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Direct access to hadronic decay parameters with twisted boundary conditions

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Our exploratory study looks for direct access to the resonant hadronic transition amplitude without resorting to the Lüscher formalism. We study the decay $\psi(3770) \to D\bar{D}$ by applying partially twisted boundary conditions (PθBCs) to the quenched charm quark, circumventing possible problems with final state interactions. If successful, we could compute the dependence of the transition amplitude on the charm-quark mass and test the predictions made by phenomenological quark-pair-creation models. Finally, we study if and to what extent an extraction of the excited state $\psi(3770)$ is necessary for this analysis.

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1. Introduction

Investigating hadronic decays from lattice QCD is notoriously challenging. Indeed, analytical properties of decay amplitudes, like the poles in the complex plane that can be identified as resonances, are not straightforward to recover from correlation functions in Euclidean space. However, Lüscher's formalism and its various generalisations have made a breakthrough in the study of processes like the ρ meson decay, or scatterings of meson-meson, meson-baryon and baryon-baryon states. As those approaches relate the decay parameters to the spectrum of interacting particles in finite volume, they require computations in several physical volumes to identify singular points on phase shift curves. As a consequence, the numerical work can be costly.

McNeile et al. [1] proposed a different method, which gives direct access to the decay matrix elements under the condition that the kinematical configuration is close to the threshold. In our exploratory work, we combine the approach in [1] with the use of PθBCs so that the threshold condition can be established off the momenta quantisation in finite volume.

2. Particle decay under investigation

To apply the proposal in [1], we study the charmonium state $\psi(3770)$, which is a vector particle with quantum numbers $\mathrm{I}^{\mathrm{G}}(\mathrm{J}^{\mathrm{PC}}) = 0^-(1^{--})$ and mass $M_{\psi(3770)} = 3773.7(4)$ MeV [2]. We consider its main decay channel, $\psi(3770) \to D\bar{D}$, depicted in fig. 1, with $(\Gamma_{D^0} + \Gamma_{D^\pm})/\Gamma_{\text{total}} > 90\%$. Since we do not account for isospin breaking (IB), we cannot distinguish between the neutral and charged D mesons in the final state. Then, the braket that we aim to compute is

$$x \equiv \left\langle D\bar{D} \middle| \psi(3770) \right\rangle . \tag{2.1}$$

This decay is ideal for probing [1] because it occurs nearly at the threshold in an experiment, $M_{\psi(3770)} - 2M_{D^0} \sim 44\,\text{MeV}$ [2]. The relevant experimental spectrum for our work appears in fig. 2. Of course, the unphysical quark masses used in the simulations and a finite lattice spacing can affect the particular values of the masses seen in fig. 2, and they might even make the $D\bar{D}$ state heavier than $\psi(3770)$. As the initial and final states, $\psi(3770)$ and $D\bar{D}$, are very close in mass, the twist angle required to fix the kinematical configuration at the threshold is expected to be small. Moreover, twisted boundary conditions boost each meson of the $D\bar{D}$ system in the opposite direction, increasing the system energy while it stays at rest.

Since we study a process with final-state interactions between the D and $\bar{D}$ mesons, we may only apply PθBC to the quenched charm quark [3], keeping finite-volume effects under control. The decay $\psi(3770) \to D\bar{D}$ has been thoroughly studied in experiments, and it offers a nice playground to test the reliability of theoretical frameworks like early effective quark models. In this project, we concentrate on the $^3P_0$ quark pair creation model [4], which was fruitful for understanding the dynamics of hadronic meson decay in two mesons [5]. It postulates an effective charm-quark mass dependence only through the wave functions that describe each of the three hadrons entering

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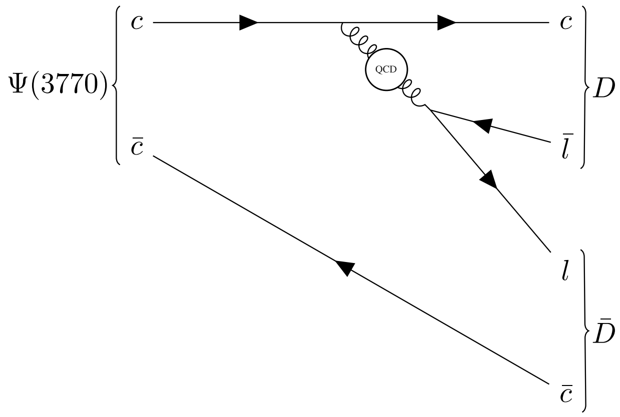

Figure 1: $\psi(3770) \rightarrow D\bar{D}$ decay Feynman diagram.

Figure 2: Experimental spectrum of vector charmonium and $D\bar{D}$ [2].

the process, and introduces a universal coupling γ. In turn, we compute the matrix element $\left\langle H_q \overline{H}_q \middle| \bar{h}h(1^-, n=3) \right\rangle_{E[H_q\overline{H}_q]=m(\bar{h}h(1^-,n=3))}$ on the lattice, and compare the dependence on the mass m_h of the heavy quark h.[1]

3. Methodology

We consider correlation functions of operators $O_{\bar{h}\bar{q}hq}$ and $O_{\bar{h}h}$, which work as interpolators of the $D\bar{D}$ and $\psi(3770)$ states, respectively. To extract the matrix element x from the lattice, we need to study the asymptotic behaviour of the correlators whose diagrams are shown in fig. 3 [6],

$$\begin{aligned} C_{\psi(3770)\rightarrow\psi(3770)}(t) &\equiv \left\langle O_{\bar{h}h}(t) O^{\dagger}_{\bar{h}h}(0) \right\rangle \longrightarrow Z^2_{\psi(3770)}\; e^{-m_{\psi(3770)}t}, \\ C_{D\bar{D}\rightarrow D\bar{D}}(t) &\equiv \left\langle O_{\bar{h}\bar{q}hq}(t) O^{\dagger}_{\bar{h}\bar{q}hq}(0) \right\rangle \longrightarrow Z^2_{D\bar{D}}\; e^{-E_{D\bar{D}}t}, \\ C_{\psi(3770)\rightarrow D\bar{D}}(t) &\equiv \left\langle O_{\bar{h}\bar{q}hq}(t) O^{\dagger}_{\bar{h}h}(0) \right\rangle \longrightarrow \sum_{t_i} Z_{\psi(3770)}\; e^{-m_{\psi(3770)}t_i}\, x\, e^{-E_{D\bar{D}}(t-t_i)}\; Z_{D\bar{D}}, \end{aligned} \tag{3.1}$$

where the full correlator for $D\bar{D} \rightarrow D\bar{D}$ process is

$$C_{D\bar{D}\rightarrow D\bar{D}} = C^2_{D\rightarrow D} + C_{D\bar{D}\rightarrow D\bar{D},\,\mathrm{box}}. \tag{3.2}$$

[1]Here, $\bar{h}h(1^-, n=3)$ is the second radial excitation of the vector quarkonium while H_q is the ground-state pseudoscalar heavy-light meson. At $m_h = m_c$, $\bar{h}h(1^-, n=3) \equiv \psi(3770)$ and $H_q \equiv D$.

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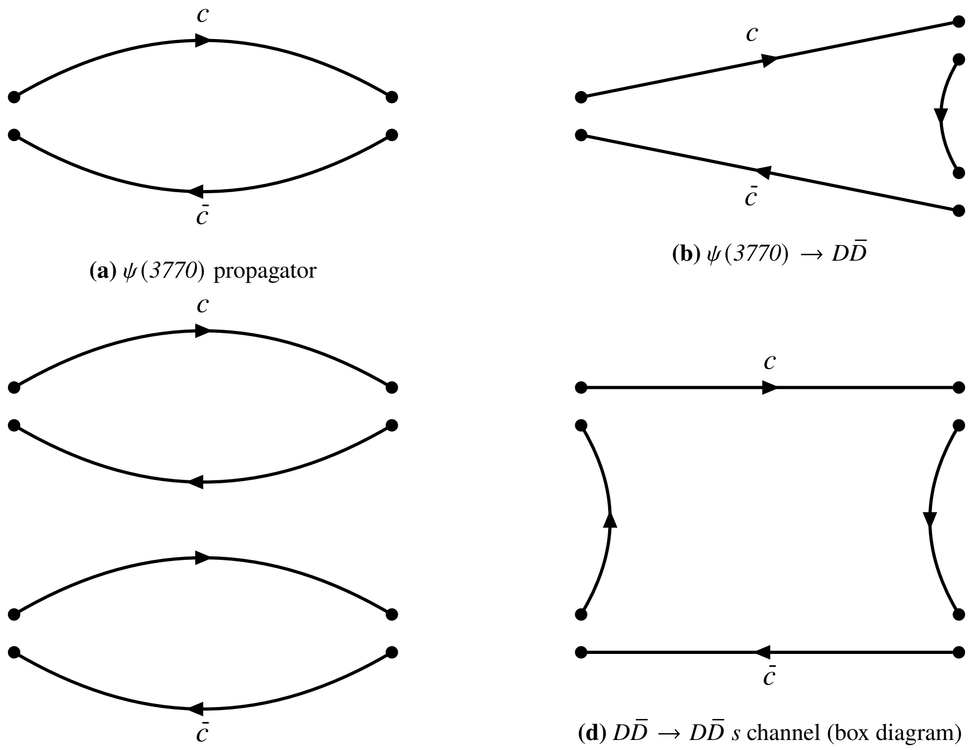

(a) $\psi(3770)$ propagator

(b) $\psi(3770) \to D\bar{D}$

(c) $D\bar{D} \to D\bar{D}$ t channel (direct component)

(d) $D\bar{D} \to D\bar{D}$ s channel (box diagram)

Figure 3: Diagrams needed for our study [6].

The asymptotic behaviour of $C_{\psi(3770)\to D\bar{D}}(t)$ can be derived using the transfer matrix formalism. Calling $\left|\psi(n)\right\rangle$ the 1^- quarkonia states and $\left|D\bar{D}(n)\right\rangle$ the $D\bar{D}$ mesons P-wave states, both series of states are eigenmodes of the transfer matrix. Then, if we assume the threshold condition $m_{\psi(n=3)} = E_{D\bar{D}(m=1)}$[2] and $\left\langle\Omega\middle|O_{\bar{h}h}\middle|D\bar{D}(n)\right\rangle = \left\langle\Omega\middle|O_{\bar{h}\bar{q}hq}\middle|\psi(m)\right\rangle = 0$, we have

$$\begin{aligned}
&\left\langle O_{\bar{h}\bar{q}hq}(t)O^\dagger_{\bar{h}h}(0)\right\rangle \\
&= \sum_{0\le t_1<t}\sum_{m,n} \left\langle\Omega\middle|O_{\bar{h}\bar{q}hq}\middle|D\bar{D}(n)\right\rangle \left\langle D\bar{D}(n)\middle|\psi(m)\right\rangle \left\langle\psi(m)|\Omega\right\rangle\, e^{-m_{\psi(m)}t_1}e^{-E_{D\bar{D}(n)}(t-t_1)} + \dots \\
&= \sum_{0\le t_1<t} \left\langle\Omega\middle|O_{\bar{h}\bar{q}hq}\middle|D\bar{D}(1)\right\rangle \left\langle D\bar{D}(1)\middle|\psi(3)\right\rangle \left\langle\psi(3)|\Omega\right\rangle\, e^{-E_{D\bar{D}(1)}t} + \dots \\
&\sim \left\langle\Omega\middle|O_{\bar{h}\bar{q}hq}\middle|D\bar{D}(1)\right\rangle \left\langle\psi(3)|\Omega\right\rangle\; x\; t\; e^{-E_{D\bar{D}(1)}t}.
\end{aligned} \tag{3.3}$$

To extract $\left|\psi(3770)\right\rangle$ and $m_{\psi(3770)}$, we solve a Generalised Eigenvalue Problem (GEVP) where we consider several Gaussian smearing levels of the quark field, covariant derivatives and Dirac structures. Setting $E_{D\bar{D}} = m_{\psi(3770)}$ allows to find the appropriate PθBC $h(x + L\vec{e}_i) = e^{i\theta_i L}h(x)$ for the heavy field obeying the threshold condition.[3]

[2] $\psi(n = 3) \equiv \psi(3770), D\overline{D}(m = 1) \equiv D\bar{D}$.

[3] Note that we have imposed isotropic twisted boundary conditions.

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id	β	$(\frac{L}{a})^3 \times \frac{T}{a}$	a [fm]	m_π [MeV]	$m_\pi L$	# cfgs	κ_ℓ	κ_s	κ_c
D5	5.3	$24^3 \times 48$	0.0653	439	4.7	150	0.13625	0.135777	0.12724
F7		$48^3 \times 96$		268	4.3	t.b.d.	0.13638	0.135730	0.12713

Table 1: Coordinated Lattice Simulations (CLS) $N_f = 2$ ensemble parameters. From left to right: Ensemble label, inverse bare coupling $\beta = 6/g_0^2$, lattice geometry and spacing, pion mass, $m_\pi L$, number of configurations in our study, and κ values for the bare light, strange and charm quarks (see main text for more details). Moreover, we know that at leading order the finite volume effect decays exponentially fast with $m_\pi L$, and $m_\pi L \geq 4$ reduces significantly its impact. Finally, we plan to extend our analysis to F7 to study the pion mass dependence of our results, but no statistics is available yet.

We may use the asymptotic behaviour in eq. (3.1) in two different ways to extract x from the lattice [1, 6, 7]. The first method is to consider the decay $\psi(3770) \to D\bar{D}$ directly,

$$\frac{C_{\psi(3770)\to D\bar{D}}(t)}{\sqrt{C_{D\bar{D}\to D\bar{D}}(t)\; C_{\psi(3770)\to\psi(3770)}(t)}} \quad \underset{t/a\gg 1}{\longrightarrow} \quad tx + \text{const}, \tag{3.4}$$

where the dominant excited-state contamination appears in the constant term. The factor

$$\left\langle \Omega \middle| O_{\bar{h}\bar{q}hq} \middle| D\bar{D}(1) \right\rangle \; \langle \psi(3)|\Omega\rangle \; e^{-E_{D\bar{D}(1)}t} \tag{3.5}$$

of $C_{\psi(3770)\to D\bar{D}}(t)$ simplifies versus the denominator on the left-hand side of eq. (3.4). The other method is to only use the information from $D\bar{D} \to D\bar{D}$,

$$\frac{C_{D\bar{D}\to D\bar{D},\text{ box}}}{C_{D\bar{D}\to D\bar{D},\text{ direct}}} \quad \underset{t/a\gg 1}{\longrightarrow} \quad \frac{1}{2}x^2t^2 + O(t). \tag{3.6}$$

Here, it is interesting to study the dependence (if any) of the residual linear term on the twist angle θ used for the boundary conditions.

4. Preliminary Findings

In this section, we show the status of our analysis on ensemble D5, an $N_f = 2$ *Coordinated Lattice Simulations* CLS simulation, whose details appear in table 1. $N_f = 2$ CLS ensembles employ the Wilson plaquette for the gauge action, as well as $O(a)$-improved Wilson quarks for the fermionic action. The gauge configurations were computed using the DD-HMC algorithm, the light-quark hopping parameter yields pion masses between 190 MeV $< m_\pi <$ 630 MeV, the strange-quark mass is fixed to its physical value by setting m_K^2/f_K^2 and m_π^2/f_K^2 to their physical values, and the charm-quark mass is chosen such that $m_{D_s} = m_{D_s,\text{phys}}$. For more details on the ensemble simulations, see [8, 9].

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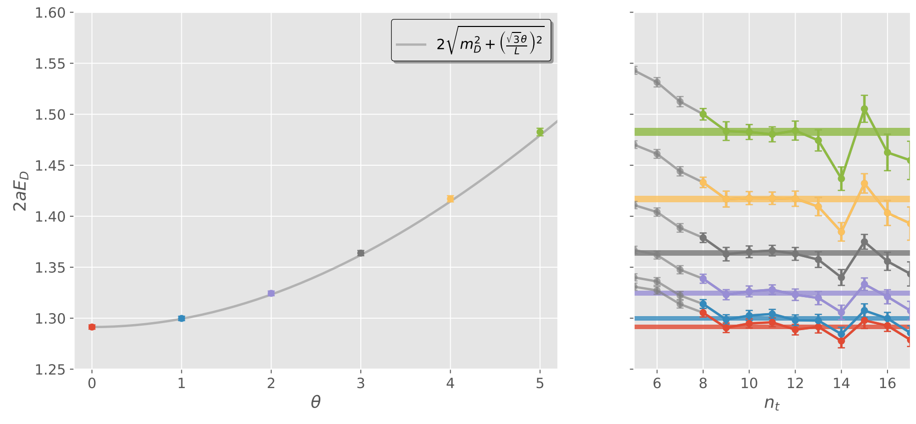

Figure 4: Left: Dispersion relation of the $D\bar{D}$ system as a function of the twist angle $\theta[1/L]$, assuming $E_{D\bar{D}} = 2E_D$ and $m_D = E_D(\theta = 0)$. Right: $D\bar{D}$ effective mass for the various twist angles plotted on the left-hand side as a function of the time-slice.

In fig. 4, we plot the $D\bar{D}$ state energy as a function of the PθBC used for the charm quark. We observe that the data is well described by the continuum dispersion relation for two free D mesons, depicted by the grey line. Note that at $\theta = 0$, corresponding to a $D\bar{D}$ system with each meson at rest, the system on ensemble D5 is heavier than its physical value. This is to be expected, due to the unphysical pion mass $m_\pi = 440\,\text{MeV}$. So far, our main obstacle has been identifying the $\psi(3770)$ state from the charmonium spectrum. We use a basis of operators belonging to the T_1^{--} representation of the H(3) symmetry group, which overlaps with states of quantum numbers $J^{PC} = 1^{--}, 3^{--}, 4^{--}, \ldots$ [10], and we employ different levels of Gaussian smearing. At this stage, we have already computed eq. (3.6) and fitted the function $a_0 t + \frac{1}{2}a_1^2 t^2$ to the result. The matrix element we seek corresponds to a_1, which is plotted together with a_0 in fig. 5 as a function of the twist angle. As expected, we observe the linear term parameter a_0 to be small. Moreover, we observe a certain behaviour of a_0 and a_1 at $\theta = 1$, which we cannot explain at this point. Once we extract $m_{\psi(3770)}$ and the twist angle at the resonance, we could determine if this behaviour is correlated to the resonance, or not. For the moment we only include statistical errors estimated using the package `pyerrors` [11], which is based on the Γ-method [12].

5. Outlook

Currently, our efforts revolve around an accurate determination of the $\psi(3770)$ state mass. We are probing different interpolators and smearing levels for the GEVP. Afterwards, we will be able to determine the resonant twist angle, fit to eq. (3.4), and extract the corresponding decay width. Then, we will extend our calculations to more ensembles, starting with F7 (see table 1), to study the quark mass and lattice spacing dependence of our results. In particular, the quark mass dependence will serve to probe the validity of the $^3P_0$ quark model described in [4].

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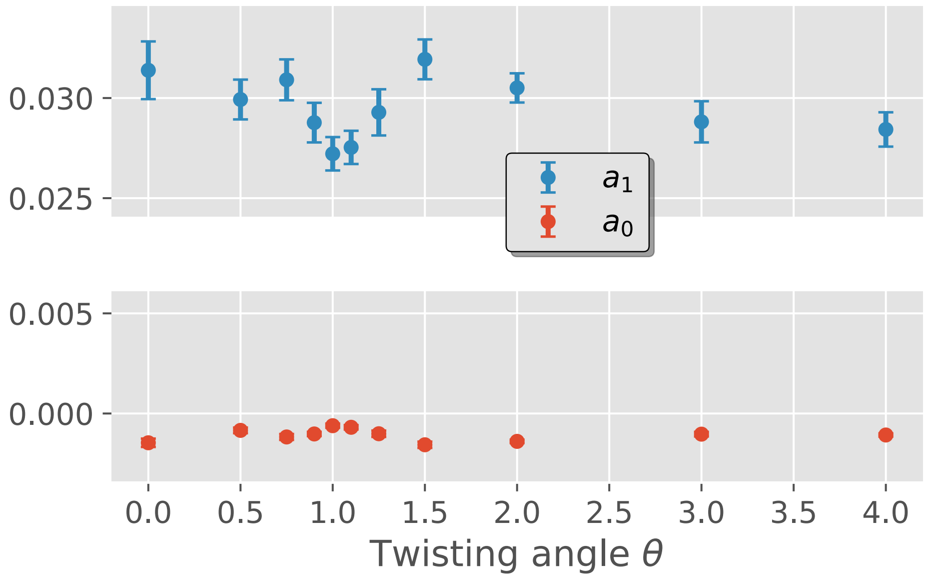

Figure 5: Fit parameters a_0 and a_1 for the model $a_0 t + \frac{1}{2} a_1^2 t^2$ used to fit the ratio $C_{D\bar{D}\to D\bar{D},\,\mathrm{box}}/C_{D\bar{D}\to D\bar{D},\,\mathrm{direct}}$ versus the twist angle $\theta[1/L]$.

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