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Study of quasi-beam function in twisted mass lattice QCD

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Constantia Alexandrou,[a,b] **Krzysztof Cichy,**[c] **Martha Constantinou,**[d] **Xu Feng,**[e,f,g] **Karl Jansen,**[h] **Chuan Liu,**[e,f,g] **Aniket Sen,**[i,*] **Gregoris Spanoudes,**[b] **Fernanda Steffens,**[i] **Jacopo Tarello**[a,b] **and Carsten Urbach**[i]

[a] *Department of Physics, University of Cyprus, P.O. Box 20537, 1678 Nicosia, Cyprus*

[b] *Computation-based Science and Technology Research Center, The Cyprus Institute, 20 Kavafi Str., Nicosia 2121, Cyprus*

[c] *Faculty of Physics, Adam Mickiewicz University, ul. Uniwersytetu Poznańskiego 2, 61-614 Poznań, Poland*

[d] *Temple University,1925 N. 12th Street, Philadelphia, PA 19122-1801, USA*

[e] *School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*

[f] *Collaborative Innovation Center of Quantum Matter, Beijing 100871, China*

[g] *Center for High Energy Physics, Peking University, Beijing 100871, China*

[h] *NIC, DESY, Platanenallee 6, D-15738 Zeuthen, Germany*

[i] *Institut für Strahlen- und Kernphysik, Rheinische Friedrich-Wilhelms-Universität Bonn, Nussallee 14-16, 53115 Bonn*

E-mail: sen@hiskp.uni-bonn.de

We present an exploratory study of the quasi-beam function using an $N_f = 2+1+1$ twisted mass lattice of size $24^3 \times 48$, with a pion mass of 350 MeV and of lattice spacing 0.093 fm. We show results for longitudinal momentum of 1.7 GeV and transverse separation of up to 0.18 fm. We also study the non-perturbative renormalization of the bare matrix element using the RI/MOM scheme.

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*Speaker

1. Introduction

The structure of hadrons is studied through objects such as form factors and parton distribution functions (PDFs) [1]. PDFs are useful for studying one dimensional structure of hadrons, since it only depends on the fraction of longitudinal momentum of the parent hadron that is carried by the constituent partons. For a three dimensional understanding of hadron structure, one needs generalized parton distributions (GPDs) [2–4] and transverse momentum dependent PDFs (TMDPDFs) [1, 5–7]. In the present work, we focus on the latter.

Experimentally, TMDPDFs can be extracted from high energy scattering processes such as Drell-Yan production and semi-inclusive deep-inelastic scattering. Historically, theoretical calculation of TMDPDFs (and PDFs in general) from first principles using lattice QCD has been challenging due to difficulties in computing the associated light-cone correlators on the lattice. Over the last several years, the development of large momentum effective theory (LaMET) [8–11] has made it possible to calculate such objects in lattice QCD. Under the framework of LaMET, the TMDPDF is defined in terms of a quasi-beam function and a soft function [10, 12]. Recently, the soft function [13, 14] and the quasi-beam [15, 16] function have been calculated, the latter with pion external states, within lattice QCD. While preparing this proceedings contribution, a work on quasi-TMDPDF with the nucleon was also put forward [17]. In this work we present an exploratory study of the quasi-beam function with nucleon external states, with an aim to combine this result with our previous soft function calculation [14] and obtain the full TMDPDF. We also study a possible non-perturbative renormalization through regularization-independent momentum subtraction (RI/MOM) scheme.

In the following section, we define the quasi-beam function in relation to the TMDPDF. In section 3, we explain the non-perturbative renormalization procedure using RI/MOM. Section 4 describes the lattice setup used in the simulation and section 5 summarizes the results.

2. The quasi-beam function

The scheme-independent TMDPDF $f^{TMD}(x, b, \mu, \zeta)$ is defined as [10]

$$f^{TMD}(x, b, \mu, \zeta) = H\left(\frac{\zeta_z}{\mu^2}\right) e^{-\ln\left(\frac{\zeta_z}{\zeta}\right)K(b,\mu)} \tilde{f}(x, b, \mu, P^z), S_r^{\frac{1}{2}}(b, \mu) + \dots \tag{1}$$

where $\tilde{f}(x, b, \mu, P^z)$ is the so-called quasi-TMDPDF, $S_r^{\frac{1}{2}}(b, \mu)$ is the reduced soft function. x is the longitudinal momentum fraction, b the transverse separation, μ defines the renormalization scale and $\zeta_z = (2xP^z)^2$ is the Collins-Soper scale of the quasi-TMDPDF with ζ being the scale for the light-cone correlation. The factor $H\left(\frac{\zeta_z}{\mu^2}\right)$ is the perturbative matching kernel and $K(b, \mu)$ is the Collins-Soper kernel. Under LaMET, the soft factor can be obtained through a ratio of a meson form factor and the TMD wave function [18]. The quasi-TMDPDF, on the other hand, is defined as

$$\tilde{f}(x, b, \mu, P^z) = \lim_{L\to\infty} \int \frac{dz}{2\pi} e^{-iz(xP^z)} Z^{\overline{MS}}(\mu, z) \frac{P^z}{E_P} B^{\Gamma}(z, b, L, P^z). \tag{2}$$

Here. $B(z, b, L, P^z)$ is known as the quasi-beam function and it is described by the bare matrix element

$$\begin{aligned} B^{\Gamma}(z, b, L, P^z) &= \langle N(P^z)|O^{\Gamma}(z, b, L)|N(P^z)\rangle \\ &= \langle N(P^z)|\bar{\psi}(x+b+z)\Gamma\mathcal{W}(x+b+z; L)\psi(x)|N(P^z)\rangle, \end{aligned} \tag{3}$$

where $N(P^z)$ is a nucleon state with momentum boost of $(0, 0, P^z)$ and $\psi(x)$ is the standard light quark doublet. For the unpolarized TMDPDF, Γ can be either γ_t or γ_z. In this work, we show results for $\Gamma = \gamma_z$. $\mathcal{W}(x+b+z; L)$ is an asymmetric staple-shaped Wilson line with L being the length of the symmetric part:

$$\mathcal{W}(x+b+z; L) = \mathcal{W}_z(x; x+L)\mathcal{W}_\perp(x+L; x+L+b)\mathcal{W}_z(x+L+b; x+b+z). \tag{4}$$

Here $\mathcal{W}_z$ defines a Wilson line along the boost direction and $\mathcal{W}_\perp$ defines one along the transverse direction. The shape of the asymmetric staple is shown in Figure 1.

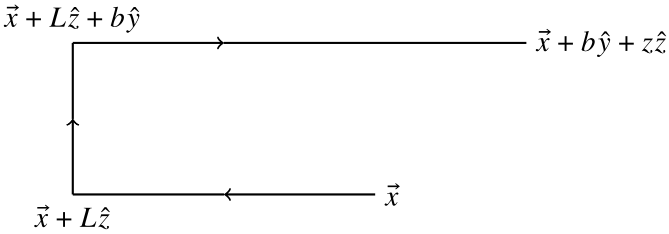

Figure 1: The shape of the asymmetric staple defined in the operator of the quasi-beam function

The bare matrix element defined in Eq. (3) is shown to have an intrinsic e^{-L} divergence arising from Wilson link self energy [10, 16], and, hence, in the literature one often defines a "subtracted" quasi-beam function as

$$\tilde{B}^{\Gamma}(z, b, P^z) = \frac{B^{\Gamma}(z, b, L, P^z)}{\sqrt{Z_E(b, 2L+z)}}. \tag{5}$$

Here $Z_E(b, 2L+z)$ is the vacuum expectation value of a rectangular Wilson loop having side length of $2L+z$ along the boost direction and b along the transverse direction. This subtraction takes care of the L-divergence in the matrix element. However, if one performs a non-perturbative renormalization by building an amputated Green's function with the operator under consideration, a similar divergence is expected and must be subtracted from there as well. Therefore, effectively, this subtraction cancels out between the bare matrix element and the renormalization factor. Since we will be using RI/MOM, we do not utilize Eq. (5).

3. Non-perturbative renormalization using RI/MOM

The bare matrix element defined in Eq. (3) must be renormalized, as it contains both logarithmic divergences arising from the endpoints of the Wilson line, as well as from the cusps associated with the quark wave function, and a linear divergence associated with the length of the Wilson line. Additionally, according to symmetry arguments, staple-shaped operators of different Dirac

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structures can and will mix; thus, an appropriate renormalization which disentangles the mixing is also needed. One commonly used method is the non-perturbative renormalization using the RI/MOM scheme [19], with a subsequent perturbative matching to the $\overline{\text{MS}}$ scheme. In this work, we show results for the non-perturbative part of the renormalization.

The RI/MOM renormalized quasi-beam function for each operator Γ is obtained through the following relation:

$$B^{\Gamma}_{RI/MOM}(z, b, P^z; \mu) = Z^{RI/MOM}_{\Gamma\Gamma'}(z, b, L; \mu) B^{\Gamma'}(z, b, L, P^z). \tag{6}$$

Since operator mixing is present, the renormalization functions take a matrix form $Z_{\Gamma\Gamma'}$. In order to determine all renormalization matrix elements, we impose the following condition using a number of different projectors Γ':

$$Z^{RI/MOM}_{\Gamma\Gamma'}(\mu) = \left[\frac{\text{Tr}[\Lambda^{\Gamma}(p)\Gamma'^{\dagger}]}{12 e^{i(p_z \cdot z + p_\perp \cdot b)} Z_q(p)} \right]^{-1} \Bigg|_{p^2=\mu^2}, \tag{7}$$

where $\Lambda^{\Gamma}(p)$ is the amputated vertex function of the staple-shaped operator O^{Γ} (see Eq. (3)),

$$\Lambda^{\Gamma}(p) = S^{-1}(p) G^{\Gamma}(p) S^{-1}(p). \tag{8}$$

$S(p)$ is the quark propagator and $G^{\Gamma}(p)$ denotes the Green's function,

$$G^{\Gamma}(p) = \langle q(p) | O^{\Gamma}(z, b, L) | q(p) \rangle. \tag{9}$$

Here $q(p)$ is an off-shell quark state defined in the Landau gauge and $\mu = \sqrt{p^2}$ defines the renormalization scale.
The quark wave function renormalization $Z_q(p)$, in the denominator of Eq. (7), is defined as

$$\begin{aligned} Z_q(p) &= \frac{1}{12} \text{Tr}[S^{-1}(p) S^{\text{tree}}(p)] \\ &= \frac{\text{Tr}[-i \sum_\mu S^{-1}(p) \gamma_\mu \sin(p_\mu)]}{12 \sum_\mu \sin(p_\mu)^2}. \end{aligned} \tag{10}$$

In some recent works [16, 20], following this scheme, different mixing patterns are considered including possible mixing among all different Dirac structures Γ. In our study, we follow a different approach by considering the minimal set of staple-shaped operators that are allowed to mix by C, P, T symmetries. A similar approach was taken for the quasi-PDF case and it was found that γ_t has no possible mixing [21, 22]. In the present case, mixing is unavoidable. It can be shown, however, through symmetry arguments, that for $\Gamma = \gamma_z$ the only operators that mix are $\{\mathbf{1}, \gamma_y, \gamma_y\gamma_z\}$ [23]. This assumes that the transverse direction in the construction of the staple is along the y-direction.

4. The lattice setup

For the lattice simulation, we use an $N_f = 2+1+1$ clover improved twisted mass fermion ensemble of size $24^3 \times 48$, produced by the Extended Twisted Mass Collaboration (ETMC) [24]. This is the same ensemble that was used in our previous study of the soft function [14], since we would like to combine our results with those of the reduced soft function. This exploratory study is done using 100 configurations with 8 source positions for each configuration. To increase statistics, the boost is taken in all possible directions, both positive and negative. For each such direction of boost, the staple is constructed in both the remaining transverse directions. This gives us 12 measurements (6 boost directions × 2 transverse directions) for each source position. The details of the lattice simulation are summarized in Table 1.

$L^3 \times T$	$a[fm]$	$a\mu_{sea}$	$m^{\pi}_{sea}[MeV]$	N_{conf}	N_{src}	N_{meas}
$24^3 \times 48$	0.093	0.00530	350	100	8	9600

Table 1: Details of the lattice ensemble used in the calculation

The bare matrix element for the quasi-beam function is calculated through a ratio of a 3-point and a 2-point function,

$$\begin{aligned} B^{\Gamma}(z,b,L,P^z) &= \frac{\langle C^{3pt}_{\Gamma}(z,b,L,P^z;t_s,\tau)\rangle}{\langle C^{2pt}(P^z;t_s)\rangle} \\ &= \frac{\sum_{\mathbf{x}} e^{-i\mathbf{P}\cdot\mathbf{x}}\langle 0|N(\mathbf{x},t_s)O^{\Gamma}(z,b,L;\tau)\bar{N}(\mathbf{0},0)|0\rangle}{\sum_{\mathbf{x}} e^{-i\mathbf{P}\cdot\mathbf{x}}\langle 0|N(\mathbf{x},t_s)\bar{N}(\mathbf{0},0)|0\rangle}. \end{aligned} \tag{11}$$

Here, τ is the insertion time of the operator O^{Γ} and t_s defines the source-sink time separation. In this work, we show results for a single source-sink separation of $t_s = 10a$.
The 3-point function is constructed for the isovector combination $u-d$, by inserting a τ_3 in flavor space. This choice ensures the elimination of the disconnected contributions and only connected diagrams need to be calculated.
The propagators are calculated from APE-smeared links. Momentum smearing [25] is also applied in order to improve the signal at large boost. It is also observed that applying stout smearing to the gauge links used in construction of the staple also reduces the statistical errors. Here, we have applied 5 steps of stout smearing to the staple shaped Wilson line.

For the renormalization, we have used Landau gauge fixed configurations from the same ensemble. The $Z^{RI/MOM}$ factors are calculated using 10 configurations at a scale of $(4,4,4,4+\frac{1}{2})$ in lattice units. The staples are similarly stout smeared for the vertex functions as well.

5. Results

In this section, we present the results for the quasi-beam functions at a boost of $\frac{6\pi}{L}$, which is about 1.7 GeV for the given lattice. In Figure 2, we present the bare and RI/MOM renormalized quasi-beam functions at different values of L and z for $b = 0.09$ fm and $b = 0.18$ fm. The values for z and $-z$ are symmetrized using the relation $B(-z) = B^{\dagger}(z)$.
One can easily observe the L-associated divergence in the bare matrix element, even though such behavior is strongly suppressed by the application of stout smearing. For the non-perturbatively

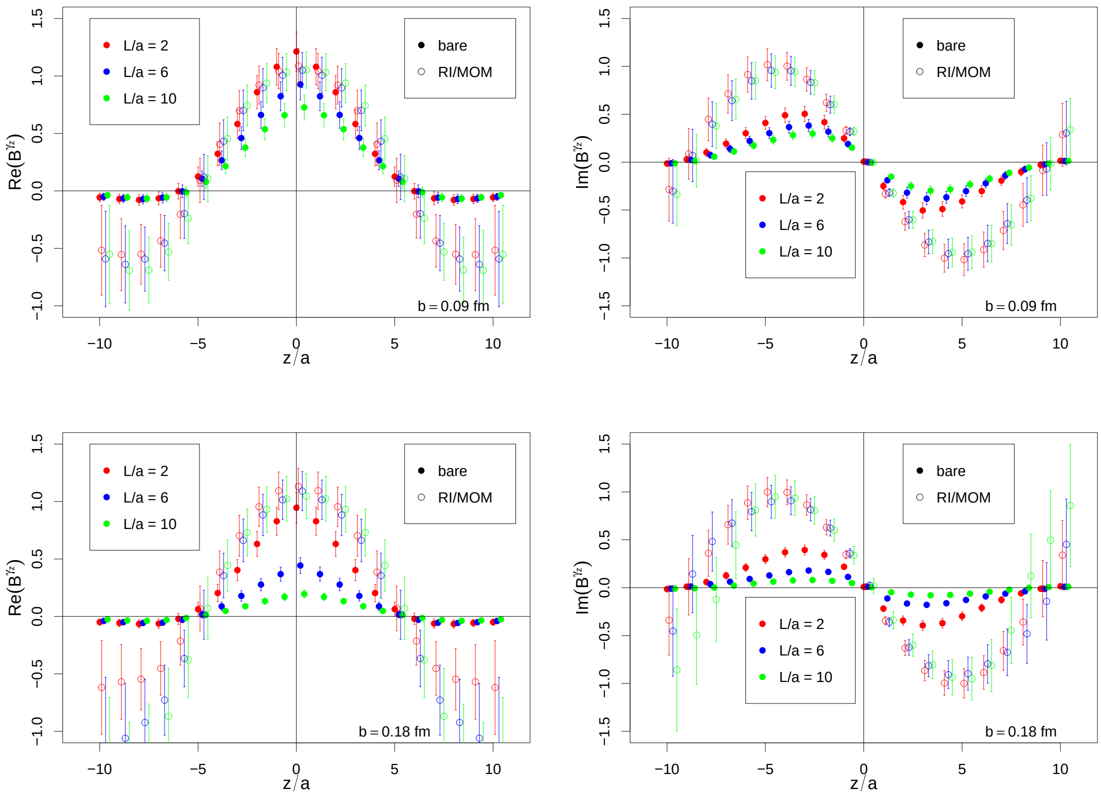

Figure 2: Real and imaginary parts of the bare and RI/MOM renormalized quasi-beam functions for different values of L and z. The plots in the first row correspond to $b = 0.09$ fm and in the second row to $b = 0.18$ fm. Note that points for different L are horizontally shifted for better visibility.

renormalized matrix elements, no residual L dependence is observed, as expected. It is also observed that the errors get significantly large with increasing b and z. So more statistics is definitely required in order to probe larger transverse separations.

In Figure 3, we plot the RI/MOM factors for the diagonal element of γ_z for different values of b. The renormalization factors increase rapidly for larger z and b. This explains the strong scaling of errors in the renormalized matrix element, particularly for larger values of z.
The divergence of RI/MOM factors with increasing b and z is problematic, since the signal from the bare matrix element decreases for such cases. For $b/a > 2$, renormalization through RI/MOM might not be feasible. Other methods of non-perturbative renormalization are currently being studied.

6. Conclusion

This work represents an exploratory study of the quasi-beam functions necessary for the construction of the TMDPDF. We showed results for the bare and non-perturbatively renormalized quasi-beam

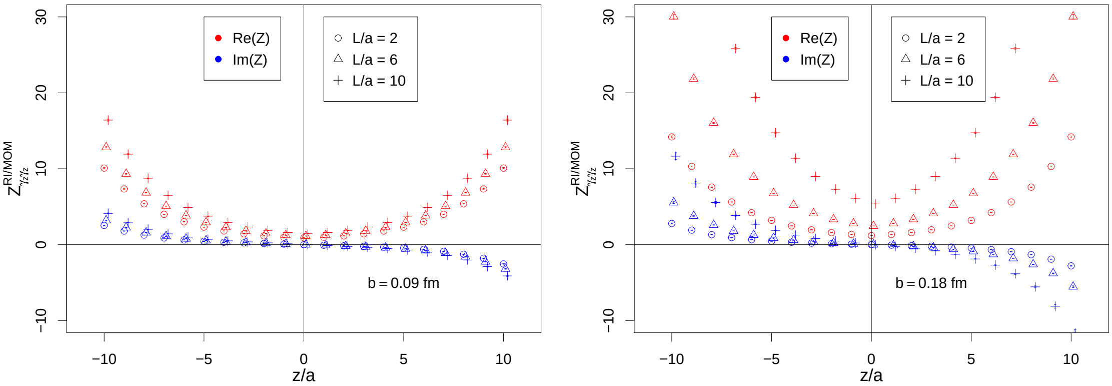

Figure 3: RI/MOM renormalization factors for γ_z

functions for transverse separation up to 0.18 fm at a boost of 1.7 GeV. The results are promising, however the renormalization procedure through RI/MOM might not be effective for large transverse separations. We are currently studying other techniques such as the ratio scheme [26] and auxiliary field [27] approach. Work is also underway for the calculation of the perturbative conversion factors. The next step is to combine this result with the soft function calculations and obtain the quasi-TMDPDF on the lattice and perform the matching to light-cone TMDPDF.

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