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# Cluster excitation and single-nucleon excitation in $^{12}$C

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We propose a new method for the classification of excitations of nuclei. Taking $^{12}$C as an example, we determine whether a given excited state is more of an $\alpha$-cluster excitation or a single-nucleon excitation. Using the so-called pinhole algorithm in the framework of nuclear lattice effective field theory, we study the distance between any two $\alpha$ clusters in the nucleus, and the separations of arbitrarily chosen nucleons inside an $\alpha$ cluster. For a state which is predominantly a cluster-type excitation (such as the Hoyle state) the distribution of $\alpha-\alpha$ separations is very different from that of the ground state. For states that are predominantly single-nucleon excitations, the two-nucleon separation inside an $\alpha$ cluster is in general increased.

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## 1. Introduction

A nucleus is a self-bound system of fermions, *i.e.* protons and neutrons. Somewhat surprisingly, already this simple fact has a profound influence on the $\alpha$-cluster structure of nuclei. Due to the Pauli principle, the four nucleons (proton spin-up, proton spin-down, neutron spin-up, and neutron spin-down) inside an $\alpha$ cluster can have the same spatial distribution, while further nucleons must occupy a different state with a different distribution. Thus, in nature we see the $^8$Be nucleus as a prominent example of clustering with two weakly interacting $\alpha$ clusters. Similarly, the famous "Hoyle state" of $^{12}$C, which is crucial for the synthesis of carbon and heavier elements in stars, is a three $\alpha$-cluster state and its geometric shape has been debated for a long time [1–17].

In the shell model, which has achieved great success in describing nuclear systems [18, 19], nucleons are described as independent particles moving in a mean field generated by all the nucleons. Either the cluster picture or the single-particle picture would likely be too simple for realistic nuclear systems, therefore the duality and interplay of these is an interesting topic [20]. In a recent work using nuclear lattice effective field theory (NLEFT) [17], the geometric shape has been studied for low-lying states of $^{12}$C, and some interesting features of cluster excitation and single-nucleon excitation can be seen from the 3D density distribution. In this contribution, we will show a more quantitative description of the excitation modes by looking at the distribution of distances between two $\alpha$ clusters and of two nucleons inside a cluster.

## 2. Theoretical framework

Here, we work within NLEFT [21, 22] and use a spatial lattice spacing $a = 1.64$ fm and a temporal lattice spacing $a_t = 0.55$ fm. The spatial lattice spacing corresponds to a momentum cut-off of 378 MeV. A periodic box with $L = 9$ is used and this corresponds to a physical box size of 14.8 fm. We use an SU(4) symmetric NN interaction:

$$V = \frac{C_2}{2!}\sum_{\mathbf{n}}\tilde{\rho}(\mathbf{n})^2 + \frac{C_3}{3!}\sum_{\mathbf{n}}\tilde{\rho}(\mathbf{n})^3, \tag{1}$$

with $C_2$ and $C_3$ the two-body and three-body coupling constants, respectively. The density operator is smeared locally and non-locally with the coefficients $s_{\mathrm{L}}$ and $s_{\mathrm{NL}}$, respectively, as

$$\tilde{\rho}(\mathbf{n}) = \sum_{i=1}^{A}\tilde{a}_i^\dagger(\mathbf{n})\tilde{a}_i(\mathbf{n}) + s_{\mathrm{L}}\sum_{|\mathbf{n}'-\mathbf{n}|=1}\sum_{i=1}^{A}\tilde{a}_i^\dagger(\mathbf{n}')\tilde{a}_i(\mathbf{n}'), \tag{2}$$

$$\tilde{a}_i(\mathbf{n}) = a_i(\mathbf{n}) + s_{\mathrm{NL}}\sum_{|\mathbf{n}'-\mathbf{n}|=1}a_i(\mathbf{n}'). \tag{3}$$

The four parameters in the interaction are determined by fitting to the ground-state energy of $^4$He and $^{12}$C, the ground-state charge radius of $^{12}$C, and to several electromagnetic transition rates [17]. The final values are

$$C_2 = -2.15\times10^{-5}, \quad C_3 = 6.17\times10^{-12}, \quad s_{\mathrm{NL}} = 0.05, \quad s_{\mathrm{L}} = 0.08. \tag{4}$$

For a projection MC (PMC) simulation with $N_{\rm ch}$ coupled channels, one starts from a set of initial trial states $|\Phi_i\rangle$ with $i = 1, 2, \ldots, N_{\rm ch}$ and each of them is a Slater determinant of single-particle wave functions. For the choices of initial trial states see the discussion in Refs. [15, 17]. One can calculate the Euclidean projection amplitudes at time step $N_t$ as

$$Z_{kl}(N_t) = \langle \Phi_k | M^{N_t} | \Phi_l \rangle, \tag{5}$$

with transfer matrix defined as

$$M =: \exp(-\alpha H) :, \tag{6}$$

where the colons denote normal ordering and $\alpha = a_t/a$. The Hamiltonian is $H = T + V$ ,with $T = \mathbf{p}^2/(2m_N)$ the kinetic energy, and $V$ is the NN interaction in Eq. (1).

The "adiabatic" transfer matrix is calculated as

$$M^{(a)}_{qq'}(N_t) = \sum_{q''} Z^{-1}_{qq''}(N_t) Z_{q''q'}(N_t + 1), \tag{7}$$

with eigenvalues

$$\lambda_i(N_t) = \exp(-\alpha_t E_i(N_t)). \tag{8}$$

The low-lying spectrum can be calculated as

$$E_i(N_t) = -\frac{\log(\lambda_i(N_t))}{\alpha_t}. \tag{9}$$

To get the geometric information of the nucleus, we adopt the so-called pinhole algorithm [23], in which the following amplitude is calculated

$$Z_{kl}(\mathbf{n}_1, \ldots, \mathbf{n}_A, N_t) = \langle \Phi_k | M^{N_t/2} \rho(\mathbf{n}_1, \ldots, \mathbf{n}_A) M^{N_t/2} | \Phi_l \rangle, \tag{10}$$

with the $A$-body density operator $\rho(\mathbf{n}_1, \ldots, \mathbf{n}_A)$ constructed from the normal-ordered product of density operators $\rho(\mathbf{n}_i) = a^\dagger_i(\mathbf{n}) a_i(\mathbf{n})$. The positions for $A$ nucleons (or "pinholes") $\mathbf{n}_i$ are sampled stochastically. The finite size of the nucleons is accounted for by a random Gaussian smearing of the nucleon positions. With the position information $r_i, i = 1, 2, \ldots A$, for all the nucleons, we can study the density distribution, transition form factors, and geometric structure as illustrated in Ref. [17].

## 3. Results and discussion

We find clusters of protons/neutrons (spin up/down) by identifying the most compact groupings of four nucleons. The rms radii of such clusters are quite similar to that of an $\alpha$ particle, both for the ground state and the Hoyle state of $^{12}$C. This shows that the $\alpha$-cluster structure is quite dominant for those states, which is not always the case for other excited states as we show in Fig. 1. In the left panel of Fig. 1, we show the distribution of the average distance between two nucleons inside an $\alpha$ cluster. In the right panel of Fig. 1, we show the distribution of the distance between two arbitrary $\alpha$ clusters in $^{12}$C.

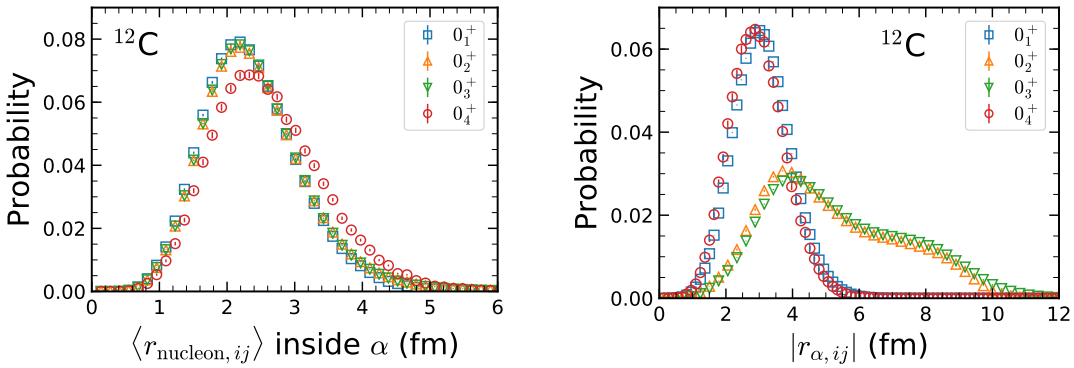


**Figure 1:** (Left) Distribution of two nucleons inside an $\alpha$ cluster for different states of $^{12}$C calculated by NLEFT. (Right) Distribution of two $\alpha$ clusters.

As discussed in Ref. [17], the ground state ($0_1^+$) is a compact equilateral triangle of three $\alpha$ clusters, the Hoyle state ($0_2^+$) is a large-angle obtuse triangle close to a linear chain, $0_3^+$ is a small-amplitude vibrational excitation of $0_2^+$, and $0_4^+$ is predominantly a two-particle-two-hole excitation. This interpretation is consistent with the characteristic features of the distance distribution in Fig. 1: On the one hand, when the excited state is predominantly a single-nucleon excitation such as $0_4^+$, the $\alpha$-structure will be reduced, and the average distance between two nucleons inside the $\alpha$ cluster will increase due to the excitation of one (or more) nucleon(s) out of the cluster. On the other hand, when the excited state is more of an $\alpha$-cluster excitation (such as $0_2^+$ and $0_3^+$), the average distance between two nucleons inside an $\alpha$ cluster is little influenced, while the spatial configuration of $\alpha$ clusters is changed.

## 4. Summary

In this contribution, we showed a model-independent way to distinguish between $\alpha$-cluster excitations and single-nucleon excitations in nuclei. Although we have used the framework of NLEFT, our method should have general applicability. This entailed a calculation of the distribution of the average distance between two nucleons inside a proton/neutron - spin up/down cluster, as well as the distribution of the two-cluster separation. When a nuclear excited state is more of a cluster excitation, the distribution of the two-nucleon distance inside a cluster was found to not change much, while that of the distance between two clusters was greatly affected. For single-nucleon excitations, this situation is reversed.

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