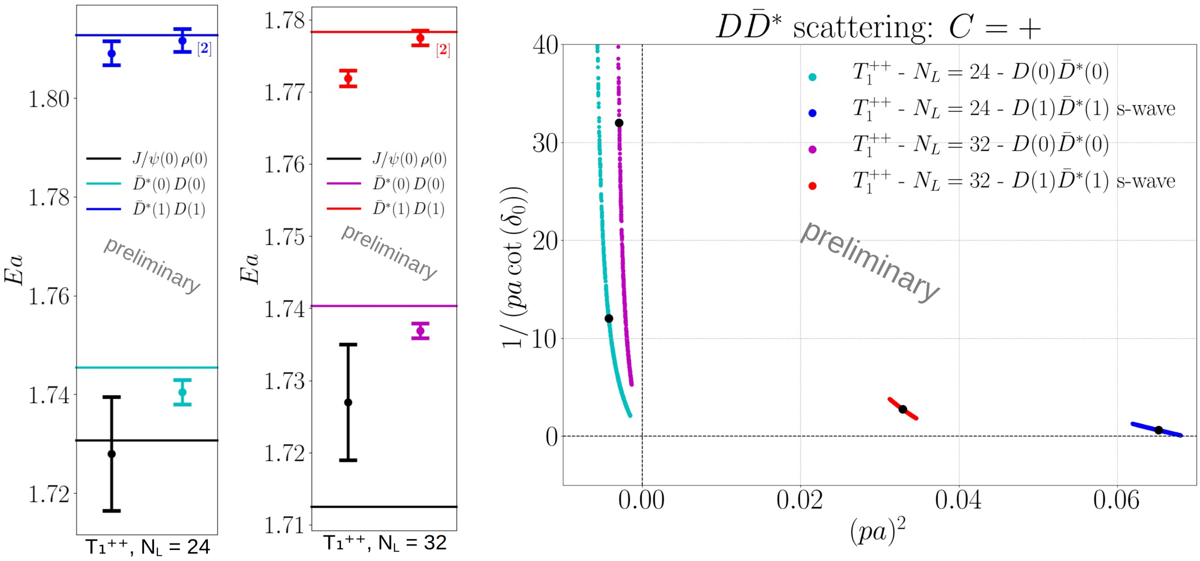

Figure 1: Results for $I(J^{PC}) = 1(1^{++})$. The two panes on the left represent the energy levels (points, E_n^{lat}) and non-interacting energies (lines, $E^{\text{lat}}_{H_i(\mathbf{p}_i)} + E^{\text{lat}}_{H_j(\mathbf{p}_j)}$). From left to right the panes represent T_1^{++} with $N_L = 24$ and T_1^{++} with $N_L = 32$, respectively. Numbers within the square brackets refer to the multiplicity of certain non-interacting levels. The plot on the right shows $1/(p\cot(\delta))$ where the colors of the states match those in the spectra, and δ is the s-wave $D\bar{D}^*$ scattering phase shift with approximations stated in subsection 4.2. Results are shown with 1σ statistical uncertainty.

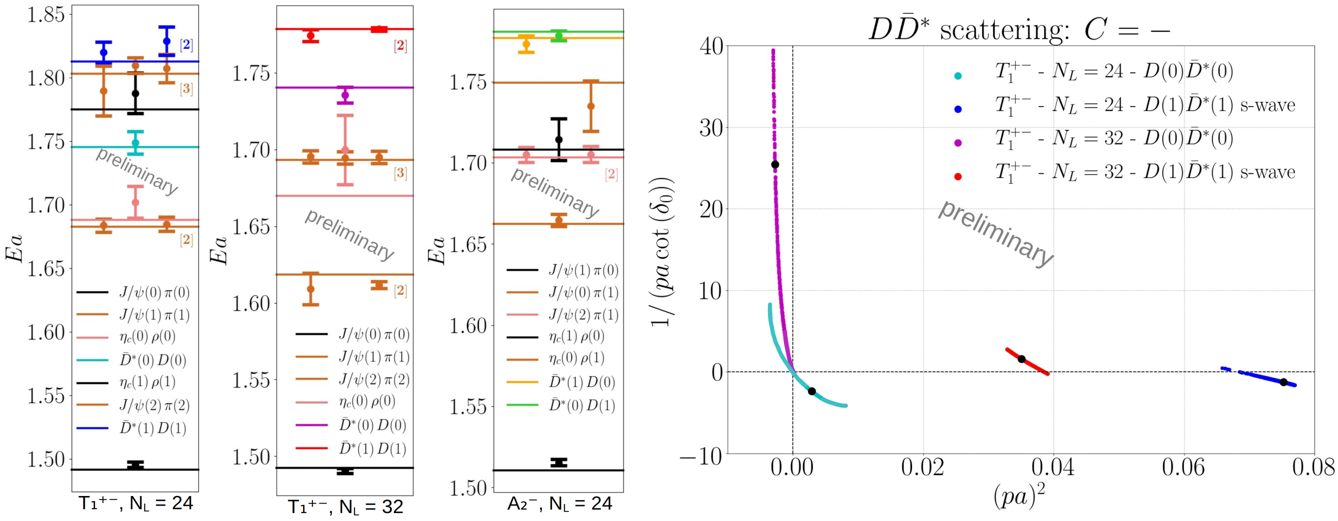

Figure 2: Results for $I(J^{PC}) = 1(1^{+-})$. The three panes on the left represent the energy levels (points, E_n^{lat}) and non-interacting energies (lines, $E^{\text{lat}}_{H_i(\mathbf{p}_i)} + E^{\text{lat}}_{H_j(\mathbf{p}_j)}$). From left to right the panes represent T_1^{+-} with $N_L = 24$, T_1^{+-} with $N_L = 32$ and A_2^- with $N_L = 24$, respectively. Numbers within the square brackets refer to the multiplicity of certain non-interacting levels. The plot on the right shows $1/(p\cot(\delta))$ where the colors of the states match those in the spectra, and δ is the s-wave $D\bar{D}^*$ scattering phase shift with approximations stated in subsection 4.2. Results are shown with 1σ statistical uncertainty.

to the infinite volume s-wave $D\bar{D}^*$ scattering phase shift δ via

$$p\cot(\delta(p)) = \frac{2\mathcal{Z}^{\mathbf{d}}_{00}(1, (\frac{pL}{2\pi})^2)}{\gamma\sqrt{\pi}L}\,, \tag{2}$$

where higher partial waves are omitted, and the momentum $p = |\mathbf{p}_{cm}|$ in the center-of-mass frame is derived from

$$E_{\text{cm}} = \sqrt{|\mathbf{p}_{cm}|^2 + m_i^2} + \sqrt{|\mathbf{p}_{cm}|^2 + m_j^2}\,, \quad \text{where} \quad E_{\text{cm}} = \sqrt{E_n^2 - |\mathbf{P}|^2}\,. \tag{3}$$

Discretization effects modify the dispersion relation, which deviates from the continuum one. To mitigate this, we use the following energies

$$E_n = E_n^{\text{lat}} + E^{\text{con}}_{H_i(\mathbf{p}_i)} + E^{\text{con}}_{H_j(\mathbf{p}_j)} - E^{\text{lat}}_{H_i(\mathbf{p}_i)} - E^{\text{lat}}_{H_j(\mathbf{p}_j)}\,, \tag{4}$$

where $E^{\text{lat}}_{H(\mathbf{p})}$ and $E^{\text{con}}_{H(\mathbf{p})} = (|\mathbf{p}|^2 + m_H^2)^{1/2}$ are single-hadron energies. Within the aforementioned approximations, the scattering amplitude can be parametrized in terms of δ

$$T = \frac{1}{p\cot(\delta(p)) - ip}\,. \tag{5}$$

Assuming elastic scattering near the threshold, one can perform the effective range expansion $p\cot(\delta(p)) = 1/a_0 + r_0 p^2/2 + O(p^4)$. Our preliminary results are presented in Figs. 1 and 2. One can infer the smallness of the interaction from the small $1/(p\cot(\delta))$ values, which are zero in the non-interacting limit.

5. Conclusion and outlook

We have extracted the spectrum of charmonium-like states with $1(1^+)$. This is the first study considering hadronic states with these quantum numbers, a non-zero total momentum and two different lattice volumes. The energy shifts are small, which is consistent with conclusions from previous lattice QCD studies using the Lüscher method. This disfavors a significant attraction between D and $\bar{D}^*$. Experimental evidence and findings from this preliminary study perhaps suggest that a significant coupling between channels causes the existence of Z_c. In the near future, we will make a comparison with phenomenological approaches and put constraints on them. In particular, we aim to compare our lattice eigen-energies with the energy levels that different models, such as [14, 17], predict.

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