PROCEEDINGS OF SCIENCE

Progress Toward Stealth Dark Matter Scattering

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On behalf of the Lattice Strong Dynamics (LSD) collaboration, we present the first stages of our SU(4) Gauge theory Stealth Dark Matter baryon scattering research program. We describe our Laplacian Heaviside (LapH) and stochastic LapH implementation in a forthcoming publicly available N_c agnostic sLapH code base.

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1. Introduction

Dark matter makes up ∼85% of the mass of the Universe. At the same time, we know from the Standard Model that nearly 99% of the mass of regular matter comes from the dynamics of the strong nuclear force, QCD. It is possible that the mass of dark matter could come from a new SU(N) gauge interaction which confines a new type of fermion analogous to a quark. In this work, we are interested in Stealth Dark Matter (SDM), an SU(4) gauge theory ($N_c = 4$) where the dark matter would be the stable, ground state spin-0 baryon. In recent years, this dark matter theory has become an active area of research. With the confined fermions we call *stealth quarks* being charged under the Standard Model electroweak theory, SDM could be detected on Earth through electromagnetic polarizabilty [1, 2]. Lattice calculations have also shown that SDM could have a first order phase transition which would yield stochastic gravitational waves during its early-Universe confinement transition [3–5].

In continuation of the SDM research program, we are interested in studying SDM self-interactions. Astrophysical constraints, such as those from the Bullet Cluster, place upper bounds on dark matter's self-interaction cross section per unit mass to be on the same order of magnitude as that of the neutron [6, 7]. In order to constrain SDM, we are performing lattice calculations of SU(4) gauge theory to study the self-interaction of SDM baryons. Since the SDM baryons are bosons rather than fermions, we cannot assume the Stealth Baryons would behave like QCD baryons, and thus we must perform lattice calculations to learn about their self-interactions.

Due to the number of colors being four instead of three as in QCD, the SDM baryon scattering problem nominally contains 576 Wick contraction diagrams, which is a factor of

$$\frac{4! \times 4!}{3! \times 3!} = 16 \tag{1}$$

more than the equivalent baryon scattering problem in QCD. To make this problem tractable, we apply Laplacian Heaviside smearing (LapH) [8, 9], and its stochastic version, sLapH [10] to our hadron operators, as is done in cutting edge QCD scattering calculations [11, 12].

In this report, we describe our implementation of LapH and sLapH as well as model averaging [13] in our preparation of the full SDM scattering problem.

In addition to the motivation provided by Stealth Dark Matter, we seek to provide publicly available software for implementing sLapH for arbitrary N_c.

2. LapH and sLapH for scattering

The Laplacian Heaviside method (LapH) [8, 9] provides a good approximation to the all-to-all propagator by smearing quark fields with the low modes of the gauge covariant lattice Laplacian. The theoretical advantage is that while preserving all of the lattice symmetries, corrections to the approximation of the all-to-all propagator lie in the high energy modes in which we are not interested. A computational advantage is that it requires a fixed number of inversions to compute the eigenvectors and perambulators, which are much lower rank than the full propagator. These LapH building blocks can then be used and reused to construct an arbitrarily large number of operators that may be used in a variational analysis.

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2.1 Mathematical formalism

The gauge covariant Laplacian is given by

$$\Delta^{ab}_{xy}(t) = \sum_{k=1}^{N_d} \left(U^{ab}_k(x,t)\delta_{y,x+\hat{k}} + U^{\dagger\,ab}_k(y,t)\delta_{y,x-\hat{k}} \right) - 2\delta_{x,y}\delta^{ab}, \tag{2}$$

where N_d is the number of spatial dimensions. One then solves for N_{vec} of the total $L_x^3 \times N_c$ eigenvectors,

$$\Delta^{ab}_{xy}(t)\ V_{x,a|i}(t) = \lambda_i(t)\ V_{y,b|i}(t). \tag{3}$$

Considering only the N_{vec} eigenvectors with the smallest eigenvalues, λ_i, the quark fields are then smeared to their low modes according to

$$\psi^a_\alpha(x,t) \to S^{ab}_{xy}(t)\ \psi^b_\beta(y,t), \tag{4}$$

with S defined as

$$S^{ab}_{xy} = \sum_{i=1}^{N_{\text{vec}}} V_{x,a|i}\ (V^\dagger)_{i|y,b}, \tag{5}$$

at each time slice. Note that if N_{vec} is maximal, this smearing matrix is just the identity operator. However, for a typical analysis, $N_{\text{vec}} \ll L_x^3 \times N_c$, and can be optimized for a particular study. Suppressing the color indices, the propagator D^{-1} is then smeared as

$$D^{-1\ \alpha\beta}(x,t|y,t_0) = \psi^\alpha(x,t)\ \bar{\psi}^\beta(y,t_0) \tag{6}$$

$$\to S_{xx'}(t)\ \psi^\alpha(x',t)\ \bar{\psi}^\beta(y',t_0)\ S^\dagger_{y'y}(t_0) \tag{7}$$

$$= \sum_i \sum_j V_{x|i}(t)\ \underbrace{(V^\dagger)_{i|x'}(t)\ \psi^\alpha(x',t)\ \bar{\psi}^\beta(y',t_0)\ V_{y'|j}(t_0)}_{\equiv \tau^{\alpha\beta}_{ij}(t,t_0)} (V^\dagger)_{j|y,b}(t_0), \tag{8}$$

$$= \sum_i \sum_j V_{x|i}(t)\ \tau^{\alpha\beta}_{ij}(t,t_0)\ (V^\dagger)_{j|y}(t_0), \tag{9}$$

where the perambulator τ is computed by summing over spatial positions and color components, completing the transformation of the propagator from a position basis to a low mode eigenbasis of the gauge covariant lattice Laplacian. It is therefore a much lower dimensional representation of the full propagator. An example pion two-point correlation function would be given by

$$C_\pi(t,t_0;) = -\operatorname{Tr}\left[D(t_0,t)\,\gamma_5\,D(t,t_0)\,\gamma_5\right] \tag{10}$$

$$\to -\operatorname{Tr}\left[\left(V_{i'}\ \tau_{i'i}(t_0,t)\ V^\dagger_i\right)\gamma_5\left(V_j\ \tau_{jj'}(t,t_0)\ V^\dagger_{j'}\right)\gamma_5\right]. \tag{11}$$

Once the eigenvectors V and perambulators τ are computed, one can construct any correlation function with a plethora of operator constructions by replacing the propagator D^{-1} as above, *without any addition inversions*.

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Stochastic LapH (sLapH) [10] is an alternative formulation where the LapH eigenbasis is approximated by a small number of stochastic noise vectors, $\rho^{(n)}$. If the noise vectors satisfy $\langle \rho_i \rangle = 0$ and $\langle \rho_i \rho_j \rangle = \delta_{ij}$, $i, j = 1, \ldots N_{\text{vec}}$, they provide an unbiased estimator of the identity operator in the LapH subspace. To construct sLapH vectors, one fixes the number of eigenvectors N_{vec} and generates $N_{\text{noise}} < N_{\text{vec}}$ noise vectors, $\rho^{(n)}$ of length N_{vec}. Then, to be used in place of $V_{x,a|i}$, a new set of sLapH vectors, $\tilde{V}_{x,a|n}$ are computed as

$$V_i \rightarrow \tilde{V}_n \equiv \sum_{i=1}^{N_{\text{vec}}} V_i \, \rho_i^{(n)}. \tag{12}$$

With $N_{\text{noise}} < N_{\text{vec}}$, one can produce a lower rank approximation to the smearing matrix, S. This greatly reduces the summations required to compute correlation functions as in Equation 11. It is especially useful to reduce the rank of S when computing a correlation function containing a large number of contractions. In the SU(4) baryon-baryon scattering problem, the creation and annihilation operators each involve $N_q = 8$ quark fields. In general, the number of contractions to compute a correlation function, N_{cont}, scales as

$$N_{\text{cont}} \sim N_{\text{vec}}^{2N_q}. \tag{13}$$

Hence, it is optimal for us to reduce the number of vectors, as can be achieved with sLapH, even at the cost of the extra statistical noise.

2.2 sLapH and excited state contamination

Regardless of whether one is using LapH or sLapH, the choice of N_{vec} determines how well the operator overlaps with the low lying spectrum that one is interested in studying. This can be characterized by looking at the effective mass for different choices of N_{vec}. Figure 1 shows this effect for an SU(4) pseudoscalar correlation function. In left-hand plot one can see that the same effective mass plateau is reached for every value of N_{vec}, but the plateau is reached sooner for smaller N_{vec} because only the lowest modes are included in the quark operators, and there is less contamination from excited states contained in the high modes. Another useful metric for choosing the number of eigenvectors is by looking at the effective mass value at a particular time slice as a function of N_{vec}. In the right-hand plot, one can see that $N_{\text{vec}} = 16$ could be an optimal choice for the number of eigenvectors because there is a minimal amount of excited state contamination, i.e. $m_{\text{eff}}^{(16)} < m_{\text{eff}}^{(12,24)}$. At the same time, although the effective mass values are similar, $N_{\text{vec}} = 16$ is probably a better choice than $N_{\text{vec}} = 24, 32$ because a smaller N_{vec} means less contractions have to be computed when using LapH.

Similarly, choosing the optimal number of stochastic vectors to use involves optimizing for the reduction of noise, excited state contamination, and number of vectors. Figure 2 shows the effect of using different choices of N_{noise}. As expected, the effective mass values at each timeslice vary for different choices of N_{vec} when using LapH (circle markers). However, using sLapH (star markers), where $N_{\text{vec}} = 48$ is fixed, one can see that their effective mass values have the same central value as the LapH $N_{\text{vec}} = 48$ points (red circles). Here, one may argue that $N_{\text{noise}} = 16$ (blue stars) is the optimal number of noise vectors, because it yields a modest increase in noise from the $N_{\text{vec}} = 48$ points, but with a large reduction in the number of vectors needed.

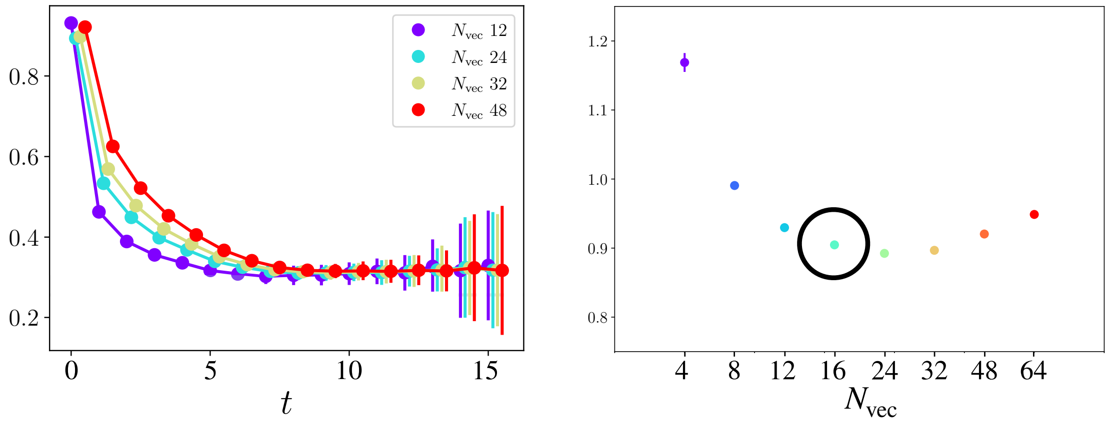

Figure 1: Left: hyperbolic cosine form of effective mass for different choices of N_{vec} for a pseudoscalar correlation function on an $L_x = 16$ SU(4) gauge field ensemble. Right: effective mass value at fixed $t = 1$ as a function of N_{vec}.

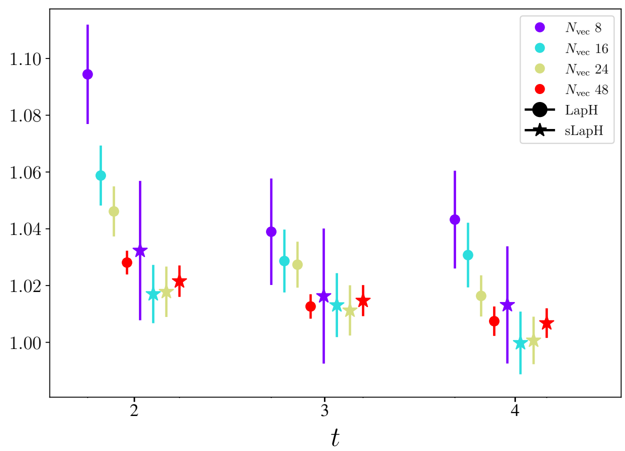

Figure 2: Example effective masses values for different choices of N_{vec} and N_{noise} using LapH and sLapH.

3. Model Averaging

In a scattering analysis, one is faced with a high dimensional parameter space over which fits are performed. Unfortunately, much of the tuning required to complete an analysis must be done by hand. But to automate and remove human bias from the procedure of choosing a fitting model, we use model averaging [13], which is sumarized as follows. One can complete a set of fits using different models, M, such as one-, two-, and three-state fits, and fits with different fitting windows, t_{start}, t_{stop}. The resulting parameters extracted from each fit will vary, but the statistical and systematic uncertainty resulting in these fits can be combined into a model averaged value, θ. This model averaged value is computed by summing over the the model parameters θ_M, weighted

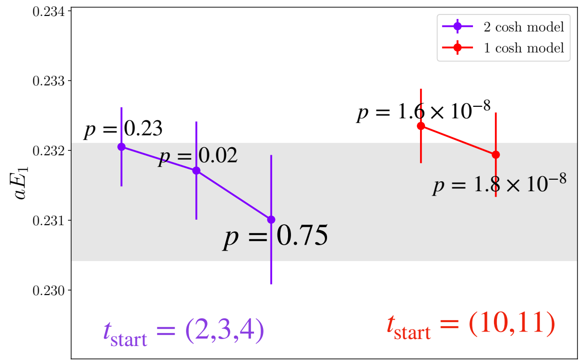

Figure 3: Model average of pseudoscalar ground state mass for an example $L = 32$ SU(4) ensemble.

by the model probability. That is,

$$\theta = \sum_M \theta_M p(M|D), \tag{14}$$

where the probability of the model M given the data D is given by

$$-2 \log p(M|D) = \chi^2 + 2k, \tag{15}$$

where the penalty $k = N_p + N_t - (t_{\text{stop}} - t_{\text{start}})$ is the total number of parameters in the fit, including the number parameters in the model, N_p, and the number of timeslices of data not included in the fit window.

Figure 3 shows the model average of the ground state energy in lattice units aE_1 of an SU(4) pseudoscalar on an $L_x = 32$ ensemble. The model average value and uncertainty is shown as the gray band. One can see the different models contributing to the model average, with the respective probabilities annotated in the plot. One can also note that the fits to one state, the “1 cosh model" fits, have vanishing probabilities because the small fit window required to reduce excited state contamination penalizes the likelihood of these fits.

We have learned that we cannot simply use model averaging blindly, but must be strict about data and fit quality. For example, we must reject unconstrained fits, which can skew the model average. Also, since the model average can be sensitive to the fit window through the penalty k, we must ensure that extremely noisy data at large times are cut. Using the model averaging procedure described above, we do this data and fit quality checking by hand, but as we proceed, we plan to adopt the more formal information criteria recently explored in [14].

4. Conclusion

We are interested in SU(4) baryon-baryon scattering for the Stealth Dark Matter research program, and LapH and sLapH are essential to make the scattering problem tractable. LapH allows for the construction of large a variational basis of operators overlapping with low lying energy states.

sLapH allows us to reduce the number of contractions significantly, at a modest cost of statistical noise. We apply model averaging to complete our data analysis procedure. As we proceed with the Stealth Dark Matter program, we continue to work to develop a publicly available N_c agnostic sLapH code base built on top of existing the QDP++ and Chroma software.

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