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Noether supercurrent operator mixing from lattice perturbation theory

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In this work we present perturbative results for the renormalization of the supercurrent operator, S_μ, in $\mathcal{N} = 1$ Supersymmetric Yang-Mills theory. At the quantum level, this operator mixes with both gauge invariant and noninvariant operators, which have the same global transformation properties. In total, there are 13 linearly independent mixing operators of the same and lower dimensionality. We determine, via lattice perturbation theory, the first two rows of the mixing matrix, which refer to the renormalization of S_μ, and of the gauge invariant mixing operator, T_μ. To extract these mixing coefficients in the $\overline{\rm MS}$ renormalization scheme and at one-loop order, we compute the relevant two-point and three-point Green's functions of S_μ and T_μ in two regularizations: dimensional and lattice. On the lattice, we employ the plaquette gluonic action and for the gluinos we use the fermionic Wilson action with clover improvement.

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1. Introduction to $\mathcal{N}=1$ SYM and definition of operators

The $\mathcal{N}=1$ Supersymmetric Yang-Mills (SYM) Lagrangian [1] connects gluon and gluino fields; it shares some of the fundamental properties of supersymmetric gauge theories containing quarks and squarks, while at the same time it is amenable to high-accuracy numerical simulations [2]. Therefore it is an ideal forerunner for future investigations of theories containing more superfields. In the Wess-Zumino (WZ) gauge, the Lagrangian is[1]:

$$\mathcal{L}_{\rm SYM} = -\frac{1}{4}u^\alpha_{\mu\nu}u^\alpha_{\mu\nu} + \frac{i}{2}\bar\lambda^\alpha\gamma^\mu\mathcal{D}_\mu\lambda^\alpha, \quad u_{\mu\nu} = \partial_\mu u_\nu - \partial_\nu u_\mu + ig[u_\mu, u_\nu], \quad \mathcal{D}_\mu\lambda = \partial_\mu\lambda + ig[u_\mu,\lambda], \tag{1.1}$$

where $u_{\mu\nu}$ is the gluon field tensor, u_μ is the gluon field, λ is the gluino field which is a Majorana spinor in the adjoint representation of the gauge group. $\mathcal{L}_{\rm SYM}$ remains invariant, up to a total derivative, under the supersymmetric transformations:

$$\begin{aligned} \delta_\xi u^\alpha_\mu &= -i\bar\xi\gamma^\mu\lambda^\alpha, \\ \delta_\xi\lambda^\alpha &= \frac{1}{4}u^\alpha_{\mu\nu}[\gamma^\mu,\gamma^\nu]\xi. \end{aligned} \tag{1.2}$$

Noether's theorem gives a supercurrent for this Lagrangian stemming from the transformations of Eq. (1.2); in Euclidean space, the supercurrent takes the form:

$$S_\mu = -\frac{1}{2}{\rm tr}_c(u_{\rho\,\sigma}[\gamma_\rho,\gamma_\sigma]\gamma_\mu\lambda)$$

In this work, we make use of the Wilson formulation on the lattice, with the addition of the clover (SW) term for gluino fields. For the lattice discretization of S_μ, the lattice version of gluon field tensor, $\hat F_{\rho\sigma}$, which we adopt, is a sum of plaguettes in the $\rho-\sigma$ plane having x as their initial and final point (see, e.g., Ref. [3] for standard notation).

A proper study of S_μ must address the fact that it mixes with a number of other operators at the quantum level. These operators must necessarily have the same transformation properties under global symmetries (e.g. Lorentz, or hypercubic on the lattice, global $SU(N_c)$ transformations, ghost number, etc.) and their dimension must be lower than or equal to that of S_μ, namely $7/2$. There are altogether four classes of such operators, as follows:

Class G: Gauge-invariant operators.

Class A: BRST variation of operators.

Class B: Operators which vanish by the equations of motion.

Class C: Other operators which do not belong to the above classes.

S_μ being gauge invariant operator belongs to Class G. In particular, S_μ mixes with another dimension $7/2$ gauge invariant operator, denoted here as:

$$T_\mu = 2{\rm tr}_c(u_{\mu\,\nu}\gamma_\nu\lambda) \tag{1.3}$$

[1] In order to quantize the theory, we fix the gauge by including a gauge-fixing term, together with the compensating ghost field (c^α) terms; these terms are the same as in the non-supersymmetric case.

A total of twelve gauge noninvariant operators could in principle mix with S_μ. These operators necessarily belong to classes A, B and C[2]:

$$\begin{aligned}
\mathcal{O}_{A1} &= \frac{1}{\alpha}\mathrm{tr}_c((\partial_\nu u_\nu)\gamma_\mu\lambda) - ig\,\mathrm{tr}_c([c,\bar{c}]\gamma_\mu\lambda) \\
\mathcal{O}_{B1} &= \mathrm{tr}_c(u_\mu \not{D}\lambda),\ \mathcal{O}_{B2} = \mathrm{tr}_c(\not{u}\gamma_\mu\not{D}\lambda) \\
\mathcal{O}_{C1} &= \mathrm{tr}_c(u_\mu\lambda),\ \mathcal{O}_{C2} = \mathrm{tr}_c(\not{u}\gamma_\mu\lambda),\ \mathcal{O}_{C3} = \mathrm{tr}_c(\not{u}\partial_\mu\lambda),\ \mathcal{O}_{C4} = \mathrm{tr}_c((\partial_\mu\not{u})\,\lambda) \\
\mathcal{O}_{C5} &= \mathrm{tr}_c((\partial_\nu u_\nu)\gamma_\mu\lambda),\ \mathcal{O}_{C6} = \mathrm{tr}_c(u_\nu\gamma_\mu\partial_\nu\lambda),\ \mathcal{O}_{C7} = i\,g\ \mathrm{tr}_c([u_\rho, u_\sigma][\gamma_\rho,\gamma_\sigma]\gamma_\mu\lambda) \\
\mathcal{O}_{C8} &= i\,g\ \mathrm{tr}_c([u_\mu,u_\nu]\gamma_\nu\lambda),\ \mathcal{O}_{C9} = i\,g\ \mathrm{tr}_c([c,\bar{c}]\gamma_\mu\lambda)
\end{aligned} \tag{1.4}$$

Among them, there is just one class A acceptable operator. Two of them are not present on-shell because are class B operators. Further, there are nine class C operators, where the first two class C operators are lower dimensional operators and they only may show up in the lattice regularization.

For a comprehensive presentation of our results, along with detailed explanations and a longer list of references, we refer to our publication [3].

2. Computational setup for the renormalization of the supercurrent operator

Our calculation set up shares a backbone of methodology, which is briefly described in three steps:

1. The first step is to produce a minimal list of all candidate mixing operators by exploiting certain symmetries of the action, valid both in the continuum and the lattice formulation of the theory. This reduces the number of the operators that can possibly mix with S_μ at the quantum level to a minimum set of 13 operators.

2. Secondly, we careful select and compute a set of Green's functions both in the continuum and the lattice regularizations. The lattice calculations are the crux of this work; and the continuum calculations serve as a necessary introductory part, allowing us to relate our lattice results to the $\overline{\mathrm{MS}}$ scheme. An unambiguous extraction of all mixing coefficients and renormalization constants of the operator S_μ entails specific choices of the external momenta for the Green's functions. In particular, we calculate two-point and three-point Green's functions of S_μ using both dimensional regularization (continuum), where we regularize the theory in D-dimensions ($D = 4 - 2\varepsilon$), and lattice regularization. The continuum Green's functions will be used in order to calculate the renormalized Green's functions in the $\overline{\mathrm{MS}}$ scheme, which are necessary ingredients for the renormalization conditions on the lattice.

3. Lastly, we apply renormalization conditions in the $\overline{\mathrm{MS}}$ scheme to the Green's functions, in order to get the results on the renormalization and the mixing coefficients.

[2]Operators $\mathcal{O}_{C5}$ and $\mathcal{O}_{C9}$, taken together with $\mathcal{O}_{A1}$, are linearly dependent; however, keeping both of them in the list affords us with additional consistency checks.

Note that the same mixing operators may mix with T_μ. We follow the same methodology and we also calculate the same one-loop Green's functions with insertion of the T_μ operator. Thus, we determine the second row of the 13×13 mixing matrix. However these results [3] are omitted here for the sake of brevity.

For off-shell matrix elements, the mixing assumes this general form:

$$S_\mu^R = Z_{SS} S_\mu^B + Z_{ST} T_\mu^B + Z_{SA1} \mathcal{O}_{A1}^B + \sum_{i=1}^{2} Z_{SBi} \mathcal{O}_{Bi}^B + \sum_{i=1}^{9} Z_{SCi} \mathcal{O}_{Ci}^B, \tag{2.1}$$

where the superscript B stands for the bare and R for renormalized quantities. In order to determine all Z-factors, we consider two-point Green's functions with one external gluino and one external gluon fields ($\langle u_\nu^{\alpha_1}(-q_1)\, S_\mu(x)\, \bar{\lambda}^{\alpha_2}(q_2)\rangle$), as well as three-point Green's functions with external gluino/gluon/gluon fields ($\langle u_\nu^{\alpha_1}(-q_1)\, u_\rho^{\alpha_2}(-q_2)\, S_\mu(x)\, \bar{\lambda}^{\alpha_3}(q_3)\rangle$) and with gluino/antighost/ghost fields ($\langle c^{\alpha_3}(q_3)\, S_\mu(x)\, \bar{c}^{\alpha_2}(q_2) \bar{\lambda}^{\alpha_1}(-q_1)\rangle$); similarly for T_μ.

In Table 1, we show the tree-level Green's functions for all operators apart from overall color and exponential factors. These functions naturally show up in the results of the bare Green's functions of S_μ^R, allowing us to deduce the corresponding mixing coefficients. Furthermore, the tree-level Green's functions with the same external fields may depend on more than one external momentum q_i; this is a consequence of the absence of momentum conservation since there is no summation/integration over the position of the operators. Although this seems to complicate things it is a way to disentangle the mixing patterns. For this reason, it is convenient to calculate the Green's functions for specific choices of the external momenta. Taking into account potential IR divergences, which may appear at exceptional values of q_i, a sufficient set of choices for external momenta are: 3 choices for the two-point Green's functions with external $u(q_1)\lambda(q_2)$: $q_2 = 0$, $q_1 = 0$, $q_2 = -q_1$, as well as a single choice for each of the two three-point Green's functions with external $u(q_1)u(q_2)\lambda(q_3)$ and $\lambda(q_1)\bar{c}(q_2)c(q_3)$: ($q_2 = 0$, $q_3 = -q_1$) and ($q_2 = q_1$, $q_3 = 0$), respectively.

Each Green's function is written as a sum of several Feynman diagrams. The one-loop Feynman diagrams (one-particle irreducible (1PI)) contributing to corresponding Green's functions are shown in Figures 1, 2, 3.

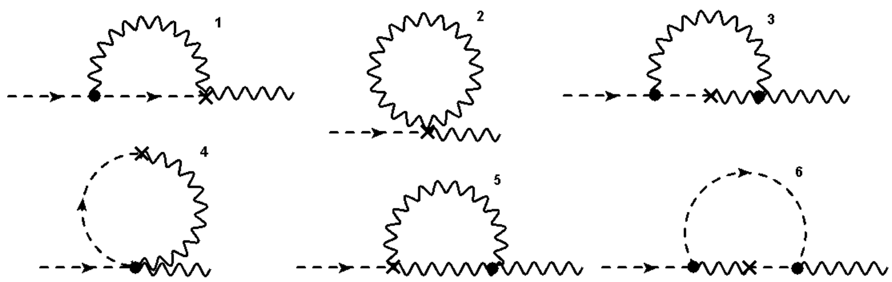

Figure 1: One-loop Feynman diagrams contributing to the two-point Green's functions $\langle u_\nu S_\mu \bar{\lambda}\rangle$ and $\langle u_\nu T_\mu \bar{\lambda}\rangle$. A wavy (dashed) line represents gluons (gluinos). A cross denotes the insertion of $S_\mu(T_\mu)$. Diagrams 2, 4 do not appear in dimensional regularization; they do however show up in the lattice formulation.

Operators	Tree Level two-point Green's function (external legs: $u_\nu(-q_1)\,\bar\lambda(q_2)$)	Tree Level three-point Green's function (external legs: $u_\nu\, u_\rho\, \bar\lambda$)	Tree Level three-point Green's function (external legs: $c\,\bar c\,\bar\lambda$)
S_μ	$-i(\not{q}_1\gamma_\nu - q_{1\nu})\gamma_\mu$	$g\,[\gamma_\nu,\gamma_\rho]\gamma_\mu/2$	0
T_μ	$i(q_{1\mu}\gamma_\nu - \not{q}_1\delta_{\mu\nu})$	$-g\,(\delta_{\mu\nu}\gamma_\rho + \delta_{\mu\rho}\gamma_\nu)$	0
$\mathscr{O}_{A1}$	$i\,q_{1\nu}\gamma_\mu/(2\alpha)$	0	$(g/2)\gamma_\mu$
$\mathscr{O}_{B1}$	$i\,\delta_{\mu\nu}\not{q}_2/2$	$-g\,(\delta_{\nu\mu}\gamma_\rho + \delta_{\rho\mu}\gamma_\nu)/2$	0
$\mathscr{O}_{B2}$	$i\,\gamma_\nu\gamma_\mu\not{q}_2/2$	$-2\,g\,\gamma_\nu\gamma_\mu\gamma_\rho$	0
$\mathscr{O}_{C1}$	$\delta_{\mu\nu}/2$	0	0
$\mathscr{O}_{C2}$	$\gamma_\nu\gamma_\mu/2$	0	0
$\mathscr{O}_{C3}$	$i\,\gamma_\nu q_{2\mu}/2$	0	0
$\mathscr{O}_{C4}$	$i\,\gamma_\nu q_{1_\mu}/2$	0	0
$\mathscr{O}_{C5}$	$i\,\gamma_\mu q_{1\nu}/2$	0	0
$\mathscr{O}_{C6}$	$i\,\gamma_\mu q_{2\nu}/2$	0	0
$\mathscr{O}_{C7}$	0	$-g\,[\gamma_\nu,\gamma_\rho]\gamma_\mu$	0
$\mathscr{O}_{C8}$	0	$-g\,(\delta_{\nu\mu}\gamma_\rho + \delta_{\rho\mu}\gamma_\nu)/2$	0
$\mathscr{O}_{C9}$	0	0	$-(g/2)\gamma_\mu$

Table 1: Two-point and three-point amputated tree-level Green's functions of S_μ and T_μ, as well as of gauge noninvariant operators which may mix with S_μ. $\langle u_\nu^{\alpha_1}(-q_1)\,\mathscr{O}_i(x)\,\bar\lambda^{\alpha_2}(q_2)\rangle$, $\langle u_\nu^{\alpha_1}(-q_1)\,u_\rho^{\alpha_2}(-q_2)\,\mathscr{O}_i(x)\,\bar\lambda^{\alpha_3}(q_3)\rangle$ and $\langle c^{\alpha_3}(q_3)\,\mathscr{O}_i(x)\,\bar c^{\alpha_2}(q_2)\bar\lambda^{\alpha_1}(q_1)\rangle$ are shown apart from overall factors of $\delta^{\alpha_1\alpha_2}e^{ix\cdot(q_1+q_2)}$, $f^{\alpha_1\alpha_2\alpha_3}e^{ix\cdot(q_1+q_2+q_3)}$ and $f^{\alpha_1\alpha_2\alpha_3}e^{ix\cdot(q_1-q_2+q_3)}$, respectively.

3. Results for Green's functions and for the mixing matrix on the lattice

Both $\overline{\text{MS}}$-renormalized and bare Green's functions have the same tensorial structures. At the one-loop order the differences between the $\overline{\text{MS}}$-renormalized and corresponding bare lattice Green's functions appear in the renormalization conditions in the $\overline{\text{MS}}$ scheme. These differences are polynomial in the external momenta and proportional to the tree-level Green's functions of the operators. Due to the fact that these differences appear in the renormalization conditions, we present them here. First of all, the resulting expression for the difference between the two-point $\overline{\text{MS}}$-renormalized and lattice bare Green's functions of S_μ for $q_2 = 0$, is:

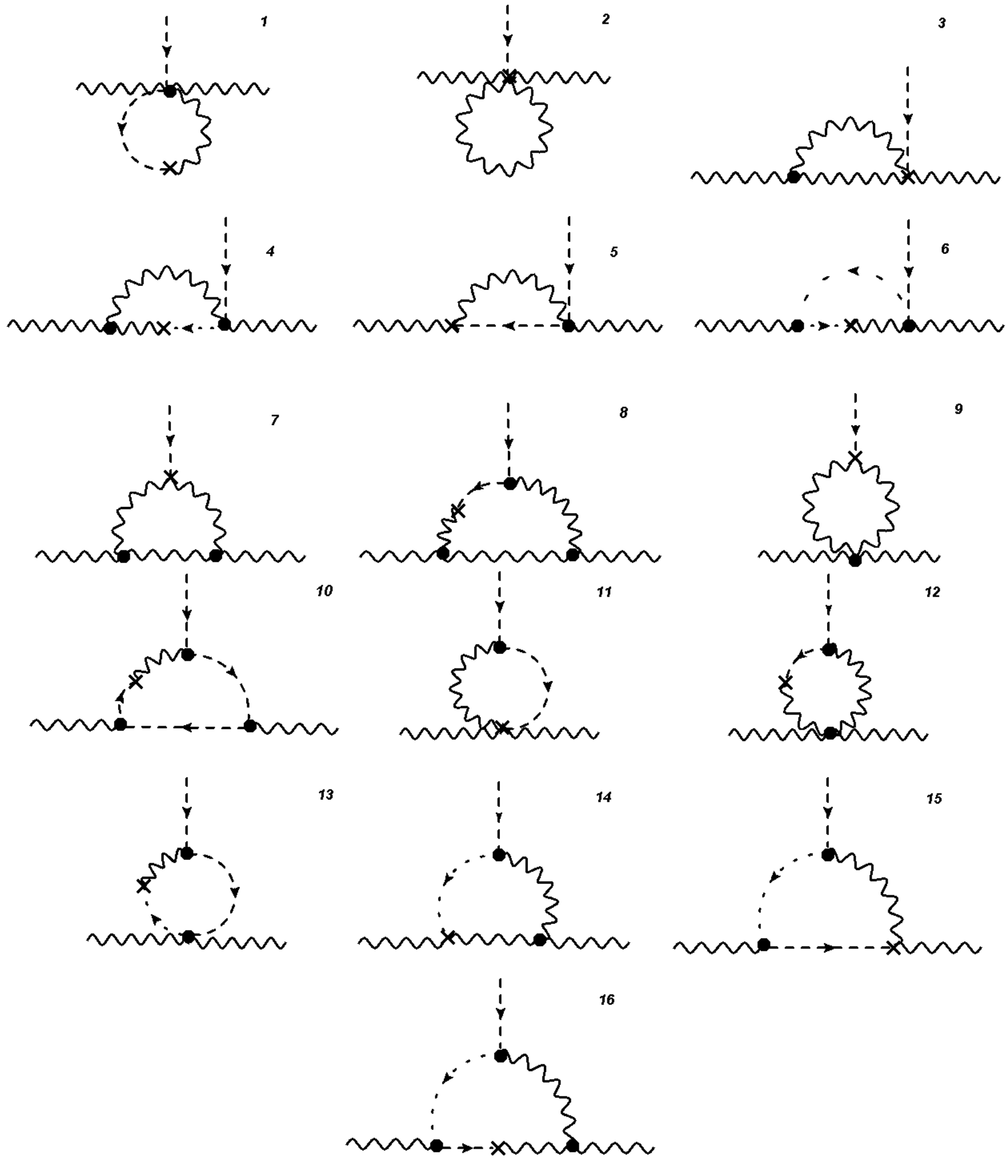

Figure 2: One-loop Feynman diagrams contributing to the three-point Green's functions $\langle u_\nu u_\rho S_\mu \bar{\lambda}\rangle$ and $\langle u_\nu u_\rho T_\mu \bar{\lambda}\rangle$. Diagrams 1, 2, 3, 5, 6, 11, and 13 do not appear in dimensional regularization but they contribute in the lattice regularization. A mirror version of diagrams 3, 4, 5, 6, 8, 10, 14, 15 and 16 must also be included.

$$\langle u_\nu^{\alpha_1}(-q_1)\, S_\mu\, \bar{\lambda}^{\alpha_2}(q_2)\big|^{\overline{\rm MS}}_{q_2=0} - \langle u_\nu^{\alpha_1}(-q_1)\, S_\mu\, \bar{\lambda}^{\alpha_2}(q_2)\rangle\big|^{LR}_{q_2=0} = i\frac{g^2}{16\pi^2}\frac{1}{2}\delta^{\alpha_1\,\alpha_2}e^{iq_1x}N_c \times \Bigg[$$
$$-5.99999\,\not{q}_1\,\delta_{\mu\,\nu} + \gamma_\nu q_{1\mu} 5.99722 + (\gamma_\nu\gamma_\mu\not{q}_1 + \gamma_\mu q_{1\nu} - 2\gamma_\nu q_{1\mu})\Bigg(\frac{39.47842}{N_c^2} - 30.57429$$
$$+5.17830\alpha - 4.55519 c_{\rm SW}^2 + 5.3771 c_{\rm SW} r + \frac{3}{2}(1-\alpha)\log\left(a^2\bar{\mu}^2\right)\Bigg)\Bigg] \qquad (3.1)$$

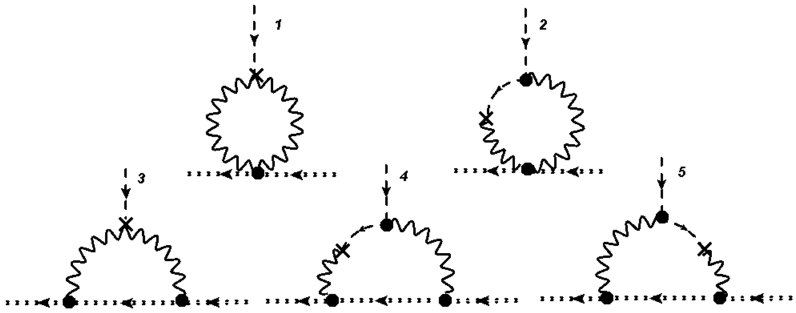

Figure 3: One-loop Feynman diagrams contributing to the three-point Green's functions $\langle c\, S_\mu\, \bar{c}\, \bar{\lambda}\rangle$ and $\langle c\, T_\mu\, \bar{c}\, \bar{\lambda}\rangle$. The "double dashed" line is the ghost field. Diagrams 1 and 2 do not appear in dimensional regularization; they do however show up in the lattice formulation.

The absence of q-independent terms means that the lower-dimensional operators, $\mathcal{O}_{C1}$ and $\mathcal{O}_{C2}$, do not mix with S_μ. The renormalization condition for the two-point Green's functions involves the renormalization factors of the external fields and of the supercurrent operator as well as the corresponding nonvanishing tree-level Green's functions along with their mixing coefficients. The condition applied to the gluino-gluon Green's function of the operator S_μ reads to one loop:

$$\begin{aligned}\langle u_\nu^R\, S_\mu^R\, \bar{\lambda}^R\rangle &= Z_\lambda^{-1/2}\, Z_u^{-1/2} \langle u_\nu^B\, S_\mu^R\, \bar{\lambda}^B\rangle \\ &= Z_\lambda^{-1/2}\, Z_u^{-1/2} Z_{SS} \langle u_\nu^B\, S_\mu^B\, \bar{\lambda}^B\rangle + Z_{ST}\langle u_\nu^B\, T_\mu{}^B\, \bar{\lambda}^B\rangle^{tree} \\ &+ Z_{SA1}\langle u_\nu^B\, \mathcal{O}_{A1}^B\, \bar{\lambda}^B\rangle^{tree} \\ &+ \sum_{i=1}^{2} Z_{SBi}\langle u_\nu^B\, \mathcal{O}_{Bi}^B\, \bar{\lambda}^B\rangle^{tree} + \sum_{i=1}^{6} Z_{SCi}\langle u_\nu^B\, \mathcal{O}_{Ci}^B\, \bar{\lambda}^B\rangle^{tree} + \mathcal{O}(g^4)\end{aligned} \tag{3.2}$$

From the choice $q_2 = 0$ we extract:

$$Z_{SS}^{LR,\overline{\rm MS}} = 1 + \frac{g^2}{16\pi^2}\left(\frac{-9.86960}{N_c} + N_c(-2.3170 + 14.49751 c_{\rm SW}^2 - 1.23662 c_{\rm SW}\, r)\right) \tag{3.3}$$

$$Z_{ST}^{LR,\overline{\rm MS}} = \frac{g^2}{16\pi^2} 3N_c \tag{3.4}$$

$$Z_{SA1}^{LR,\overline{\rm MS}} = Z_{SC1}^{LR,\overline{\rm MS}} = Z_{SC2}^{LR,\overline{\rm MS}} = Z_{SC4}^{LR,\overline{\rm MS}} = Z_{SC5}^{LR,\overline{\rm MS}} = 0 \tag{3.5}$$

An important feature of the supercurrent operator is that its renormalization is finite: this is in line with its classical conservation. The mixing with T_μ on the lattice is related to the γ-trace anomaly of the supercurrent operator and it is in agreement with older results in the literature [4]. Further, there is no mixing with $\mathcal{O}_{A1}$, $\mathcal{O}_{C4}$ and $\mathcal{O}_{C5}$ operators.

Since for the choice $q_2 = 0$ the tree-level two-point Green's functions of $\mathcal{O}_{B1}, \mathcal{O}_{B2}, \mathcal{O}_{C3}, \mathcal{O}_{C6}$

vanish, we evaluate the two-point Green's functions at $q_1 = 0$, leading to this expression:

$$\langle u_\nu^{\alpha_1}(-q_1)\, S_\mu\, \bar\lambda^{\alpha_2}(q_2)\rangle\big|_{q_1=0}^{\overline{\rm MS}} - \langle u_\nu^{\alpha_1}(-q_1)\, S_\mu\, \bar\lambda^{\alpha_2}(q_2)\rangle\big|_{q_1=0}^{LR} = i\frac{g^2 N_c}{16\pi^2}\times$$
$$\frac{1}{2}\delta^{\alpha_1\,\alpha_2} e^{iq_2 x}\times\left[\gamma_\nu\gamma_\mu \not{q}_2\left(0.80802-\frac{1}{2}\log\left(a^2\bar\mu^2\right)\right) - \not{q}_2\delta_{\mu\,\nu}\left(0.38395+\log\left(a^2\bar\mu^2\right)\right)\right] \tag{3.6}$$

From this choice ($q_1 = 0$) we determine the logarithmically divergent mixings with class B operators as well as with $\mathscr{O}_{C3}$ and $\mathscr{O}_{C6}$.

$$Z_{SB1}^{LR,\overline{\rm MS}} = \frac{g^2}{16\pi^2} N_c\left(-0.38395-\log\left(a^2\bar\mu^2\right)\right) \tag{3.7}$$
$$Z_{SB2}^{LR,\overline{\rm MS}} = \frac{g^2}{16\pi^2} N_c\left(0.80802-\frac{1}{2}\log\left(a^2\bar\mu^2\right)\right) \tag{3.8}$$
$$Z_{SC3}^{LR,\overline{\rm MS}} = Z_{SC6}^{LR,\overline{\rm MS}} = 0 \tag{3.9}$$

All the previous results are consistent with the choice $q_2 = -q_1$. The expression for the corresponding difference at $q_2 = -q_1$ is:

$$\langle u_\nu^{\alpha_1}(-q_1)\, S_\mu\, \bar\lambda^{\alpha_2}(q_2)\rangle\big|_{q_2=-q_1}^{\overline{\rm MS}} - \langle u_\nu^{\alpha_1}(-q_1)\, S_\mu\, \bar\lambda^{\alpha_2}(q_2)\rangle\big|_{q_2=-q_1}^{LR} = i\frac{g^2}{16\pi^2}\frac{1}{2}\delta^{\alpha_1\,\alpha_2} N_c\times\Bigg[$$
$$0.80802\gamma_\mu q_{1\nu} + 4.38396\gamma_\nu q_{1\mu} + \frac{1}{2}\gamma_\nu\gamma_\mu\not{q}_1\log\left(a^2\bar\mu^2\right) + \left(\gamma_\nu\gamma_\mu\not{q}_1 + \gamma_\mu q_{1\nu} - 2\gamma_\nu q_{1\mu}\right)\Bigg(\frac{39.47842}{N_c^2}$$
$$-31.38231 + 5.17830\alpha - 4.55519 c_{\rm SW}^2 + 5.37708 c_{\rm SW} r + 2\log\left(a^2\bar\mu^2\right) - \frac{3}{2}\alpha\log\left(a^2\bar\mu^2\right)\Bigg)$$
$$+\not{q}_1\delta_{\mu\,\nu}\left(-5.61605+\log\left(a^2\bar\mu^2\right)\right)\Bigg] \tag{3.10}$$

In order to determine the mixing coefficients with the operators $\mathscr{O}_{C7}$, $\mathscr{O}_{C8}$ and $\mathscr{O}_{C9}$, we also need to impose a set of renormalization conditions on three-point Green's functions. The first one involves two external gluons and one gluino:

$$\begin{aligned}\langle u_\nu^R u_\rho^R S_\mu^R\,\bar\lambda^R\rangle &= Z_\lambda^{-1/2} Z_u Z_{SS}\langle u_\nu^B u_\rho^B S_\mu^B\,\bar\lambda^B\rangle + Z_{ST}\langle u_\nu^B u_\rho^B T_\mu^B\,\bar\lambda^B\rangle^{tree}\\ &+ \sum_{i=1}^{2} Z_{SBi}\langle u_\nu^B u_\rho^B\,\mathscr{O}_{Bi}^B\,\bar\lambda^B\rangle^{tree} + \sum_{i=7}^{8} Z_{SCi}\langle u_\nu^B u_\rho^B\,\mathscr{O}_{Ci}^B\,\bar\lambda^B\rangle^{tree} + \mathscr{O}(g^4)\end{aligned} \tag{3.11}$$

The second one involves external gluino/antighost/ghost fields:

$$\begin{aligned}\langle c^R\, S_\mu^R\,\bar c^R\,\bar\lambda^R\rangle &= Z_c^{-1} Z_\lambda^{-1/2} Z_{SS}\langle c^B S_\mu^R \bar c^b\bar\lambda^B\rangle + Z_{ST}\langle c^B T_\mu^B \bar c^B\bar\lambda^B\rangle^{tree}\\ &+ Z_{SA1}\langle c^B\,\mathscr{O}_{A1}^B\,\bar c^B\bar\lambda^B\rangle^{tree} + Z_{SC9}\langle c^B\,\mathscr{O}_{C9}^B\,\bar c^B\bar\lambda^B\rangle^{tree} + \mathscr{O}(g^4)\end{aligned} \tag{3.12}$$

The result for the difference of the first three-point Green's function is:

$$\langle u_\nu^{\alpha_1}(-q_1)u_\rho^{\alpha_2}(-q_2)\,S_\mu\,\bar{\lambda}^{\alpha_3}(q_3)\rangle\big|^{\overline{\rm MS}}_{q_2=0,q_3=-q_1} - \langle u_\nu^{\alpha_1}(-q_1)u_\rho^{\alpha_2}(-q_2)\,S_\mu\,\bar{\lambda}^{\alpha_3}(q_3)\rangle\big|^{LR}_{q_2=0,q_3=-q_1} =$$
$$\frac{g^3 N_c}{16\pi^2} f^{\alpha_1\alpha_2\alpha_3}\Bigg[\left(\delta_{\nu\rho}\gamma_\mu - \gamma_\nu\gamma_\rho\gamma_\mu\right)\Bigg(\frac{19.73920}{N_c^2} - 12.48660 + 3.28231\alpha$$
$$-2.27761c_{\rm SW}^2 + 2.68854c_{\rm SW}r + \frac{1-2\alpha}{2}\log\left(a^2\bar{\mu}^2\right)\Bigg)\Bigg] \tag{3.13}$$

With this result we conclude that there are no mixings with $\mathcal{O}_{C7}$ ad $\mathcal{O}_{C8}$. The lattice Green's functions containing gluino-ghost-antighost external fields are identical to the continuum ones at one loop order; thus, there is also no mixing of S_μ with $\mathcal{O}_{C9}$.

4. Summary – Conclusion

To summarize the main points, we have seen that the supercurrent operator suffers from mixing with both gauge invariant and noninvariant operators. We have calculated the first two rows of the mixing matrix; these correspond to the two gauge invariant operators which are involved. The results are important in order to have a thorough picture of the mixing pattern when gauge noninvariant and off-shell Green's functions are employed. The novelty in our one-loop results is that we calculate the complete mixing patterns of the supercurrent operator perturbatively. More precisely, we use gauge-variant off-shell Green's functions; we obtain analytic expressions for the renormalization factors and mixing coefficients, where the number of colors N_c, the coupling constant g, the gauge parameter α, the clover/Wilson parameters $c_{\rm SW}/r$ (on the lattice) are left unspecified.

In our ongoing investigations, the renormalization of S_μ is also deduced by calculating exclusively gauge-invariant Green's functions; in this case the mixing of gauge-noninvariant operators is irrelevant and there is no need to fix the gauge or introduce ghost fields, leaving only the effective 2×2 space of S_μ and T_μ mixings. Therefore, this Gauge-Invariant Scheme (GIRS) is more accessible via non-perturbative calculations. For detailed information on non-perturbative results, see the proceedings by I. Soler in this conference [5].

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