

Exploring conformality in lattice $\mathcal{N} = 4$ supersymmetric Yang–Mills

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Maximally supersymmetric Yang–Mills theory ($\mathcal{N} = 4$ SYM) is conformal for any value of the coupling. Lattice regularization breaks conformality through the introduction of a non-zero lattice spacing and a finite lattice volume. This proceedings presents ongoing computations of conformal scaling dimensions in lattice $\mathcal{N} = 4$ SYM, based on a lattice formulation that exactly preserves a supersymmetry sub-algebra at non-zero lattice spacing. The main targets are the non-trivial anomalous dimension of the Konishi operator, as well as a mass anomalous dimension extracted from the eigenvalue mode number of the fermion operator. The latter is expected to vanish in the conformal continuum theory, providing insight into the interplay of lattice discretization and conformality.

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1. Introduction

Maximally supersymmetric Yang–Mills theory ($\mathcal{N} = 4$ SYM) is widely studied in theoretical physics. It is arguably the simplest non-trivial quantum field theory in four dimensions, especially in the large- N planar limit of its $SU(N)$ gauge group, thanks to its many symmetries. It plays a key role in holographic duality, provided early insight into S-duality, and continues to inform modern analyses of scattering amplitudes.

Lattice regularization of $\mathcal{N} = 4$ SYM is an active area that provides both a non-perturbative definition of the theory as well as a way to numerically predict its behavior from first principles, even at strong coupling and away from the planar limit. See Ref. [1] for a recent review. Lattice formulations of $\mathcal{N} = 4$ SYM necessarily break symmetries of the continuum theory, and most of the recent activity has relied on an approach that preserves a closed supersymmetry subalgebra at non-zero lattice spacing $a > 0$ [1, 2]. Despite preserving only a single one of the 16 supersymmetries, this single exact supersymmetry significantly simplifies the lattice theory, making it possible to ensure the recovery of the other 15 in the $a \rightarrow 0$ continuum limit.

This proceedings focuses on a different challenge to lattice studies of $\mathcal{N} = 4$ SYM. Although the continuum theory is conformal for every value of the 't Hooft coupling $\lambda = Ng_{\text{YM}}^2$, numerical lattice field theory calculations require both a non-zero lattice spacing corresponding to a UV cutoff $\sim 1/a$, as well as a finite lattice volume $(L \cdot a)^4$ introducing an IR cutoff. Both of these explicitly break conformal scale invariance, complicating lattice analyses of the spectrum of λ -dependent conformal scaling dimensions, key information about the theory.

After a brief review of lattice $\mathcal{N} = 4$ SYM in the next section, I will summarize recent and ongoing numerical investigations of two scaling dimensions of particular interest. First, Section 3 presents results from Ref. [3] for a mass anomalous dimension obtained by analyzing the eigenmode number of the fermion operator. This anomalous dimension is expected to vanish, $\gamma_* = 0$, for all values of λ , which allows numerical results to reveal the effects of breaking conformality (and supersymmetry). Second, in Section 4 I show preliminary results from ongoing lattice studies of the non-trivial Konishi scaling dimension $\Delta_K(\lambda)$. Section 5 concludes with a brief discussion of next steps for this work.

2. Lattice $\mathcal{N} = 4$ SYM in a nutshell

The lattice formulation used in this work is based on so-called topological twisting, which organizes the 16 supercharges of the theory into integer-spin representations of the twisted rotation group $SO(4)_{\text{tw}} \equiv \text{diag} [SO(4)_{\text{euc}} \otimes SO(4)_R]$, where $SO(4)_{\text{euc}}$ is the Lorentz group Wick-rotated to euclidean space-time and $SO(4)_R$ is a subgroup of the $SU(4)$ R-symmetry. The twisted-scalar supersymmetry Q is nilpotent, preserving the subalgebra $\{Q, Q\} = 0$ even at non-zero lattice spacing where the other 15 supersymmetries are broken by the lattice discretization of space-time.

The lattice theory is formulated on the A_4^* lattice (which distinguishes λ_{lat} from λ [4]), with an

action that features the same \mathcal{Q} -exact and \mathcal{Q} -closed terms as the twisted continuum theory:

$$S = \frac{N}{4\lambda_{\text{lat}}} \sum_n \text{Tr} \left[\mathcal{Q} \left(\chi_{ab}(n) \mathcal{D}_a^{(+)} \mathcal{U}_b(n) + \eta(n) \overline{\mathcal{D}}_a^{(-)} \mathcal{U}_a(n) - \frac{1}{2} \eta(n) d(n) \right) \right] - \frac{N}{16\lambda_{\text{lat}}} \sum_n \text{Tr} \left[\varepsilon_{abcde} \chi_{de}(n + \widehat{\mu}_a + \widehat{\mu}_b + \widehat{\mu}_c) \overline{\mathcal{D}}_c^{(-)} \chi_{ab}(n) \right]. \quad (1)$$

Here η , ψ_a and $\chi_{ab} = -\chi_{ba}$ are 1-, 5- and 10-component fermions respectively associated with the lattice sites, links and oriented plaquettes. The five-component complexified gauge links \mathcal{U}_a and $\overline{\mathcal{U}}_a$ appearing in the finite-difference operators $\mathcal{D}_a^{(+)}$ and $\overline{\mathcal{D}}_a^{(-)}$ contain both the gauge and scalar fields.

The complexified gauge links imply $U(N) = SU(N) \times U(1)$ gauge invariance. In order to carry out numerical calculations, flat directions in both the $SU(N)$ and $U(1)$ sectors need to be regulated. We achieve this by adding two deformations to Eq. 1, following Ref. [5].¹ First, the double-trace scalar potential

$$S_{\text{scalar}} = \frac{N}{4\lambda_{\text{lat}}} \mu^2 \sum_n \sum_a \left(\frac{1}{N} \text{Tr} \left[\mathcal{U}_a(n) \overline{\mathcal{U}}_a(n) \right] - 1 \right)^2 \quad (2)$$

regulates the $SU(N)$ flat directions while softly breaking the \mathcal{Q} supersymmetry. The second deformation is \mathcal{Q} -exact, and replaces the term

$$\mathcal{Q} \left(\eta(n) \overline{\mathcal{D}}_a^{(-)} \mathcal{U}_a(n) \right) \longrightarrow \mathcal{Q} \left(\eta(n) \left[\overline{\mathcal{D}}_a^{(-)} \mathcal{U}_a(n) + G \sum_{a \neq b} (\det \mathcal{P}_{ab}(n) - 1) \mathbb{I}_N \right] \right) \quad (3)$$

in Eq. 1. This picks out the $U(1)$ sector through the determinant of the plaquette oriented in the a – b plane, $\mathcal{P}_{ab}(n)$, which is an $N \times N$ matrix at each lattice site n . The tunable parameter μ needs to be sent to zero in the continuum limit in order to restore supersymmetry, where the $U(1)$ deformation involving G can be expected to decouple.

Using parallel software that we make publicly available² and presented in Ref. [7], we have used the rational hybrid Monte Carlo (RHMC) algorithm to generate many ensembles of field configurations. We have also publicly implemented the stochastic estimation of the eigenmode number discussed in the next section, for which we rescale some of the fermion field components in order to put the fermion operator into its most symmetric form. See Ref. [3] for further details about this rescaling.

3. Mass anomalous dimension from fermion eigenmode number

Building on the early work of Ref. [8], the recent study Ref. [3] uses efficient modern techniques to evaluate the eigenmode number of the fermion operator, which is related to a mass anomalous dimension γ_* [9–12]. Starting from the spectral density of the squared massless fermion operator $D^\dagger D$,

$$\rho(\omega^2) = \frac{1}{V} \sum_k \langle \delta(\omega^2 - \lambda_k^2) \rangle \quad (4)$$

¹There is ongoing exploration of alternative lattice actions, including Ref. [6].

²github.com/daschaich/susy

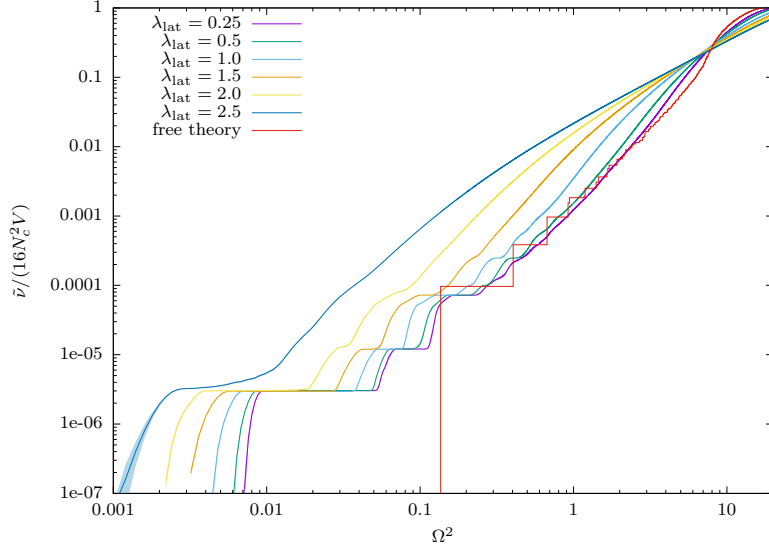


Figure 1: From Ref. [3], the normalized eigenmode number for U(2) 16^4 lattices, including both the free theory (red) and the interacting theory with $0.25 \leq \lambda_{\text{lat}} \leq 2.5$, plotted vs. the energy scale Ω^2 on log–log axes.

(where the eigenvalues λ_k of D should not be confused with the 't Hooft coupling λ), the eigenmode number $\nu(\Omega^2)$ is the number of eigenvalues λ_k^2 of the non-negative operator $D^\dagger D$ that are smaller than Ω^2 :

$$\nu(\Omega^2) = \int_0^{\Omega^2} \rho(\omega^2) d\omega^2 \propto (\Omega^2)^{2/(1+\gamma_*)}. \quad (5)$$

Note that the antisymmetry of the lattice $\mathcal{N} = 4$ SYM fermion operator D ,

$$\Psi^T D \Psi = \chi_{ab} \mathcal{D}_{[a}^{(+)} \psi_{b]} + \eta \mathcal{D}_a^{\dagger(-)} \psi_a + \frac{1}{2} \varepsilon_{abcde} \chi_{ab} \mathcal{D}_c^{\dagger(-)} \chi_{de},$$

ensures that the eigenvalues of D come in $\pm\lambda_k$ pairs.

In order to efficiently evaluate the spectral density and eigenmode number over the full spectral range of each lattice ensemble, we adopt the method of stochastically estimating the Chebyshev expansion [13]

$$\rho_r(x) \approx \sum_{n=0}^P \frac{2 - \delta_{n0}}{\pi \sqrt{1-x^2}} c_n T_n(x). \quad (6)$$

For lattice ensembles with gauge groups U(2), U(3), U(4) and lattice volumes up to 16^4 , we retain $5000 \leq P \leq 10000$ terms in the Chebyshev expansion. In Ref. [3] we checked selected results against both direct iterative evaluation of the low-lying eigenvalues as well as stochastic projection computations. Both of those methods are much more computationally expensive than the stochastic Chebyshev expansion.

Figure 1, from Ref. [3], presents results for the normalized eigenmode number $\nu(\Omega^2)$ from 16^4 lattices with gauge group U(2) and a range of 't Hooft couplings. In particular, we evaluated the eigenmode number for the free theory, shown by the red line in this figure, which shows the

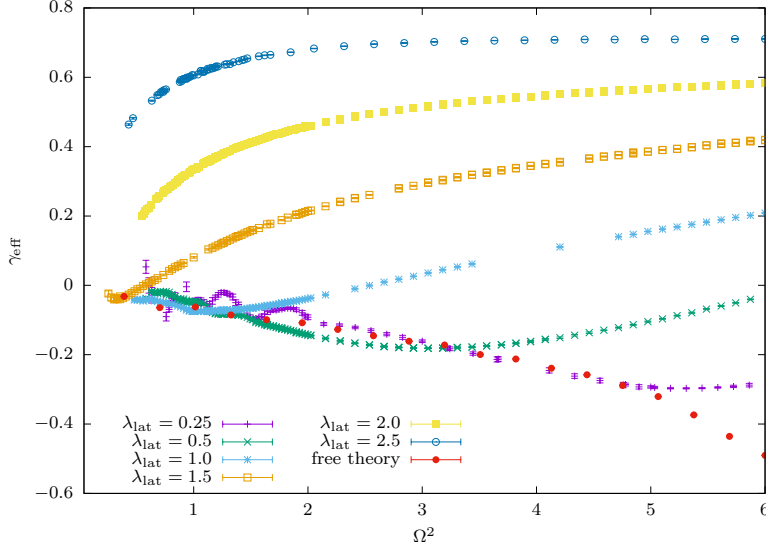


Figure 2: From Ref. [3], the effective anomalous dimension γ_{eff} obtained from the $U(2)$ 16^4 results shown in Fig. 1, again plotted vs. the energy scale Ω^2 (on linear axes).

effects of the finite, discrete lattice space-time. The smoother curves come from ensembles of field configurations generated with $0.25 \leq \lambda_{\text{lat}} \leq 2.5$.

According to Eq. 5, the slope of $\nu(\Omega^2)$ on the log–log axes of Fig. 1 is directly related to the mass anomalous dimension γ_* . Due to the explicit breaking of conformality, we are only able to access a scale-dependent effective anomalous dimension $\gamma_{\text{eff}}(\Omega^2)$. We extract this effective anomalous dimension by fitting our data to Eq. 5 in windows $[\Omega^2, \Omega^2 + \ell]$ with $\ell \in [0.03, 1]$ fixed for each ensemble. The $U(2)$ 16^4 results for $\gamma_{\text{eff}}(\Omega^2)$ from these fits are collected in Fig. 2. From this figure we can see that the true $\gamma_* = 0$ is correctly being recovered in the IR, $\Omega^2 \ll 1$. We can also see that even the free theory on a 16^4 lattice suffers from significant lattice artifacts as Ω^2 increases. Finally, these artifacts clearly increase rapidly as the lattice ’t Hooft coupling becomes stronger, quantifying the challenge of successfully recovering the superconformal continuum field theory.

4. Konishi and SUGRA scaling dimensions

While the expectation $\gamma_* = 0$ for the mass anomalous dimension discussed above makes it a useful tool with which to explore the breaking of supersymmetry and conformality in lattice studies of $\mathcal{N} = 4$ SYM, it’s also important to pursue non-trivial scaling dimensions using lattice field theory. A compelling target is the scaling dimension of the ‘Konishi’ operator

$$\mathcal{O}_K(x) = \sum_I \text{Tr} [\Phi^I(x)\Phi^I(x)], \quad (7)$$

where Φ^I are the six real scalar fields of the continuum theory. This is the simplest conformal primary operator of $\mathcal{N} = 4$ SYM, with non-trivial scaling dimension $\Delta_K(\lambda) = 2 + \gamma_K(\lambda)$ that has been predicted with weak-coupling perturbation theory [14–16], from holography at strong

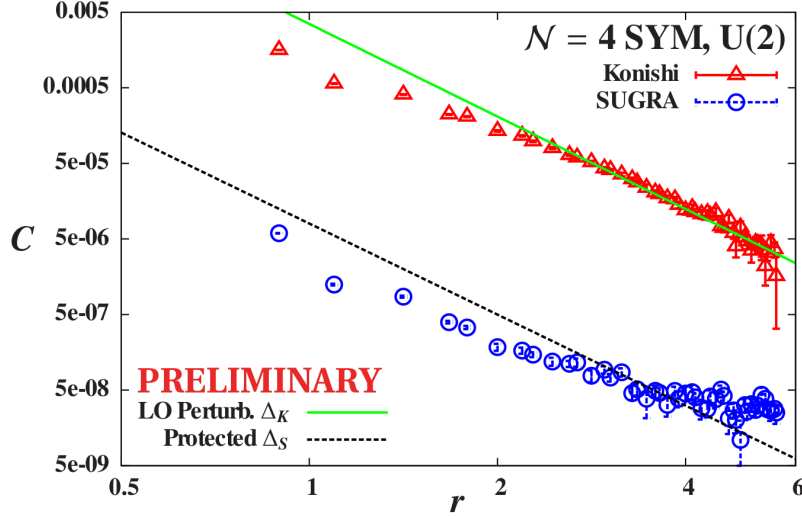


Figure 3: The Konishi and SUGRA two-point correlators for a $U(2)$ 16^4 ensemble with $\lambda_{\text{lat}} = 0.5$, plotted vs. the distance r on log–log axes. The solid and dotted lines are not fits, but expectations based on leading-order perturbation theory and the exact value $\Delta_S = 2$, respectively.

couplings $\lambda \rightarrow \infty$ with $\lambda \ll N$ [17], and for all couplings in the $N = \infty$ planar limit [18]. Thanks to S-duality, which predicts an invariant spectrum of $\mathcal{N} = 4$ SYM anomalous dimensions under the interchange $\frac{4\pi N}{\lambda} \longleftrightarrow \frac{\lambda}{4\pi N}$, the perturbative results are also relevant in the alternate strong-coupling regime $\lambda \gg N$ [19]. In addition, the superconformal bootstrap program has been applied to analyze the Konishi anomalous dimension, with initial bounds on the maximum value γ_K could reach across all λ [20, 21] recently being generalized to λ -dependent constraints [22].

In order to analyze the Konishi scaling dimension from our lattice calculations, we first need to isolate scalar fields $\varphi_a(n)$ that have been twisted into the complexified gauge links. We do so using a polar decomposition,

$$\mathcal{U}_a(n) = e^{\varphi_a(n)} U_a(n). \quad (8)$$

Because the Konishi operator is a scalar under the twisted rotation group, it picks up a non-zero vacuum expectation value (vev) that needs to be subtracted,

$$O_K^{\text{lat}}(n) = \sum_a \text{Tr} [\varphi_a(n) \varphi_a(n)] - \text{vev}. \quad (9)$$

Figure 3 shows preliminary results for the Konishi correlator

$$C_K(r) = O_K^{\text{lat}}(n+r) O_K^{\text{lat}}(n) \propto r^{-2\Delta_K}, \quad (10)$$

along with the corresponding correlator of the ‘SUGRA’ operator

$$O_S^{IJ}(x) = \text{Tr} [X^{\{I}(x) X^{J\}}(x)] \quad [O_S^{\text{lat}}]^{ab}(n) = \text{Tr} [X^{\{a}(n) X^{b\}}(n)]. \quad (11)$$

In the continuum theory, O_S transforms in the symmetric traceless $20'$ representation of the $SO(6)$ R-symmetry, and it has no anomalous dimension: $\Delta_S = 2$ for all 't Hooft couplings. Considering gauge group $U(2)$ on a 16^4 lattice volume with $\lambda_{\text{lat}} = 0.5$, the figure shows a range of r in which

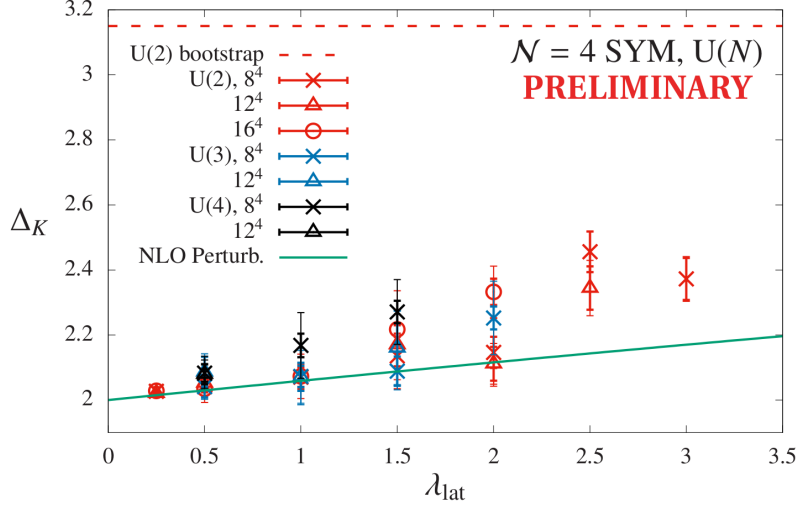


Figure 4: Preliminary results for the Konishi scaling dimension $\Delta_K(\lambda_{\text{lat}})$ from MCRG analyses for gauge groups up to $U(4)$ (different colors) and lattice volumes up to 16^4 (different symbols). The thin and thick error bars on each point correspond to statistical and total uncertainties, respectively. The dashed line at the top of the plot is an upper bound on $\Delta_K(\lambda_{\text{lat}})$ from conformal bootstrap analyses.

the Konishi correlator shows a clear power law — a straight line on these log–log axes, with slope in agreement with the leading-order perturbative prediction for $\Delta_K(\lambda)$. The SUGRA correlator is noisier, due to being $\sim 100\times$ smaller in magnitude, but behaves consistently with its protected scaling dimension $\Delta_S = 2$.

Work remains in progress to quantify the systematic uncertainties that come into play when extracting Δ_K from correlators of this sort, which include sensitivity to the power-law fit range and the subtraction of the vev in Eq. 9, along with finite-volume and discretization artifacts. So far we have had more success determining Δ_K through Monte Carlo renormalization group (MCRG) stability matrix analyses [23]. Treating the lattice system as a formally infinite sum of operators, $\sum_i c_i \mathcal{O}_i$, essentially all of which are irrelevant in the RG sense, an RG blocking R_b step defines a new system

$$H^{(n)} = R_b H^{(n-1)} = \sum_i c_i^{(n)} \mathcal{O}_i^{(n)} \quad (12)$$

based on n -times blocked operators $\mathcal{O}_i^{(n)}$ with RG-flowed coefficients $c_i^{(n)}$.

For a conformal system like $\mathcal{N} = 4$ SYM at any value of the 't Hooft coupling λ , the conformal fixed point is characterized by $H^* = R_b H^*$ with couplings c_i^* . Linearizing around this fixed point,

$$c_i^{(n)} - c_i^* = \sum_k \left. \frac{\partial c_i^{(n)}}{\partial c_k^{(n-1)}} \right|_{H^*} (c_k^{(n-1)} - c_k^*) \equiv \sum_k T_{ik}^* (c_k^{(n-1)} - c_k^*), \quad (13)$$

defines the stability matrix T_{ik}^* . In lattice calculations, the matrix elements of T_{ik}^* come from correlators of the operators \mathcal{O}_k and \mathcal{O}_k evaluated after different numbers of RG blocking steps, while its eigenvalues provide the corresponding scaling dimensions.

The lattice $\mathcal{N} = 4$ SYM RG blocking transformation that we use was introduced by Ref. [24] and produces the results for the Konishi scaling dimension $\Delta_K(\lambda_{\text{lat}})$ shown in Fig. 4 for different

gauge groups $U(2)$, $U(3)$ and $U(4)$ along with different lattice volumes up to 16^4 . In these analyses, we have used both $\mathcal{O}_K^{\text{lat}}$ and $\mathcal{O}_S^{\text{lat}}$ to construct the stability matrix, using APE-like smearing to expand the basis of operators. We have also imposed $\Delta_S = 2$ as a constraint, which ensures physical results for $\Delta_K \geq 2$ corresponding to $\gamma_K > 0$. By varying the operator basis in the analyses we have obtained initial estimates for systematic uncertainties. The thin error bars on each point combine statistical and systematic uncertainties in quadrature, while the thick error bars show the statistical uncertainties themselves.

For the range of 't Hooft couplings we have analyzed so far, $\lambda_{\text{lat}} \leq 3$ our results for the Konishi scaling dimension are consistent with perturbation theory, and well below the $U(2)$ bootstrap bound on the maximum value of Δ_K across all λ [20, 21]. This is not surprising, given that the perturbative expansion parameter is $\lambda/4\pi^2$. It is a longstanding challenge to reach stronger couplings [6], and another challenge comes from the twisted formulation we employ. Because the twisted rotation group involves only an $SO(4)_R$ subgroup of the full $SO(6)_R$ R-symmetry, the SUGRA operator \mathcal{O}_S is projected from the $20'$ representation of $SO(6)$ to a collection of $SO(4)$ representations. In particular, these include an $SO(4)_R$ -singlet piece that mixes with our lattice Konishi operator $\mathcal{O}_K^{\text{lat}}$ and may require variational analyses to disentangle.

5. Conclusions and next steps

This proceedings has explored the role of broken conformality in lattice studies of $\mathcal{N} = 4$ SYM by analyzing two conformal scaling dimensions. The first, a mass anomalous dimension related to the eigenmode number of the fermion operator, is expected to vanish, $\gamma_* = 0$, for all values of λ , making it a useful tool with which to quantify the effects of broken conformality. By stochastically reconstructing the Chebyshev expansion of the spectral density, we can extract a scale-dependent effective anomalous dimension that correctly converges to the true $\gamma_* = 0$ in the IR. Even the free theory exhibits significant lattice artifacts at higher energy scales, which increase rapidly as the lattice 't Hooft coupling λ_{lat} becomes stronger.

Second, turning to the non-trivial, λ -dependent scaling dimension of the Konishi operator, we have been able to verify conformal power-law scaling of the Konishi correlator with an appropriate value of Δ_K . Further analyses using the MCRG stability matrix method have produced preliminary results for the scaling dimension that are consistent with continuum perturbation theory for $\lambda_{\text{lat}} \leq 3$. Work is in progress to finalize these results, and in the longer term to push to stronger 't Hooft couplings, with particular interest in the self-dual point $\lambda = 4\pi N$. We are also exploring potential applications of the gradient flow to lattice $\mathcal{N} = 4$ SYM, which could include testing the extent to which broken conformality results in a non-zero β -function, as well as an alternate means of analyzing conformal scaling dimensions [25].

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