

## Ab-initio study of dibaryons with highest bottom number

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We present the first lattice study of dibaryons with highest bottom number. Utilizing a set of state-of-the-art lattice QCD ensembles and methodologies, we determine the ground state of dibaryon composed of two  $\Omega_{bbb}$  baryons. We extract the related scattering amplitude in the  $^1S_0$  channel and find a sub-threshold pole, which signifies an unambiguous evidence for a deeply bound  $\Omega_{bbb} - \Omega_{bbb}$  dibaryon. The binding energy of such a state as dictated by this pole singularity is found to be  $-81^{+14}_{-16}$  MeV. We quantify various systematic uncertainties involved in this determination, including those related to the excited state contamination and Coulomb repulsion between the bottom quarks.

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## 1. Introduction

Understanding baryon-baryon interactions from first-principles is of prime interest in nuclear and astrophysics, as they form the foundation towards a fundamental explanation of why atomic nuclei exist. Simplest systems in which such interactions can be studied transparently are dibaryons, which have a baryon number 2. Despite decades of experimental efforts, only one bound dibaryon has been established (Deuteron). Even so, there might be other bound dibaryons, particularly in the heavy sector, that exist in Nature and are yet to be discovered.

As of today, there are a handful of lattice QCD investigations of dibaryons in the light and the strange sector, mostly performed at unphysically heavy quark masses, covering channels such as Deuteron, dineutron, H-dibaryon, etc. *c.f.* Ref [1]. An important lesson learned from these studies is that discretization effects could be substantial in these systems, which when unaddressed can lead to large discrepancies. Among the heavy baryons,  $\Omega_{ccc}$  and  $\Omega_{bbb}$  serves a promising domain to study the nonperturbative features of QCD, free of the light quark dynamics. On the same ground, a system of two  $\Omega_{ccc}$  and two  $\Omega_{bbb}$  baryons are interesting as they serve as useful platforms in understanding the baryon-baryon interactions in chiral dynamics free environment. Such a system in the light sector is more complicated, since the  $\Delta$  baryon is a resonance and that there are kinematically allowed four particle final states. Scattering of  $\Omega_{sss}$  baryons has been addressed in a few lattice QCD works [2–4] following different methodologies. Although they arrive at conflicting conclusions on whether the interaction between  $\Omega_{sss}$  baryons is weakly attractive or weakly repulsive, a common consensus from these studies is that the interactions are very weak<sup>1</sup> in Nature. Another recent lattice study of  $\Omega_{ccc}$  baryons scattering reported a very shallow bound state in the  $^1S_0$  channel [5]. While all these investigations suggest weak interactions in systems with quark masses ranging from light to charm, several lattice studies in the recent years on heavy dibaryons [6, 7] and heavy tetraquarks [8, 9] have shown that multi-hadron systems with multiple bottom quarks can have deep binding. Naturally, an investigation of interactions between  $\Omega_{bbb}$  baryons would be very timely, as this can shed light on the behaviour of such interactions in the heavy sector.

In this work, we perform a lattice QCD calculation of scalar dibaryons with highest number of bottom quarks. We name such a dibaryon system as  $\mathcal{D}_{6b}$ . The elastic threshold corresponds to the  $\Omega_{bbb} - \Omega_{bbb}$  channel.

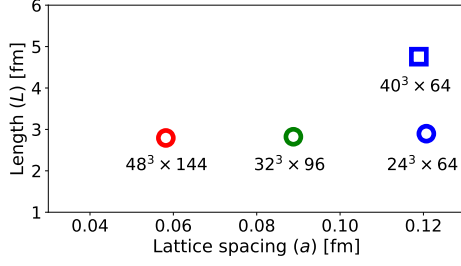
## 2. Methodology

*Ensembles:* We utilize four ensembles with dynamical u/d, s, and c quark fields generated by the MILC collaboration, with HISQ fermion action [10]. The specifics of the lattice ensembles are as shown in Figure 1.

*Quark propagators:* Quark propagators were computed using the time evolution of an NRQCD Hamiltonian, including improvements up to  $O(\alpha_s v^4)$  on Coulomb gauge fixed wall sources and multiple source timeslices. The bare quark mass was tuned using the spin averaged kinetic mass of 1S bottomonia states and was found to reproduce the 1S bottomonia hyperfine splitting accurately.

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<sup>1</sup>The word ‘weak’ used throughout this article refers to the strength of the interaction, and has no connection to the real weak interaction in the Standard Model of Particle Physics.



**Figure 1:** Lattice QCD ensembles utilized in this work presented in the  $L$  (spatial extent) versus  $a$  (lattice spacing) landscape.

*Two point correlation functions:* Hadron spectroscopy programs in lattice QCD such as ours proceed through the computation of two point correlation functions of the form

$$C_O(t_f - t_i) = \sum_{\vec{x}_f} e^{-i\vec{p}\cdot\vec{x}_f} \langle 0 | O(\vec{x}_f, t_f) \bar{O}(t_i) | 0 \rangle, \quad (1)$$

where  $O$  are the interpolating operators with the desired quantum numbers. At the sink time slice, we utilize several different quark field smearing procedures to identify the reliable ground state plateau and quantify any possible excited state contamination (see Ref. [11] for more details). The single and the dibaryon ground state energies in the finite-volume are obtained by fitting the averages of correlation function with a single exponential at large times.

*Interpolating operators:* A nondisplaced nonrelativistic operator with  $J^P = 3/2^+$  [2] was utilized for  $\Omega_{bbb}$  baryon. The color space is trivially antisymmetric for a baryon. The flavor and the spatial structure of the operator is trivially symmetric. The remaining spin space for a nonrelativistic operator allows only  $J=3/2$  spin, which is symmetric as shown below.

$O_{\Omega_{bbb}}$	$ s_1 s_2 s_3\rangle$	$ J, J_z\rangle$
$\chi_1$	$ 111\rangle$	$ 3/2, +3/2\rangle$
$\chi_2$	$ 112\rangle_S$	$ 3/2, +1/2\rangle$
$\chi_3$	$ 122\rangle_S$	$ 3/2, -1/2\rangle$
$\chi_4$	$ 222\rangle$	$ 3/2, -3/2\rangle$

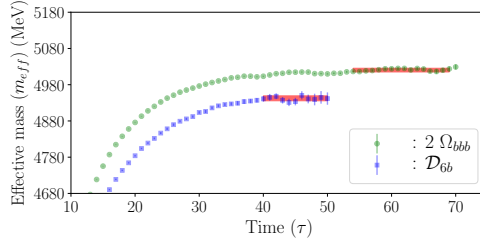
**Table 1:** Four rows of the  $O_{\Omega_{bbb}}$  baryon operator in the  $^1H^+$  finite-volume irrep.  $\chi_i$  refers to different azimuthal components and  $s_i$  refer to the spin components of the  $i^{\text{th}}$  quark. For NRQCD action, only the upper two components of the Dirac spinor are nonzero and are referred using 1 and 2 respectively. All spin components of the total operator is symmetrized, and are highlighted with a subscript  $S$ . The third column shows the total spin and the azimuthal component of each rows of the operator.

Assuming dominance of s-wave scattering in the  $\Omega_{bbb}$  interactions near the two baryon threshold, we build the two hadron operator relevant for scalar  $\mathcal{D}_{6b}$  channel using simple spin algebra as follows.

$$O_{\mathcal{D}_{6b}^{J=0}} = \frac{1}{2} \left[ \chi_{\frac{3}{2}} \chi_{-\frac{3}{2}} + \chi_{-\frac{1}{2}} \chi_{\frac{1}{2}} - \chi_{\frac{1}{2}} \chi_{-\frac{1}{2}} - \chi_{-\frac{3}{2}} \chi_{\frac{3}{2}} \right]. \quad (2)$$

### 3. Results

*Signal quality and fit estimates:* We present the signal quality and the ground state saturation in baryon and the dibaryon correlation functions in terms of the effective mass, defined as  $am_{eff} = \log[C(\tau)/C(\tau + 1)]$ , in Figure 2. Here  $C(\tau)$  is the correlation function at source sink separation time  $\tau$ . Signal saturation and a clear energy gap between the noninteracting two-baryon level and the interacting dibaryon system in the finest ensemble is evident. This pattern is observed on all four ensembles we study, which unambiguously point to a finite volume energy level in the interacting dibaryon system below the threshold. In order to ensure the identification of the real ground state plateau, we perform the same analysis using different smearing procedures (see Ref [11] for details). This helps us estimating the errors related to possible excited state contamination arising from incorrect identification of the ground state plateau.



**Figure 2:** Effective masses of the non-interacting two-baryon (green) and dibaryon (blue) ground states in the finite volume on the finest lattice ensemble. A negative energy shift in the interacting case is clear, indicating attractive interactions. The solid bands present the fit results and fitting windows.

Single exponential fits to  $C(\tau)$  yield the mass estimates in lattice units for the baryon and dibaryon ground states. The use of NRQCD formulation for quark fields implies an additive normalization proportional to the number of heavy quarks within the hadron realized. To this end, we determine the energy splittings between the baryon and dibaryon ground state mass estimates. Note that by taking such energy differences, it automatically removes of the additive normalization in the energy estimates intrinsic to the NRQCD formulation. This energy splitting  $\Delta E = M_{D_{6b}} - 2M_{\Omega_{bbb}}$  in physical units as determined on different lattice are tabulated in Table 2.

Ensemble	$\Delta E$	Ensemble	$\Delta E$
$24^3 \times 64$	-61(11)	$40^3 \times 64$	-62(7)
$32^3 \times 96$	-68(9)	$48^3 \times 144$	-71(7)

**Table 2:** Energy splittings  $\Delta E = M_{D_{6b}} - 2M_{\Omega_{bbb}}$  in MeV on different ensembles.

*Scattering analysis:* The existence and the properties of a hadron from finite volume spectrum is determined in terms of pole singularities in the relevant scattering amplitudes across the complex Mandelstam  $s$ -plane. To this end, we follow Lüscher’s finite-volume formalism. The  $s$ -wave scattering amplitude is given by  $t = (\cot \delta_0 - i)^{-1}$ , and a pole in  $t$  related to a bound state happens when  $k \cdot \cot \delta_0 = -\sqrt{-k^2}$ . Here  $\delta_0(k)$  is the two spin  $\Omega_{bbb}$  scattering phase shift leading to  $J^P = 0^+$  is related to the finite-volume energy eigenvalues as

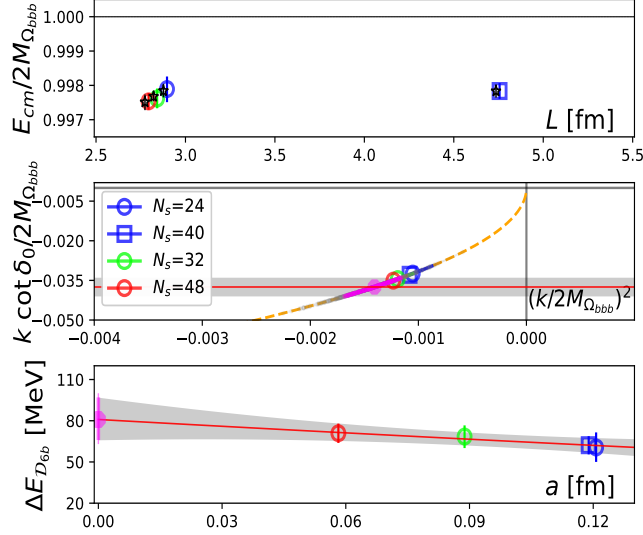
$$k \cot(\delta_0(k)) = \frac{2Z_{00}(1; (\frac{kL}{2\pi})^2)}{L\sqrt{\pi}}. \quad (3)$$

Here  $k$  is the momentum of the  $\Omega_{bbb}$  baryon in the centre of momentum frame and is given by  $k^2 = \frac{\Delta E}{4} (\Delta E + 4M_{\Omega_{bbb}}^{phys})$ , and  $M_{\Omega_{bbb}}^{phys}$  is the mass of  $\Omega_{bbb}$  baryon in the continuum limit determined independently from the respective correlation functions.  $Z_s$  are the Lüscher's zeta functions [12].

We parameterize  $k \cot\delta_0$  either as a constant or as a constant plus a linear term in the lattice spacing to determine possible cutoff systematics in our finite volume treatment. We perform several different fits involving different subsets of the four energy splittings listed earlier, with the two different fits forms. All of the fits indicate the existence of a deeply bound state pole in this channel. The best fit is found to be the one that incorporates all the energy splittings, and incorporates the lattice spacing dependence of the scattering length  $a_0$  with a linear parametrization  $k \cot\delta_0 = -1/a_0^{[0]} - a/a_0^{[1]}$ . The fit quality turns out to be  $\chi^2/d.o.f = 0.7/2$  and the estimates in this case and the resultant binding energy are

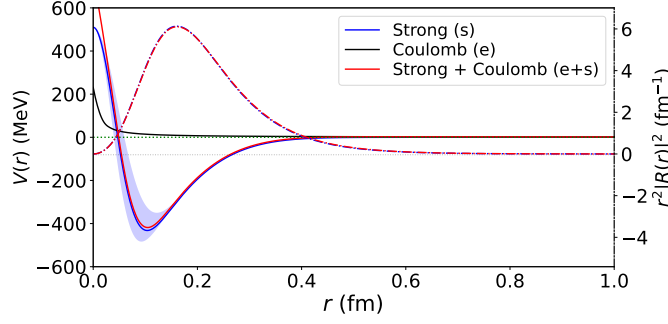
$$a_0^{[0]} = 0.18_{(-0.02)}^{(+0.02)} \text{ fm}, \quad a_0^{[1]} = -0.18_{(-0.11)}^{(+0.18)} \text{ fm}^2, \quad \& \quad \Delta E_{\mathcal{D}_{6b}} = -81_{(-16)}^{(+14)} \text{ MeV}. \quad (4)$$

We present the details of our main results in the below figure.



**Figure 3:** Results from the finite-volume scattering analysis. Top: Comparison of the simulated energy levels (large symbols) with the energy levels (black stars) analytically reconstructed using Eq. (4), indicating the quality of the scattering analysis fit. Middle:  $k \cot\delta_0$  versus  $k^2$  in units of energy of the threshold ( $2M_{\Omega_{bbb}}$ ) and information on poles in  $t$  indicated by magenta symbols. Bottom: Continuum extrapolation of the binding energy determined from the fitted scattering amplitude (see Eq. (4)).

Various systematics: Now we discuss various systematic uncertainties related to this calculation. We use state-of-the-art lattice QCD ensembles in this study, together with an NRQCD Hamiltonian with improvement coefficients up to  $\mathcal{O}(\alpha_s v^4)$  for the time evolution of the bottom quark fields. This setup has been demonstrated to reproduce the bottomonium hyperfine splittings correctly with an uncertainty of about 6 MeV [11]. A faithful reproduction of the hyperfine splitting reflects small and controllable discretization effects. Note that the energy splittings that we compute and utilize in our scattering analysis have reduced systematics. For the heavy dibaryons, statistical errors, fit-window errors and possible excited state contamination in the identified energy



**Figure 4:** Coulombic potential ( $V_e$ ), the parameterized  $\Omega_{bbb}$  baryon interaction potentials ( $V_s$ ) and their sum are shown by the solid black, blue and red curves, respectively. The shaded band is the variation of  $V_s$  with respect to its parameters. The radial probability densities of the quantum mechanical ground state wave-functions of  $V_s$  and combined potentials are represented by the dotted curves.

plateau are the main sources of error. We arrive at a robust identification of the ground state plateau by studying the ground state signals for the same state with different smearing programs. The systematics arising from excited state effects are gauged from the variation in the continuum limit estimates determined from different smearing and analysis procedures utilized.

This dibaryon being extremely heavy, bound and having an electric charge -2, effects from Coulombic repulsion could be substantial. We investigate possible corrections due to this following an analysis procedure as in Ref. [5]. The interaction between the  $\Omega_{bbb}$  baryons is modelled with an attractive multi-Gaussian potential  $V_s$  with its parameters tuned to reproduce the quantum mechanical bound state with binding energy  $-81^{(+14)}_{(-16)}$  MeV. The Coulombic interaction is further modelled as in Ref. [5], and its effects are determined by studying the ground state solutions of the total potential. In Figure 4, we show the parameterized potentials  $V_s$  and Coulombic interaction and their combination, together with the radial probabilities of the ground state wave-functions in  $V_s$  and combined potentials. It is evident that the Coulombic interaction hardly affects  $\Omega_{bbb}$  baryon interaction potential  $V_s$  and its ground state radial probability densities. We find that the deviations in binding energy with inclusion of such Coulombic interaction are found to be between 5 – 10 MeV.

The uncertainties related to extrapolation to the continuum are gauged from variation in the results obtained with different possible fit forms. To gauge the uncertainties related to scale setting and the bottom quark mass tuning we investigate the system with perturbed values of the scale and bare bottom quark masses. Electromagnetic effects are quantified based on possible corrections arising from higher order radiative corrections in the NRQCD Hamiltonian. All of these systematic uncertainties, are found to be less than or equal to 12 MeV, and are detailed in the Supplemental material of Ref. [11].

#### 4. Summary and conclusions

We present a first investigation of interactions between  $\Omega_{bbb}$  baryons. We find a deeply bound dibaryon  $\mathcal{D}_{6b}$  in the  $^1S_0$  channel. The relevant scattering amplitude is extracted following the Lüscher's formalism which features a bound state pole with binding energy  $-81^{(+14)}_{(-16)}$ (14) MeV. Complementary measurement and analysis procedures are utilized in identifying the real ground

state plateau to ensure the robustness of our results. All possible systematics, including but not limited to fitting-window errors and excited state effects are studied. The resultant uncertainties added in quadrature are quantified in the second parenthesis. Unlike the light dibaryons with a handful of lattice calculations with conflicting observations, our conclusion on the existence of a deeply bound state  $\mathcal{D}_{6b}$  would be robust to calculations such as those using a variational approach. Considering the observation of weakly interacting nature in equivalent systems ( $\mathcal{D}_{6s}$  and  $\mathcal{D}_{6c}$ ) [2–4], future studies of quark mass dependence of this system would be very appealing and could shed further light into the change in quark dynamics at different energy scales.

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